Questions, Answers and Minimal Erotetic Semantics

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Outline

Types of questions

According to David Harrah, the following labels for question-types are used by most theorists (examples are also taken from Harrah [Har02], pp.1-2):

LABEL	EXAMPLE
whether	Is two even or odd?
yes-no	Is two a prime number?
which	Which even numbers are prime?
what	What is Church's Thesis?
who	Who is Bourbaki?
why	Why does two divide zero?
deliberative	What shall I do now?
disjunctive	How long is your new proof, or do you have a shorter one?
hypothetical	If you had a proof, how long would it be?
conditional	If you now have a proof, how long is it?
given-that	Given that Turing's Conjecture is provable, is Church's Thesis provable?

Types of questions

- The list is by no means exhaustive. One can easily add to it *when*, *where*, *how*, etc.
- Linguists tend to speak about *constituent*, *alternative*, and *polar* questions.
- Many Polish logicians follow Ajdukiewicz in distinguishing *complementation questions* and *decision questions* as the basic types.
- The logic of questions is sometimes called *erotetic logic* (from Greek "erotema", which means "question"). The name was coined by Prior & Prior in their 1955 paper.

History

- In the late fifties/early sixties of the 20th century the conceptual apparatus of modern formal logic begun to be extensively applied in the area of questions and questioning. Gerold Stahl, Tadeusz Kubiński, David Harrah, Nuel D. Belnap, and Lennart Åqvist established the first widely elaborated logical theories of questions.
- For decades research on questions focussed on their representation as well as the answerhood problem.
- The priorities started to change in the 1980's, beginning with Hintikka's *Interrogative Model of Inquiry*. Generally speaking, research on how questions *function* (in inquiry, dialogues, reasoning, issue management, and so forth) gradually overshadowed research upon what questions *are*.
- The interest in questions and questioning is currently growing. In particular, questions became a full-fledged category in dynamic epistemic logic and in belief revision theory. Theories of questions became indispensable constituents of dialogue theories. And, last but not least, research on questions is an important part of the inquisitive semantics programme.
- But so far there is no paradigmatic logical theory of questions.

Questions as troublemakers

- It is surprising, but current theories of questions disagree even with respect to the answer to the question:
- What is a question?

- The term "question" can be understood:
  1. syntactically: as referring to a sentence of a particular kind, that is, to an *interrogative sentence*,
  2. semantically,
  3. pragmatically.
- These concepts differ not only in contents, but also in scopes.
- Some theories ignore the pragmatic level, some other overestimate it. In most theories, however, all the levels are taken into consideration, although with the emphasis put on one of them.

The paraphrase approach

- Some theorists adopt the *paraphrase approach*. The basic idea is: the *meaning* of an interrogative sentence can be adequately characterized by a paraphrase that specifies:
  - the typical use of the sentence, or
  - the relevant illocutionary act performed in uttering the sentence.
- When the paraphrase approach is adopted, it is natural to shape interrogatives by means of expressions of a theory which characterizes the basic elements of the paraphrase.

Imperative-epistemic paraphrases

- The following are paraphrases proposed in Åqvist [Åqv65] and Hintikka [Hin76], respectively:

Let it be the case that I know . . . (1)

Bring it about that I know . . . (2)

- The dots should be filled by an embedded interrogative sentence. So we have e.g.:

Let it be the case that I know where you bought your car.

Bring it about that I know where you bought your car.

An example: Hintikka's account

- An expression of the form:

  Bring it about that I know γ

  (where γ is an embedded interrogative) splits into the imperative operator (i.e. "Bring it about that") and the *desideratum*. The latter describes the epistemic state of affairs the questioner wants the respondent to bring about.
- Desiderata of various questions contain such epistemic expressions as "know whether", "know where", "know who", "know when", etc. The corresponding concepts of knowledge are explicated by Hintikka in terms of the concept of "knowing that".
- In doing this he makes use of his earlier results in epistemic logic, but also introduces some modifications and novelties to them.

Speech acts

- Sometimes the proposed paraphrase directly describes the relevant illocutionary act performed. Thus we have:

$$\textit{I hereby ask you} \ldots \tag{3}$$

or

$$\textit{(Please) tell me truly} \ldots \tag{4}$$

and so on.

- A paraphrase specifies the meaning of an interrogative sentence. Roughly, questions are thus viewed as *speech acts* of a special kind, that is, *interrogative acts*.

An example: Vanderveken's account

- An elementary illocutionary act has two semantic constituents: the *illocutionary force* and the *propositional content*.
- As for an interrogative act, the illocutionary force amounts to a request of the speaker to the hearer.
- What a hearer is supposed to do is specified by the propositional content of an interrogative act. Roughly, he or she is requested to perform a future speech act which conveys information of the required kind, that is, provides a correct answer to the question under consideration.
- For example, when we have: *Does John like Mary?*, its analysis amounts, informally, to: *I request that you assert that John likes Mary or deny that John likes Mary*
- Similarly (*) *Who likes Mary?*, roughly, amounts to: (**) *You tell me who likes Mary*.
- But one may doubt whether the meaning (**) can be characterized without referring, in some way or another, to the meaning of (*).

The independent meaning thesis

- According to the *independent meaning thesis*, the meaning/ semantic content of an interrogative sentence is specific, that is, it cannot be adequately characterized in terms of semantics of expressions that belong to other categories.
- On the syntactic level interrogatives (of formal languages) are also specific: their form differs from these used to formalize declarative sentences, or (epistemic) imperatives, and so forth.

Hamblin's *dictum* and Belnap's Manifesto

"*Knowing what counts as an answer is equivalent to knowing the question.*" Hamblin [Ham58]

"Interrogatives deserve a compositional semantics that is not piggy-backed on the semantical correlates of declaratives. But if truth conditions won't do, what else should a semantic theory for interrogatives draw on? The answer goes back at least to Hamblin (. . .) instead of truth conditions, interrogatives need answerhood conditions (. . .) If you are persuaded that there is enlightenment to be had about a declarative by learning how its truth conditions arise out of the meanings of its constituents and its structure (. . .) you should expect to find corresponding enlightenment in seeing how what counts as an answer to an interrogative arises out of the meanings of its constituents and its structure." (. . .) Belnap [Bel90], pp. 13–14

Subjects and requests: Belnap’s approach

- The basic idea of Belnap’s theory of questions is: a question “presents” a set of “alternatives” together with some suggestions or indications as to what kind of choice or selection among them should be made.
- Questions are abstract entities, roughly: the meanings of interrogatives.
- For brevity, let me concentrate upon the so-called *elementary interrogatives.*
- An elementary interrogative consists of the question mark ‘?’, the *subject*, and the *request*.
  - The function of a subject is to offer the relevant (nominal) alternatives.
  - The role of a request is to characterize the required kind of selection.

Subjects and requests: Belnap's approach

To be more precise, an elementary interrogative falls under the schema:

$$? \ \mathbf{scd} \ (\eta)$$

where:

- η is the *subject*: its role is to determine the relevant alternatives;
- **s** is the *selection-size specification*: its role is to characterize the quantity of alternatives expected to occur in a direct answer (i.e. "exactly *n*", "at least *n*", "at most *m*", "at least *n* but at most *m*", etc.);
- **c** and **d** are the *completeness-claim* and the *distinctness-claim*, respectively. Roughly, **c** specifies "how many" (in the qualitative sense) of the assumed as true alternatives should occur in a direct answer, whereas a (non-empty) **d** acts to the effect that a direct answer should involve semantically distinct alternatives.

Questions as intensions of interrogatives

- The development of Montague Semantics and intensional logics in general has resulted in elaborating intensional theories of questions.
- An intensional approach distinguishes between extensions/denotations and intensions/meanings of expressions, and evaluates them relative to possible worlds. Propositions are most often conceived as sets of possible worlds or characteristic functions of such sets. The ontology is usually rich and involves objects of diverse types, defined recursively out of some basic types.
- Probably the most influential papers on questions written within the intensional paradigm were Hamblin [Ham73] and Karttunen [Kar77]. Generally speaking, in both cases the denotation of an interrogative (in a world) is viewed as a set of propositions, and its intension/meaning is a function from possible worlds to denotations.

Questions as intensions of interrogatives

- The crucial difference, however, lies in the fact that Hamblin regards the relevant propositions as expressing possible answers, while for Karttunen they express true answer(s). Consider:

  (#) *Who likes Mary?*

- According to Karttunen, the denotation of (#) in the actual world is the set of propositions that are true in the world and state of a person that he/she likes Mary. More generally, the denotation of (#) in a world w is:[1]

  $$\lambda p(\exists x(p = \lambda w(\texttt{likes-Mary}(w)(x))) \land p(w))$$

  where $p(w)$ says that proposition p is true in w. On Hamblin's account, the denotation of an interrogative in a world need not consist of propositions which are true in the world.

[1] For brevity, I do not use the original PTG notation; moreover, "`likes-Mary`" is conceived as a one-place predicate.

Questions as intensions of interrogatives

- The so-called categorial (or functional) approach conceives meanings of interrogatives as functions from categorial answers to propositions.
- To be more precise, the meaning of an interrogative is a function that assigns propositions to the meanings of categorial answers, where categorial answers are non-sentential utterances/expressions (noun phrases, etc.).
- For example, the meaning of *Who likes Mary?* can be characterized by:

$$\lambda w(\lambda x(\texttt{likes-Mary}(w)(x)))$$

- When *w* is the actual world, the domain of the relevant function is the set of persons.

Questions as intensions of interrogatives

- The *structured meaning approach* (cf. e.g. Krifka [Kri01]) makes this explicit.
- On the extensional level *Who likes Mary?* is represented by the ordered pair:

$$\langle \lambda x(\texttt{likes-Mary}(x)), \texttt{Persons} \rangle$$

- This can be written concisely as:

$$\lambda x \in \texttt{Persons}(\texttt{likes-Mary}(x))$$

- On the intensional level we get:

$$\lambda w(\lambda x \in \texttt{Persons}(w)(\texttt{likes-Mary}(w)(x)))$$

Questions as propositional abstracts: Ginzburg

- The old idea, according to which questions are akin to “open propositions”, is conceptualized by Ginzburg[2] by conceiving questions as *propositional abstracts*.
- At the start, one considers a universe which contains among its members a class of entities called *situations* (these are partial, temporally located, actual entities) and a class of entities called *states-of-affairs* (hereafter: SOA’s). In addition, the universe comprises relations as well as possibilities, facts, and outcomes.
- A SOA is a structured object constructed from a relation and an assignment of entities to the argument roles of the relation. (Relations are not conceived as sets of ordered tuples, but as unstructured atomic individuals.) Here is an example of SOA (the notation is self-explanatory):

$$\langle\langle \texttt{Like,\{likes:John,is liked:Mary\}} \rangle\rangle.$$

[2] Cf. Ginzburg [Gin95]; see also Ginzburg & Sag [GS00].

Questions as propositional abstracts: Ginzburg

- The role of SOAs is to designate properties that situations might possess. SOA's may be *supported* by situations; for instance, each situation in which John likes Mary supports the above SOA.
- *Atomic propositions*, in turn, are defined in terms of situations and SOA's: PROP(s,α) is the proposition that s is a situation of the type designated by SOA α. The set of *propositions* includes atomic propositions and is closed under certain operations of negation, meet and join.
- The next step is to define a certain semantic operation of abstraction; the abstracts, then, enrich the universe. There is no time for going into details. Let me only mention that the relevant concept of abstraction diverges from these known from Montague semantics or Lambda Calculus: it is a *simultaneous* abstraction.

Questions as propositional abstracts: Ginzburg

- Questions are then conceived as *abstracts from propositions*. Thus, for example, we have:

$$\lambda\{\ \}\mathrm{PROP}(s, \langle\langle \texttt{Like;\{likes:John,is liked:Mary\}} \rangle\rangle$$

$$\lambda\{x\}\mathrm{PROP}(s, \langle\langle \texttt{Like;\{likes:}x\texttt{,is liked:Mary\}} \rangle\rangle$$

$$\lambda\{x, y\}\mathrm{PROP}(s, \langle\langle \texttt{Like;\{likes:}x\texttt{,is liked:}y\texttt{\}} \rangle\rangle$$

  for *Does John like Mary?*, *Who likes Mary?* and *Who likes what?*, respectively.

Questions as partitions of the logical space: Groenendijk and Stokhof

- Groenendijk & Stokhof [GS84], [GS97] provide an intensional analysis of questions.
- Generally speaking, on the G&S account a question is a set of propositions (i.e. sets of possible worlds) which constitutes a partition (not necessarily a bipartition) of the logical space of possible worlds.
- As being blocks of a partition, the propositions belonging to a question are mutually exclusive.
- As an illustration, let us consider the case of yes-no questions.
- We assume that interrogatives for yes-no questions have the form $?A$.

Questions as partitions

- Let:

  $$[A]_{\mathcal{M},w}$$

  stand for the *extension* of A in world w of model $\mathcal{M}$, that is, the truth value assigned to A in world w of $\mathcal{M}$.

- The set of worlds:

  $$[A]_{\mathcal{M}} = \{w \in \mathcal{M} : w(A) = \mathbf{1}\}$$

  is the *intension* of A in $\mathcal{M}$.

- The *extension* of interrogative $?A$ in world w of model $\mathcal{M}$, $[?\ A]_{\mathcal{M},w}$, is the set of worlds:

  $$\{w^* \in \mathcal{M} : [A]_{\mathcal{M},w^*} = [A]_{\mathcal{M},w}\}$$

  that is, roughly, the set of all the worlds of $\mathcal{M}$ in which A has the same truth value as in w.

Questions as partitions

- The *intension* of $?A$ in $\mathcal{M}$, $[?A]_{\mathcal{M}}$, is the set of possible extensions of the interrogative in $\mathcal{M}$, i.e. the set:

$$\{[?A]_{\mathcal{M},w} : w \in \mathcal{M}\}$$

- Now *question* expressed by $?A$ in $\mathcal{M}$ is identified with the intension of $?A$ in $\mathcal{M}$.
- A semi-formal example may be helpful. Given the definitions introduced above, we get:

$$[\textit{? Does John like Mary}]_{\mathcal{M}} =$$

$$= \{[\textit{John likes Mary}]_{\mathcal{M}}, \mathcal{M} \setminus [\textit{John likes Mary}]_{\mathcal{M}}\}$$

Questions as partitions

- The intension/question is thus a two-element set of sets of possible worlds. Since a set of possible worlds is (or may be counted as) a proposition, a question is a set of propositions. Moreover, the propositions in the set are mutually exclusive, and they exhaust the logical space consisting of all the possible worlds in $\mathcal{M}$.
- To put it in the form of a slogan: *a yes-no question is a bipartition of the logical space*.
- Other questions are viewed in a similar manner. They are still partitions, but not necessarily bipartitions. Of course, the notions of model, extension and intension are more complicated when wh-interrogatives and quantified formulas are taken into consideration.

Inquisitive Semantics: Groenendijk again

- In view of Inquisitive Semantics[3] (hereafter: *Inq*) the meaning of a sentence has two aspects: *informative content* and *inquisitive content*. Roughly, the former is identified with the information provided, while the latter is the issue raised.
- Being a question is a semantic *property* of a sentence. Questions are not semantic objects.
- The general setting does not require that a language under consideration involves a separate syntactic category of interrogatives.
- Sometimes, however, interrogatives are syntactically defined at the outset, and only they "are" questions.
- In what follows I will concentrate on the basic system of *Inq*, dubbed Inq_B, which does not provide syntactic distinction between interrogatives and declaratives. I will remain at the propositional level.

[3] See e.g. [Gro09], [GR09], [CR11], [Gro11].

Inq_B

- We consider a (non-modal) propositional language over a set of propositional variables $\mathcal{P}$. The following are primitive logical constants of the language: $\bot, \vee, \wedge, \to$. Well-formed formulas (wffs) are defined as usual.
- We supplement the language with a possible-world semantics; a possible world is conceived either as a subset of $\mathcal{P}$ or as a valuation of $\mathcal{P}$.
- Regardless of which of these accounts is adopted, the set $\mathcal{W}$ of suitable worlds for the language is determined. The concept of truth of a wff in a possible world is defined accordingly.
- A *state* is a subset of $\mathcal{W}$. States are thus sets of possible worlds. One can think of such sets as modelling information states.

Inq_B: being a question

- In the general setting of *Inq* the *informative content* of a wff is defined as the union of all states that *support* the wff (see below).
- In the case of Inq_B, however, the informative content of a wff A is just the *truth set*, $|A|$, of A, defined by:

$$|A| = \{w \in \mathcal{W} : w(A) = \mathbf{1}\}$$

- A wff A is *informative* iff $|A| \neq \mathcal{W}$, and is *non-informative* otherwise.
- The property of *being a question* is just non-informativeness.
- It follows that each question is a classical tautology.
- Some questions are inquisitive, but some are not.

Inq_B: support

- In order to define the concept of inquisitive content (and of informative content in the general setting) we need the concept of *support*. Support is a relation between a state and a wff. The inscription $s \succ A$ reads: state s supports wff A. In the case of Inq_B support, $\succ$, is defined as follows:
- $s \succ \mathrm{p}$ iff $w(\mathrm{p}) = \mathbf{1}$ for any $w \in s$, and any propositional variable $\mathrm{p} \in \mathcal{P}$,
- $s \succ \bot$ iff $s = \emptyset$,
- $s \succ (A \wedge B)$ iff $s \succ A$ and $s \succ B$,
- $s \succ (A \vee B)$ iff $s \succ A$ or $s \succ B$,
- $s \succ (A \rightarrow B)$ iff for each $t \subseteq s$: if $t \succ A$ then $t \succ B$.
- Inquisitive negation is introduced by the following definition:

$$\neg A =_{df} (A \rightarrow \bot)$$

Thus we get:

  - $s \succ \neg A$ iff for each $t \subseteq s$ such that $t \neq \emptyset : t \not\succ A$.

Inq_B: informative content and inquisitive content

- Let me stress that support in a state does not amount to truth in each world of the state: the clauses for disjunction and implication (and also negation) are more demanding.
- The *inquisitive content* of a wff A, $[A]$, is defined by:

$$[A] = \{s \subseteq info(A) : s \succ A\}$$

- When the concept of support is defined (in some way or another), the *informative content* of A, $info(A)$, is defined by:

$$info(A) = \bigcup \{s \subseteq \mathcal{W} : s \succ A\}$$

- As for Inq_B, we have:

$$info(A) = |A|$$

- Thus in the case of Inq_B inquisitive content amounts to:

$$[A] = \{s \subseteq |A| : s \succ A\}$$

Inq_B: inquisitiveness

- In the general setting a wff is called inquisitive just in case its informative content does not belong to its inquisitive content. To be more precise, a wff A is *inquisitive* iff

$$info(A) \not\models A$$

- For Inq_B, however, this reduces to:

$$|A| \notin [A]$$

  Wffs which are not inquisitive are called *assertions*.

- The underlying intuitions are: the informative content of a wff being an assertion is sufficient to settle the issue the wff raises. On the other hand, if a wff is inquisitive, something more than the information the wff provides is needed to settle the issue it raises.

- Both $p \vee r$ and $p \vee \neg p$ are inquisitive. But $p \vee \neg p$, unlike $p \vee r$, is also non-informative. Hence we are justified in saying that $p \vee \neg p$ is an inquisitive *question*.

Inq_B: interrogative operator

- The interrogative operator ? is introduced by the following definition:

$$?A =_{df} A \vee \neg A$$

- One should not confuse the above definition with a syntactic characterization of questions/interrogatives. Also, it is not claimed that each question is a yes-no question. Here are examples of (inquisitive!) questions that do not fall under the schema $A \vee \neg A$:
  - $p \rightarrow ?r$
  - $?p \vee ?r$
- Observe that both have constituents which are questions themselves.

The answer vs. a possible answer

- Most theorists pay at least as much attention to the answers to questions as to the questions themselves. The analysed answers are usually *possible answers*.
- The relevant erotetic concept of possibility has never been made precise.
- However, it is clear that "possible" does not yield "true": some possible answers are true and some are not. Thus, in most theories, the phrase "an answer to a question" does not amount to "the true answer to a question".
- Moreover, "possible" presupposes neither "being known" nor "being believed in", although, of course, some possible answers happen to be carriers of items of knowledge or belief.
- Finally, and most importantly, these answers are supposed to be expressions which are somehow "about" the corresponding questions, addressing their topics/requests. But the relevant concept of aboutness needs an explication either.

The answer vs. a possible answer

- By the way, it is useful to distinguish between *responses* and *answers*. A response is a (possible) reaction to an interrogative speech act. A response need not be an answer.
- It is not the case that all possible answers are equally interesting to erotetic logicians or linguists addressing the subject matter. Usually a certain kind of possible answers occupies a distinguished position. They are called *direct*, or *conclusive*, or *proper*, or *sufficient*, or *exhaustive*, or *complete*, or *congruent*, etc.
- These labels are not synonymous and, as a matter of fact, the corresponding concepts are defined differently in different theories. However, the proposals share an underlying idea: the distinguished category comprises *potentially resolving answers*.
- Since the terminology is still diverse, we need a cover term. The expression *principal possible answers* (ppa's for short) seems appropriate here, so we will be using it as the cover term.

The answer vs. a possible answer

- Depending on a theory, ppa's are either syntactic or semantic entities.
- Moreover, it is not commonly accepted that all ppa's are full (declarative) sentences or are expressed by such sentences. Sometimes certain subsentential expressions or their semantic counterparts are taken into account as ppa's; more often, however, these "short answers" are regarded as coded ppa's.
- Yet when it comes to applications, one needs *nominal ppa's*, that is, formulas which either express the relevant ppa's, if these are understood semantically, or stand for them if ppa's are viewed syntactically.
- The ways in which different theories assign ppa's to questions/ interrogatives diverge. Sometimes this is done purely syntactically; in most cases stronger tools are (also) used.
- Besides ppa's, other categories of (possible) answers are characterized as well. Sometimes they are defined in terms of, int. al., ppa's. Sometimes, however, a more general concept of answer is introduced first, and ppa's are just very special cases.
- The "non-ppa" answers are labelled with adjectives like "partial", "incomplete", "indirect", "corrective", "eliminative", etc.

NLQ's

- Theories of questions, both logical and linguistic, aim at giving an account of natural-language questions (hereafter: NLQ's).
- Interrogatives and/or questions considered within a theory are objects supposed to represent the corresponding NLQ's.
- Similarly, ppa's to questions/interrogatives that represent NLQ's should represent ppa's to the corresponding NLQ's.
- However, the concept of principal possible answer to a NLQ is vague. The underlying intuitions are expressed by using, among others, pragmatic terms.

NLQ's

- A **direct answer** "*gives exactly what the question calls for. (...) The label 'direct' (...) connotes both logical sufficiency and immediacy.*" (Harrah [Har02], p. 1);
- **direct answers** "*are directly and precisely responsive to the question, giving neither more nor less information than what is called for.*" (Belnap [Bel69], p. 124);
- **direct answers** are "*these sentences which everybody who understands the question ought to be able to recognize as the simplest, most natural, admissible answers to the question*" (Kubiński [Kub80], p.12);
- a potential conclusive answer is "*an answer which would satisfy the questioner if it were true and if he were in a position to trust the answer. By a **conclusive answer**, I mean a reply which does not require further backing to satisfy the questioner.* (Hintikka [Hin78], p. 287);
- ppa's form the *class of responses that a querier would consider optimal* (Ginzburg [Gin95], p. 461).

NLQ's

- On the other hand, NLQ's permit multiple readings. Or, to put it differently, in many cases contextual and/or pragmatic factors co-determine what is "directly and precisely responsive to the question, giving neither more nor less information than what is called for", or what is a just-sufficient (i.e. immediate and sufficient) possible answer, etc.
- So what we in fact get is:

(♠) *An interrogative/question of a formal language Q represents a NLQ* Q^* CONSTRUED IN SUCH A WAY *that possible answers to* Q^* *having the desired semantic and/or pragmatic properties are represented by ppa's to Q*.

Two options for a logician

- A logician interested in questions and questioning most often starts with a formal (or formalized) language which initially does not contain direct counterparts of questions.
- Generally speaking, there are two ways of incorporating questions into a formal language (the names are telling):
  1. the “define within” approach,
  2. the “enrich with” approach.
- Regardless of which approach is adopted, one ends with a class of *erotetic formulas* or *e-formulas* for short.

E-formulas

- E-formulas of a formal language can be identified with *questions* of the language.
- This does not presuppose that e-formulas/questions of formal languages are *defined* in purely syntactic terms.
- They can be characterized in semantic terms, as the well-formed formulas that “correspond” to questions semantically construed or have the semantic property of being a question.
- In general, the sets of e-formulas and declarative well-formed formulas (*d-wffs* for short) need not be disjoint. Usually they overlap when the “define within” approach is adopted, and are disjoint otherwise.
- From now on I will be using the term “question” as synonymous to “e-formula”. But please remember: we are considering (unless otherwise stated) questions/e-formulas of formal languages.

Minimal Erotetic Semantics: the idea

- Minimal Erotetic Semantics (MiES for short) enables an introduction of some important semantic notions pertaining to e-formulas/questions regardless of whether – and if yes, how – the semantics of questions themselves has been previously elaborated.
- In defining them, one makes use of the existence of an assignment of nominal ppa's to questions, and of semantic concepts pertaining to the (sentential) ppa's.
- MiES combines some ideas present in Belnap's erotetic semantics (cf. [BS76]) with certain insights to be found in Shoesmith & Smiley [SS78].
- Of course, MiES also goes beyond them.

MiES: a precondition

- The precondition of applicability of MiES is the following: we deal with a formal language in which declarative well-formed formulas (d-wffs) and e-formulas/questions occur.
- If $\mathcal{L}$ is such language, then

$$\mathit{Form}_{\mathcal{L}} = \mathcal{D}_{\mathcal{L}} \cup \mathcal{E}_{\mathcal{L}}$$

  where $\mathcal{D}_{\mathcal{L}}$ is the set of d-wffs of $\mathcal{L}$ and $\mathcal{E}_{\mathcal{L}}$ is the set of e-formulas of $\mathcal{L}$.
- In the simplest case, $\mathcal{D}_{\mathcal{L}}$ is the set of wffs of (the language of) a logic ℓ.
- Sometimes, however, the construction is more complex.

Partitions of a language

- Languages of the analysed kind are diverse.
- For the sake of uniformity, in the general considerations the concept of a partition of a language is used.
- I follow here the idea of Shoesmith & Smiley [SS78], adjusting it a little bit for my purposes.

Partitions of a language

- Let $\mathcal{L}$ be a language of the considered kind. Thus

$$\mathit{Form}_{\mathcal{L}} = \mathcal{D}_{\mathcal{L}} \cup \mathcal{E}_{\mathcal{L}}$$

- (PARTITION OF THE SET OF D-WFFS) By a *partition of* $\mathcal{D}_{\mathcal{L}}$ we mean an ordered pair:

$$\mathsf{P} = \langle \mathsf{T}_{\mathsf{P}}, \mathsf{U}_{\mathsf{P}} \rangle$$

  where:

  - $\mathsf{T}_{\mathsf{P}} \cap \mathsf{U}_{\mathsf{P}} = \emptyset$, and
  - $\mathsf{T}_{\mathsf{P}} \cup \mathsf{U}_{\mathsf{P}} = \mathcal{D}_{\mathcal{L}}$.

- (PARTITION OF A LANGUAGE) A *partition of* $\mathcal{L}$ is simply a partition of $\mathcal{D}_{\mathcal{L}}$.

Comments

- Note that I have used the term “partition” as pertaining to the set of d-wffs only. What is “partitioned” is neither the “logical space” nor the set of e-formulas.
- Usually the sets of questions and d-wffs are supposed to be disjoint. If this is so, then a question is neither in T_P nor in U_P, for any partition P. In general, MiES does not presuppose that questions are true or false.
- There exist strange partitions, for example:

$$\langle \mathcal{D}_{\mathcal{L}}, \emptyset \rangle$$

$$\langle \{A\}, \mathcal{D}_{\mathcal{L}} \setminus \{A\} \rangle$$

- We should distinguish a class of *admissible partitions*, being a non-empty subclass of the class of all partitions of the language.

Admissible partitions and entailment

- This step allows us to define a series of semantic concepts needed, in particular the concept of entailment.
- Let X stand for a set of d-wffs of a language $\mathcal{L}$ of the considered kind, and let A be a d-wff of $\mathcal{L}$. *Entailment in* $\mathcal{L}$, symbolized by $\models_{\mathcal{L}}$, is defined as follows ($\subset$ is the sign of inclusion):

Definition (*Entailment*)

$X \models_{\mathcal{L}} A$ iff there is no admissible partition $\mathsf{P} = \langle \mathsf{T}_{\mathsf{P}}, \mathsf{U}_{\mathsf{P}} \rangle$ of $\mathcal{L}$ such that $X \subset \mathsf{T}_{\mathsf{P}}$ and $A \in \mathsf{U}_{\mathsf{P}}$.

- What remains to be done is to characterize the class of admissible partitions.
- There are (at least) three methods of doing it: the direct one, the indirect one, and the minimalistic one.

The indirect method

- For the reasons of time I will concentrate upon the indirect method.
- Generally speaking, the indirect method applies a full-fledged semantics of the declarative part of a language as the basis.
- It is assumed that the declarative part is the language of a given logic.

Language $\mathcal{L}^{+}_{CPC}$

- We enrich a language $\mathcal{L}_{CPC}$ of Classical Propositional Calculus (CPC) with e-wffs. The new language is dubbed $\mathcal{L}^{+}_{CPC}$.
- We add the following signs to the vocabulary of $\mathcal{L}_{CPC}$: ?, {, }, and the comma.

$$Form_{\mathcal{L}^{+}_{CPC}} = \mathcal{D}_{\mathcal{L}_{CPC}} \cup \mathcal{E}_{\mathcal{L}^{+}_{CPC}}$$

- $\mathcal{E}_{\mathcal{L}^{+}_{CPC}}$ comprises questions of the form:

$$?\,\{A_1,\ \ldots, A_n\}$$

  where $n > 1$ and $A_1,\ \ldots, A_n$ are nonequiform, that is, pairwise syntactically distinct CPC-wffs.
- If $?\,\{A_1,\ \ldots, A_n\}$ is a question, then each of the d-wffs $A_1,\ \ldots, A_n$ is a *principal possible answer* (ppa) to the question, and these are the only ppa's.

Some comments

- Note that any question of $\mathcal{L}^{+}_{CPC}$ has at least two ppa's.
- Observe that the set of ppa's to a question of $\mathcal{L}^{+}_{CPC}$ is always finite.
- Let me emphasize that questions of $\mathcal{L}^{+}_{CPC}$ are not sets of d-wffs, but expressions of a strictly defined form. In particular, $?\{p, q\} \neq ?\{q, p\}$.

Remarks on questions of $\mathcal{L}^{+}_{CPC}$

- Any question of the form $?\,\{A_1, \ldots, A_n\}$ can be read:

  $$\textit{Is it the case that } A_1,\ \textit{or } \ldots, \textit{or is it the case that } A_n?$$

- However, sometimes a different reading can be recommended.
- Let:

  $$?\,A =_{df} ?\,\{A, \neg A\}$$

- $?\,A$ is thus a *simple yes-no question*. One can read it as: *Is it the case that A?*
- Let:

  $$?\pm \mid A, B \mid =_{df} ?\,\{A \wedge B, A \wedge \neg B, \neg A \wedge B, \neg A \wedge \neg B\}$$

- $?\pm \mid A, B \mid$ can be read: *Is it the case that A and is it the case that B?*
- The schema $?\,\{A_1, \ldots, A_n\}$ is general enough to capture most (if not all) of propositional questions studied in the literature.

Admissible partitions of $\mathcal{L}^+_{CPC}$

- Recall that the set $\mathcal{D}_{\mathcal{L}^+_{CPC}}$ of d-wffs of $\mathcal{L}^+_{CPC}$ is the set of CPC-wffs.
- A *CPC-valuation* is a function $v : \mathcal{D}_{\mathcal{L}^+_{CPC}} \mapsto \{\mathbf{1}, \mathbf{0}\}$ defined in the usual manner.

Definition (*Admissible partitions of $\mathcal{L}^+_{CPC}$*)

A partition $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ of $\mathcal{L}^+_{CPC}$ is *admissible* iff for some CPC-valuation v:

- $\mathsf{T}_\mathsf{P} = \{A \in \mathcal{D}_{\mathcal{L}^+_{CPC}} : v(A) = \mathbf{1}\}$, and
- $\mathsf{U}_\mathsf{P} = \{B \in \mathcal{D}_{\mathcal{L}^+_{CPC}} : v(B) = \mathbf{0}\}$.

- Thus the set of "truths" of an admissible partition equals the set of d-wffs which are true under the corresponding CPC-valuation.

Entailment in $\mathcal{L}^{+}_{CPC}$

- Observe that all the usual semantic properties are retained, but now they can be rephrased in terms of admissible partitions. For example, there is no admissible partition P of $\mathcal{L}^{+}_{CPC}$ such that both A and $\neg A$ belong to $\mathrm{T_P}$. Moreover, $A \wedge B$ belongs to $\mathrm{T_P}$ if, and only if A is in $\mathrm{T_P}$ and B is in $\mathrm{T_P}$, and analogous classical clauses hold for other connectives.
- Note finally that entailment in $\mathcal{L}^{+}_{CPC}$ reduces to CPC-entailment. The following is true:

Corollary

$X \models_{\mathcal{L}^{+}_{CPC}} A$ iff there is no CPC-valuation v such that $v(B) = \mathbf{1}$ for all $B \in X$, and $v(A) = \mathbf{0}$.

Modal propositional languages with questions

- Modal propositional languages with questions are constructed similarly as the language $\mathcal{L}^{+}_{CPC}$. The difference lies in taking the language of a modal propositional logic ℓ, $\mathcal{L}_{\ell}$, as the point of departure.
- We enrich the vocabulary with the signs: ?, {, }, and the comma.
- The d-wffs of $\mathcal{L}^{+}_{\ell}$ are the wffs of $\mathcal{L}_{\ell}$.
- Questions of $\mathcal{L}^{+}_{\ell}$ are defined analogously as in the case of $\mathcal{L}^{+}_{CPC}$. Similarly for their ppa's.

Admissible partitions of $\mathcal{L}^{+}_{\mathbf{S4}}$

- As an illustration let us consider the case of **S4**.
- At the first step we make use of the standard relational semantics of **S4**.
- A **S4**-model is an ordered triple:

$$\langle W, R, V \rangle$$

where $W \neq \emptyset$, $R \subset W \times W$ is both reflexive and transitive in W, and $V : \mathcal{D}_{\mathcal{L}_{\mathbf{S4}}} \times W \mapsto \{\mathbf{1}, \mathbf{0}\}$ satisfies the usual conditions.

- Admissible partitions of $\mathcal{L}^{+}_{\mathbf{S4}}$ are defined as follows:

Definition (*Admissible partitions of* $\mathcal{L}^{+}_{\mathbf{S4}}$)

A partition $\mathsf{P} = \langle \mathsf{T}_{\mathsf{P}}, \mathsf{U}_{\mathsf{P}} \rangle$ of $\mathcal{L}^{+}_{\mathbf{S4}}$ is *admissible* iff for some **S4**-model $\langle W, R, V \rangle$ and for some $w \in W$:

- $\mathsf{T}_{\mathsf{P}} = \{A \in \mathcal{D}_{\mathcal{L}^{+}_{\mathbf{S4}}} : V(A, w) = \mathbf{1}\}$ and
- $\mathsf{U}_{\mathsf{P}} = \{B \in \mathcal{D}_{\mathcal{L}^{+}_{\mathbf{S4}}} : V(B, w) = \mathbf{0}\}$.

Admissible partitions of $\mathcal{L}^{+}_{\mathbf{S4}}$

- Thus the set of "truths" of an admissible partition consists of all the d-wffs which are true in the corresponding world of a given model.
- Note that the theses of **S4** are always included in T_{P}, for any admissible partition P of the language. This is how it should be.
- Moreover, the following holds:

Corollary

*$X \models_{\mathcal{L}^{+}_{\mathbf{S4}}} A$ iff there is no **S4**-model $\langle W, R, V \rangle$ such that for some $w \in W$, $V(C, w) = \mathbf{1}$ for each $C \in X$, and $V(A, w) = \mathbf{0}$.*

Thus entailment in $\mathcal{L}^{+}_{\mathbf{S4}}$ reduces to the so-called local entailment in **S4**.

$\mathcal{L}^{+}_{mFOL}$: A first-order language enriched with questions

- Now let us take the language $\mathcal{L}_{mFOL}$ of Monadic Predicate Calculus with Identity as the starting point. For simplicity, we assume that the vocabulary of $\mathcal{L}_{mFOL}$ contains an infinite list of individual constants, but does not contain function symbols. *Well-formed formulas* (wffs) of $\mathcal{L}_{mFOL}$ are defined in the standard way. A *sentential function* is a wff in which a free variable occurs; otherwise a wff is a *sentence*.
- We construct a second language, $\mathcal{L}^{+}_{mFOL}$, which has a declarative part and an erotetic part.
- We enrich the vocabulary with the signs: ?, {, }, **S**, **U**, and the comma.

$$Form_{\mathcal{L}^{+}_{mFOL}} = \mathcal{D}_{\mathcal{L}_{mFOL}} \cup \mathcal{E}_{\mathcal{L}^{+}_{mFOL}}$$

Questions of $\mathcal{L}^{+}_{mFOL}$

- The set of questions, $\mathcal{E}_{\mathcal{L}^{+}_{mFOL}}$, comprises:
  - *whether-questions*. These are of the form:

    $$?\,\{A_1,\ \ldots,A_n\}$$

    where $n > 1$ and $A_1,\ \ldots,A_n$ are nonequiform (i.e. pairwise syntactically distinct) sentences of $\mathcal{L}_{mFOL}$.
  - *existential wh-questions* of the form:

    $$?\,\mathbf{S}(Ax)$$

    where x stands for an individual variable and Ax is a sentential function of $\mathcal{L}_{mFOL}$ which has x as the only free variable.
  - *general wh-questions* falling under the schema:

    $$?\,\mathbf{U}(Ax)$$

    where Ax is a sentential function of $\mathcal{L}_{mFOL}$ which has x as the only free variable.

Ppa's

- By a *principal possible answer* (ppa) to $?\,\mathbf{S}(Ax)$ we mean a sentence of $\mathcal{L}_{mFOL}$ which is an instantiation (by an individual constant) of the sentential function Ax. Thus the ppa's are sentences of the form $A(x/c)$, where c is an individual constant.
- A question of the form $?\,\mathbf{S}(Ax)$ can be read:

Which x is such that Ax?

- Ppa's to a general which-question $?\,\mathbf{U}(Ax)$ are sentences of $\mathcal{L}_{mFOL}$ having the form:

$$A(x/c_1) \wedge \ldots \wedge A(x/c_n) \wedge \forall x(Ax \rightarrow x = c_1 \vee \ldots \vee x = c_n) \quad (5)$$

where $n \geq 1$ and $c_1, \ldots, c_n$ stand for distinct individual constants.

- A question $?\,\mathbf{U}(Ax)$ can read:

What are all of the x's such that Ax?

Remarks

- The symbols **S** and **U** belong to the vocabulary of the object-level language $\mathcal{L}^{+}_{mFOL}$. However, we can introduce them to the metalanguage as well (but with different meanings). We can assume that on the metalanguage level $\mathbf{S}(Ax)$ designates the set of all the sentences of the form $A(x/c)$, whereas $\mathbf{U}(Ax)$ designates the set of all the sentences of the form (5). Now we are justified in saying that each question of $\mathcal{L}^{+}_{mFOL}$ consists of the sign ? followed by an (object-level language) expression which is equiform to a metalanguage expression that designates the set of ppa's to the question.
- This, however is not tantamount to a *reduction* of questions to sets of d-wffs. Questions are still linguistic expressions of a strictly defined form.
- What we gain, however, is transparency: it is easy to say what counts as a ppa to a question.
- Nothing prevents us from taking a richer first-order language (or a higher-order language) as the point of departure, and from introducing other categories of wh-questions according to the above pattern. For possible developments, see Wiśniewski [Wiś95], Chapter 3.

Admissible partitions of $\mathcal{L}^{+}_{mFOL}$

- By a *model* of $\mathcal{L}^{+}_{mFOL}$ we mean an ordered pair $\langle M, f \rangle$ such that M is a non-empty set, and f is a function which assigns an element of M to each individual constant of $\mathcal{L}^{+}_{mFOL}$, and a subset of M to each unary predicate of $\mathcal{L}^{+}_{mFOL}$. The concepts of valuation, satisfaction and truth are defined accordingly.
- In the second step we define the class of normal models. Roughly, a model $\mathcal{M} = \langle M, f \rangle$ of $\mathcal{L}^{+}_{mFOL}$ is *normal* just in case all the elements of M are named by individual constants of $\mathcal{L}^{+}_{mFOL}$.
- To be more precise, by a normal model of $\mathcal{L}^{+}_{mFOL}$ we mean a model $\mathcal{M} = \langle M, f \rangle$ of the language such that for each $y \in M$ we have: $y = f(c_i)$ for some individual constant c_i of $\mathcal{L}^{+}_{mFOL}$.
- As long as normal models are concerned, the truth of an existential generalization $\exists x A x$ warrants the existence of a true direct answer to the corresponding existential which-question $?\, \mathbf{S}(Ax)$. This is why we have distinguished these models here.

Admissible partitions of $\mathcal{L}^{+}_{mFOL}$

- Now we are ready to introduce:

Definition (*Admissible partitions of* $\mathcal{L}^{+}_{mFOL}$)

A partition $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ of $\mathcal{L}^{+}_{mFOL}$ is *admissible* iff for some normal model $\mathcal{M} = \langle M, f \rangle$ of $\mathcal{L}^{+}_{mFOL}$:

- $\mathsf{T}_\mathsf{P} = \{A \in \mathcal{D}_{\mathcal{L}^{+}_{mFOL}} : \mathcal{M} \models A\}$, and
- $\mathsf{U}_\mathsf{P} = \{B \in \mathcal{D}_{\mathcal{L}^{+}_{mFOL}} : \mathcal{M} \not\models B\}$.

- Hence the set of “truths” of an admissible partition equals the set of d-wffs which are true in the corresponding normal model.

Remarks

- The reference to normal models is the key feature of the above construction. We have distinguished them for “erotetic” reasons. However, when we deal with a first-order (or a higher-order) language enriched with questions, normal models can be distinguished for many reasons and in different manners.
- For example, one can define them as models of a theory expressed in the declarative part of the language[4], or as models which make true some definition(s). It is also permitted to consider all models as normal.
- Each decision determines the corresponding entailment relation.

[4] That is, models of the language in which all the theorems are true.

Multiple-conclusion entailment

- We need the concept of multiple-conclusion entailment (cf. Shoesmith & Smiley [SS78]). Multiple-conclusion entailment is a semantic relation between sets of d-wffs. The entailed set is conceived as, intuitively speaking, setting out the field within which the truth must lie if the premises are all true.
- The relation $||\!\models_{\mathcal{L}}$ of *multiple-conclusion entailment in* $\mathcal{L}$ (i.e. a language of the considered kind) is defined as follows:

Definition (*Multiple-conclusion entailment*)

$X \, ||\!\models_{\mathcal{L}} \, Y$ iff there is no admissible partition $\mathsf{P} = \langle \mathsf{T}_{\mathsf{P}}, \mathsf{U}_{\mathsf{P}} \rangle$ of $\mathcal{L}$ such that $X \subset \mathsf{T}_{\mathsf{P}}$ and $Y \subset \mathsf{U}_{\mathsf{P}}$.

Comments

- Hence:

Corollary

$X \Vert\!\models_{\mathcal{L}} Y$ iff the following condition holds:

- *for each admissible partition $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ of $\mathcal{L}$: if $X \subset \mathsf{T}_\mathsf{P}$, then $Y \cap \mathsf{T}_\mathsf{P} \neq \emptyset$.*

  - Mc-entailment is a non-trivial generalization of entailment. It happens that a set of d-wffs X mc-entails a set of d-wffs Y, but no element of Y is entailed by X. For example, we have:

$$\{p,\ p \to q \vee r\} \Vert\!\models_{\mathcal{L}^+_{CPC}} \{q, r\}$$

$$\{p,\ p \to q \vee r\} \not\models_{\mathcal{L}^+_{CPC}} q$$

$$\{p,\ p \to q \vee r\} \not\models_{\mathcal{L}^+_{CPC}} r$$

Eliminating

Although negation connective occurs in any of the exemplary languages considered so far, this is not a general rule. Hence we need a technical (semantic) concept of elimination.

Definition (*Elimination*)

Let C be a d-wff of $\mathcal{L}$, and let X, Y be a sets of d-wffs of $\mathcal{L}$.

(1) X eliminates Y iff for each admissible partition $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ of $\mathcal{L}$:

- if $X \subset \mathsf{T}_\mathsf{P}$, then $Y \subset \mathsf{U}_\mathsf{P}$

(2) X eliminates C iff X eliminates $\{C\}$,

(3) C eliminates Y iff $\{C\}$ eliminates Y.

Eliminating

- Clearly eliminating is weaker than contradicting: it happens that *B* eliminates *C*, and both *B* and *C* are false in an admissible partition.
- For example, take language $\mathcal{L}^{+}_{CPC}$ and the following d-wffs of the language:

$$p \wedge q \tag{6}$$

$$p \wedge \neg q \tag{7}$$

- (6) eliminates (7), but (6) and (7) are false in any (admissible) partition of $\mathcal{L}^{+}_{CPC}$ in which p is false.

Assumptions

- From now on, I will be considering (unless otherwise stated) an arbitrary but fixed formal language $\mathcal{L}$ of the analyzed kind; by d-wffs and questions I will mean d-wffs and e-wffs of the language, respectively.
- Language $\mathcal{L}$ is supposed to satisfy the following conditions:
  - (*a*) it has questions and d-wffs among well-formed expressions,
  - (*b*) for any question of the language, the set of ppa's to the question is defined,
  - (*c*) ppa's are d-wffs, and
  - (*d*) the class of admissible partitions of $\mathcal{L}$ is defined.
- For the sake of brevity, in what follows the specifications "of $\mathcal{L}$" and "in $\mathcal{L}$" will be omitted. Similarly, I write $\models$ instead of $\models_{\mathcal{L}}$, and $||\!\!\models$ instead of $||\!\!\models_{\mathcal{L}}$.
- $\mathbf{d}Q$ will stand for the set of all the ppa's to question Q.

Soundness of a question

- MiES does not presuppose that questions are true or false. Instead, the concept of soundness of a question is used.
- The underlying intuition is: a question Q is sound if, and only if at least one principal possible answer (ppa) to Q is true.[5]
- So, for example:

$$\textit{Who is the only author of Principia Mathematica?} \tag{8}$$

  is not sound, whereas:

$$\textit{Who were the authors of Principia Mathematica?} \tag{9}$$

  is sound.
- Of course, when a formal language is concerned, the concept of soundness needs a relativization.

[5] The basic idea of this definition was suggested by Bromberger [Bro92], p.146.

Soundness of a question

Definition (*Soundness*)

A question Q is sound in a partition P iff $\mathbf{d}Q \cap \mathsf{T}_{\mathsf{P}} \neq \emptyset$.

- Thus a question is sound in a partition if at least one principal possible answer to the question is true in the partition.

Safety

- It can happen that a question is sound in one partition and is not sound in some other(s). If, however, a question is sound in each admissible partition of a language, we call it a safe question.[6]
- More formally:

Definition (*Safety*)

A question Q is safe iff $\mathbf{d}Q \cap \mathsf{T}_{\mathsf{P}} \neq \emptyset$ for each admissible partition P.

- Observe that a question can be safe although no ppa to it is valid, that is, true in each admissible partition of a language.
- For example, the following questions of $\mathcal{L}^{+}_{CPC}$ are safe, but no ppa to them is valid:

$$?\, p \tag{10}$$

$$?\pm \mid p, q \mid \tag{11}$$

[6] This idea, in turn, comes from Belnap. See e.g. Belnap & Steel [BS76], p.130.

Riskiness

- A question which is not safe is called risky.[7]
- To be more precise:

Definition (*Riskiness*)

A question Q is risky iff $\mathbf{d}Q \cap \mathsf{T}_{\mathsf{P}} = \emptyset$ for some admissible partition P.

- Thus a risky question is a question which has no true principal possible answer in at least one admissible partition of the language.
- Here are simple examples of risky questions of the language $\mathcal{L}^{+}_{CPC}$:

$$?\, \{p, q\} \tag{12}$$

$$?\, \{p \wedge q, p \wedge \neg q\} \tag{13}$$

[7] Again, this is Belnap's term.

A comment

- Safety and riskiness correspond to non-informativeness resp. informativeness in the sense of Groenendijk & Stokhof [GS97]. In view of their analysis, each question (semantically construed) is non-informative.
- In general, when questions are conceptualized semantically as partitions of the logical space, there is no room for risky questions.
- Let me add that non-informativeness is the definitory property of questions in inquisitive semantics.

Presuppositions of a question

- Following Belnap's proposal[8], we define the concept of a presupposition of a question by:

Definition (*Presupposition*)

A d-wff B is a presupposition of a question Q iff $A \models B$ for each $A \in \mathbf{d}Q$.

- Thus a presupposition of a question is a d-wff which is entailed by each ppa to the question.
- Observe that each presupposition of a question which is sound (in an admissible partition) is true (in the partition), and that a question which has a false presupposition cannot be sound (again, with respect to a given admissible partition).

[8] See e.g. Belnap & Steel [BS76], pp.119-120. Belnap expresses this definition differently, however.

Examples

- For instance, the following:
$$p \vee q \tag{14}$$
is an example of a presupposition of question $?\{p, q\}$.
- Here are examples of presuppositions of question $?\{p \wedge q, p \wedge \neg q\}$:

$$(p \wedge q) \vee (p \wedge \neg q) \tag{15}$$

$$p \wedge (q \vee \neg q) \tag{16}$$

$$q \vee \neg q \tag{17}$$

$$p \tag{18}$$

Prospective presuppositions

- The truth of a presupposition of a question need not warrant the soundness of the question. For instance, r is a presupposition of the following question of $\mathcal{L}^{+}_{CPC}$:

$$? \{p \wedge r, q \wedge r\} \tag{19}$$

but the question is not sound in an admissible partition in which r is true and both p and q are not true.

- A presupposition whose truth warrants soundness of the question is called a prospective presupposition. More precisely:

Definition (*Prospective presupposition*)

A presupposition B of question Q is prospective iff $B \mid\!\models \mathbf{d}Q$.

Prospective presuppositions

- A prospective presupposition is thus a presupposition whose truth (in an admissible partition) is both necessary and sufficient for the soundness of the question (in the partition).
- There is no warranty that each question of a language has prospective presupposition(s)!
- The following d-wff:

$$r \wedge (p \vee q) \tag{20}$$

  is a prospective presupposition of question $?\{p \wedge r, q \wedge r\}$.
- Of course, prospective presuppositions are not unique. The following d-wff is *also* prospective presuppositions of $?\{p \wedge r, q \wedge r\}$:

$$(r \wedge p) \vee (r \wedge q)$$

Prospective presuppositions

- In general, a prospective presupposition of a question of the form:

$$? \{A_1, \ldots, A_n\}$$

(of $\mathcal{L}^{+}_{CPC}$ or $\mathcal{L}^{+}_{mFOL}$) is either a d-wff of the form:

$$A_1 \vee \ldots \vee A_n \tag{21}$$

or a d-wff which is equivalent to it (by "equivalence" we mean here mutual entailment in a language).

Prospective presuppositions

- One can show that the existential generalization:

$$\exists x A x \tag{22}$$

  is a prospective presupposition of the corresponding existential which-question:

$$?\, \mathbf{S}(Ax)$$

  of $\mathcal{L}^{+}_{mFOL}$.

- This, however, is due to the fact that admissible partitions of $\mathcal{L}^{+}_{mFOL}$ are determined by models, in which each individual has a name.
- The remaining prospective presuppositions of $?\, \mathbf{S}(Ax)$ are equivalent with (22).

Prospective presuppositions

- When we have a general which-question:

$$?\,\mathbf{U}(Px)$$

  of $\mathcal{L}^{+}_{mFOL}$ (where P stands for a predicate), the existential generalization $\exists xPx$ is the strongest presupposition of the question, but is still not a prospective presupposition. The reason is simple: one cannot express in $\mathcal{L}_{mFOL}$ that P denotes a finite property.

- This is not to say that each general which-question has no prospective presupposition. A tricky counterexample: a general which-question which has a validity among the ppa's.

Notation

- The set of presuppositions of a question Q will be referred to as **Pres**Q,
- The set of prospective presuppositions of a question Q will be designated by **PPres**Q.

Types of answers. Just-sufficient answers

- So far I have operated with only one category of answers, that is, ppa's. However, the conceptual apparatus of MiES allows us to define further types of answers.
- The following definition introduces a concept that is not superfluous when ppa's are defined syntactically.

Definition (*Just-complete answer*)

A d-wff B is a just-complete answer to a question Q iff $B \notin \mathbf{d}Q$, and for some $A \in \mathbf{d}Q$, both $B \models A$ and $A \models B$ hold.

- Roughly, just-complete answers are equivalent with ppa's, but are not ppa's. By equivalence we mean mutual entailment.
- It is convenient to introduce the following notational convention:

$$[[\mathbf{d}Q]] = \{B\text{: } \textit{for some } A \in \mathbf{d}Q, B \models A \textit{ and } A \models B\}$$

Partial answers

- Partial answers are defined by:

Definition (*Partial answer*)

A d-wff A is a partial answer to a question Q iff $A \notin [[\mathbf{d}Q]]$, but for some non-empty proper subset Y of $\mathbf{d}Q$:

(1) $A \,||\!\models Y$, and

(2) for each $B \in Y$: $B \models A$.

- A partial answer is a d-wff that is neither a ppa nor a just-complete answer, but which is true if, and only if a true principal possible answer belongs to some specified *proper subset* of the set of all the ppa's to the question.
- This condition holds for each admissible partition; of course, different elements of the proper subset Y can be true in different partitions in which the partial answer is true.

Eliminative answers

- Generally speaking, an eliminative answer, if true, eliminates at least one of the possibilities offered by a question. More formally:

Definition (*Eliminative answer*)

A d-wff A is an eliminative answer to a question Q iff

(1) $A \notin [[\mathbf{d}Q]]$, and

(2) $A \in \mathsf{T}_{\mathsf{P}}$ for some admissible partition P, and

(3) there exists $B \in \mathbf{d}Q$ such that A eliminates B.

- When the classical negation[9] occurs in a language, an eliminative answer can also be defined as a consistent d-wff which entails the negation of at least one ppa, but is not equivalent with any ppa.

[9]More generally, a negation for which a counterpart of the “law of contradiction” holds.

Eliminative answers vs. partial answers

- There are eliminative answers which are not partial answers, and there are partial answers which are not eliminative.
- Consider:

$$? \{p, q, r\} \tag{23}$$

  The d-wff:

$$p \vee q$$

  is a partial answer to question (23), but is not an eliminative answer to the question. The d-wff:

$$\neg r$$

  is an eliminative answer to question (23), but is not a partial answer to it.
- So one cannot *identify* partial answers with eliminative answers.

Eliminative answers vs. partial answers

- The categories of partial answers and eliminative answers are not disjoint, however.
- For example, in the case of the following question of $\mathcal{L}^{+}_{CPC}$:

$$? \pm \mid p, q \mid$$

each of the d-wffs p, $\neg p$, q, $\neg q$ is both a partial answer and an eliminative answer to the question.

Corrective answers

- Roughly, a corrective answer is a consistent d-wff which eliminates the whole *set* of ppa's. This intuition is expressed by:

Definition (*Corrective answer*)

A d-wff A is a corrective answer to a question Q iff

(1) $A \in \mathsf{T}_{\mathsf{P}}$ for some admissible partition P, and

(2) A eliminates $\mathbf{d}Q$.

- Thus if a corrective answer is true in an admissible partition, no ppa is true in the partition.

Corrective answers vs. eliminative answers

- Clearly, each corrective answer is an eliminative answer.(As a matter of fact, a “maximal” one: it eliminates all the ppa’s.)
- On the other hand, there are eliminative answers that are not corrective. As an illustration, consider the following question of $\mathcal{L}^{+}_{CPC}$:

$$? \{p \wedge q, p \wedge r\} \tag{24}$$

- The d-wffs $\neg q$ and $\neg r$ are eliminative answers to question (24), but are not corrective answers to the question. Here are examples of corrective answers to question (24):

$$\neg p$$

$$\neg(q \vee r)$$

- One can easily show that the set of partial answers to a question and the set of corrective answers to the question are disjoint.

The applicability issue

- Providing an adequate semantic analysis of NLQ's is a difficult task. It is not by accident that theories of questions are still diverse.
- Minimal Erotetic Semantics is, in a sense, neutral here. If only questions are represented/formalized by e-formulas, some assignment of (nominal and sentential) ppa's to e-formulas is given (regardless of how ppa's are conceptualized in detail, and how the assignment is determined/defined: syntactically, semantically, or both), and the class of admissible partitions of the relevant formal language is determined, the "erotetic" concepts defined within MiES become applicable.
- To be more precise, they are directly applicable to e-formulas with well-defined/determined sets of ppa's, and indirectly - to NLQ's represented by them.

The applicability issue

- Moreover, it is irrelevant whether e-formulas/questions have been introduced into a formal language according to the “define within” method or the “enrich with” method.
- In the former case the “original” semantic concepts pertaining to other formulas pertain to e-formulas as well. But nothing prevents us from using, in addition, a new collection of concepts, the MiES-concepts. They would constitute the “second collection” of semantic concepts pertaining directly to e-formulas and indirectly to NLQ’s.
- The situation is analogous when e-formulas are defined syntactically, but their semantic analysis is also provided.
- If, however, e-formulas are defined only syntactically, then, assuming that some assignment of ppa’s to e-formulas is given, MiES shows how to deal with questions semantically without providing a semantics for questions/e-formulas themselves.

Further readings

- The survey paper Harrah [Har02] provides a comprehensive exposition of logical theories of questions elaborated till late 1990s. Supplementary information about more linguistically oriented approaches can be found, e.g., in Groenendijk & Stokhof [GS97] (reprinted as [GS11]) and Krifka [Kri11]. The paper Ginzburg [Gin11] provides a survey of recent developments in the research on questions, both in logic and in linguistics. A general overview of approaches to questions and their semantics can also be found in Wiśniewski [Wiś14].
- For the reasons of time, I did not present here all the erotetic concepts already defined within MiES. Similarly, I skipped the corollaries characterizing relations between concepts. More information on MiES can be found in Wiśniewski [Wiś96] and Wiśniewski [Wiś01]. A model-theoretic variant of MiES is presented in Wiśniewski [Wiś97], and in Chapter 4 of the book Wiśniewski [Wiś95]. Recently Michal Peliš [Pel11] proposed an account of MiES based on the notion of model as the basic one. His account includes definitions of some new concepts as well as simplifications of definitions of certain old concepts.

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