# An Introduction to Inferential Erotetic Logic

Andrzej Wiśniewski

Department of Logic and Cognitive Science
Institute of Psychology
Adam Mickiewicz University
Poznań, Poland
Andrzej.Wisniewski@amu.edu.pl

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# Outline

# What is Inferential Erotetic Logic?

- By and large, Inferential Erotetic Logic (IEL for short) is a logic that analyses inferences in which questions perform the role of conclusions, and proposes criteria of validity for these inferences.
- The idea of IEL originates from late 1980s, but IEL was developed in depth in the 1990s.[1]
- IEL provided an alternative to the received view in the logic of questions, which situated the answerhood problem in the center of attention, and to the Interrogative Model of Inquiry, elaborated by Jaakko Hintikka.

[1] Cf. Wiśniewski [Wiś86], [Wiś89], [Wiś90a], [Wiś90b], [Wiś91], [Wiś94], [Wiś95], [Wiś96], [Wiś01].

# Erotetic inferences

- IEL starts with a trivial observation that before a question is asked or posed, a questioner must arrive at it.
- In many cases arriving at questions resembles coming to conclusions: there are premises involved and some inferential thought processes take place.
- If we admit that a conclusion need not be "conclusive", we can say that sometimes questions play the role of conclusions.
- But questions can also perform the role of premises: it often happens that an agent arrives at a question when looking for an answer to another question.
- Thus the concept of an *erotetic inference* is introduced.

# Erotetic inferences

- As a first approximation an erotetic inference may be defined as a thought process in which one arrives at a question on the basis of some previously accepted declarative sentence or sentences and/or a previously posed question.
- There are erotetic inferences of (at least) two kinds: the key difference between them lies in the type of premises involved.

# E-inferences of the first kind: examples

*Andrew always comes in time, but now he is late.*

---

*What has happened to him?*

*Mary is Peter's mother.*
*If Mary is Peter's mother, then John is Peter's father or George is Peter's father.*

---

*Who is Peter's father: John or George?*

- The set of premises comprises declarative sentence(s). The conclusion is a question.

# E-inferences of the second kind: examples

*Is Andrew lying?*
*Andrew lies if, and only if he speaks very slowly.*

---

*Is Andrew speaking very slowly?*

*Where did Andrew leave for: Paris, London or Moscow?*

*If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.*
*If Andrew departed in the morning, then he left for Paris or London.*
*If Andrew departed in the evening, then he left for Moscow.*

---

*When did Andrew depart: in the morning, or in the evening?*

*Where does Andrew live?*

*Andrew lives in a university town in Western Poland.*

---

*Which towns in Western Poland are university towns?*

# E-inferences of the second kind

- The conclusion is a question. There occurs a single interrogative premise, being also a question. The remaining premises, if any, are declarative sentences.
- The set of "declarative" premises can be empty, as in:

*Did Andrew fly by BA, or by Ryanair, or by neither?*

---

*Did Andrew fly by BA?*

*Is Andrew silly and ugly?*

---

*Is Andrew ugly?*

# Validity of erotetic inferences

- As long as we are concerned with inferences which have only declaratives as premises and conclusions, validity amounts to the transmission of truth: if the premises are all true, the conclusion must be true as well.
- However, it is doubtful whether it makes any sense to assign truth or falsity to questions and thus one cannot apply the above concept of validity to erotetic inferences.
- But in the case of questions the concept of soundness seems to play an equally important role as the concept of truth in the realm of declaratives. Recall that a question $Q$ is *sound* if at least one principal possible answer (ppa) to $Q$ is true, and unsound otherwise.
- This concept is extensively used in the analysis of validity proposed by IEL, but with a certain refinement.

# Validity of erotetic inferences

- Ppa's to natural language questions are supposed to be *direct answers*, that is, possible and just-sufficient answers, where "just-sufficient" means "satisfying the request of a question by providing neither less nor more information than it is requested".
- There are erotetic inferences of (at least) two kinds, and the conditions of validity are distinct for each kind.
- In the general setting an erotetic inference of the first kind is conceived as an ordered pair $\langle X, Q \rangle$, where $X$ is a finite and non-empty set of declarative sentences, and $Q$ is a question. The elements of $X$ are the premises, and $Q$ is the conclusion.
- Similarly, an erotetic inference of the second kind is identified with an ordered triple $\langle Q, X, Q_1 \rangle$, where $Q$, $Q_1$ are questions and $X$ is a finite (possibly empty) set of declarative sentences.
- When formal languages with questions are dealt with, $X$ is a set of declarative well-formed formulas.

# Validity of erotetic inferences of the first kind

- It seems natural to impose the following necessary condition of validity on erotetic inferences of the first kind:

  ($\mathbf{C}_1$) (TRANSMISSION OF TRUTH INTO SOUNDNESS). *If the premises are all true, then the question which is the conclusion must be sound.*

- Is this sufficient? Certainly not. For if ($\mathbf{C}_1$) were sufficient, the following inferences would be valid:

  *Andrew is rich.*
  *Andrew is happy.*
  ———————————
  *Is Andrew happy?*

# Validity of erotetic inferences of the first kind

*If Andrew is rich, then he is happy.*
*Andrew is rich.*

---

*Is Andrew happy?*

- What is wrong with the above inferences?
- The question which is the conclusion has a direct answer which provides us with information that is already present (directly or indirectly) in the premises. In other words, the question which is the conclusion is logically redundant and thus not informative.

# Validity of erotetic inferences of the first kind

- So IEL imposes the following additional necessary condition of validity on erotetic inferences of the first kind:

  ($\mathbf{C}_2$) (INFORMATIVENESS). *A question which is the conclusion must be informative relative to the premises.*

- Informativeness is then explicated as the lack of entailment of any direct answer from the premises; the applied concept of entailment need not be classical.

# Validity of erotetic inferences of the first kind

- Thus the following inferences are regarded as valid:

*Someone stole the necklace.*

---

*Who did it?*

*Mary is Peter's mother.*
*If Mary is Peter's mother, then John is Peter's father or George is Peter's father.*

---

*Who is Peter's father: John or George?*

# Validity of erotetic inferences of the first kind

- Validity is a normative notion.
- In the case of erotetic inferences the appropriate notion of validity is neither given by God nor by Tradition. So some more or less arbitrary decisions have to be made.
- IEL decides to regard as valid these erotetic inferences of the first kind which have the features described by conditions ($\mathbf{C}_1$) and ($\mathbf{C}_2$).
- IEL offers more than just formulating them.
- A semantic concept of *evocation* of questions by sets of d-wffs is introduced, and validity of e-inferences of the first kind is then defined in terms of evocation.

# Validity of erotetic inferences of the second kind

- An erotetic inference of the second kind can be viewed as an ordered triple $\langle Q, X, Q_1 \rangle$, where $Q$, $Q_1$ are questions and $X$ is a finite (possibly empty) set of declarative sentences.
- The first condition of validity is:

  ($\mathbf{C}_3$) (TRANSMISSION OF SOUNDNESS/TRUTH INTO SOUNDNESS). *If the initial question is sound and all the declarative premises are true, then the question which is the conclusion must be sound.*

- This condition is essential when one allows for risky (i.e. "non-partition") questions. IEL allows for such questions.
- By the way, ($\mathbf{C}_3$) is a natural generalization of the standard condition of validity.

# An example

- Consider:

  *Where did Andrew leave for: Paris, London or Moscow?*

  *If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.*
  *If Andrew departed in the morning, then he left for Paris or London.*
  *If Andrew departed in the evening, then he left for Moscow.*

  ---

  *When did Andrew depart: in the morning, or in the evening?*

- The initial question need not be sound. The declarative premises need not be true. But if they are true and the initial question is sound, the question-conclusion must be sound.

# Validity of erotetic inferences of the second kind

- IEL regards ($\mathbf{C}_3$) only as a necessary condition of validity of erotetic inferences of the second kind. Why? If condition ($\mathbf{C}_3$) had been sufficient, then, for instance, the following would have been valid inferences:

*Is Andrew a logician?*
*Some philosophers are logicians, and some are not.*

---

*Is Andrew a philosopher?*

*Is Coco a human?*
*Humans are mammals.*

---

*Is Coco a mammal?*

- The problem here is that the questions which are conclusions have direct answers that are cognitively useless: these answers, if accepted, would not contribute to finding answers to initial questions.

# Validity of erotetic inferences of the second kind

- The second necessary condition of validity imposed by IEL is:

  ($\mathbf{C}_4$) (OPEN-MINDED COGNITIVE USEFULNESS). *For each direct answer $B$ to the question which is the conclusion there exists a non-empty proper subset $Y$ of the set of direct answers to the initial question such that the following condition holds*:

  (♣) *if $B$ is true and all the declarative premises are true, then at least one direct answer $A \in Y$ to the initial question must be true*.

- Note that the condition is very demanding: it requires each direct answer to the question-conclusion "to do the job".
- What is the job? The initial "search space" should be *narrowed down*.
- As we will see, this is not tantamount to *elimination*.

# An example

- Consider:

  *Where did Andrew leave for: Paris, London or Moscow?*

  *If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.*
  *If Andrew departed in the morning, then he left for Paris or London.*
  *If Andrew departed in the evening, then he left for Moscow.*

  ---

  *When did Andrew depart: in the morning, or in the evening?*

- $Y_{\texttt{morning}} = \{\texttt{Paris, London}\}$
- $Y_{\texttt{evening}} = \{\texttt{Moscow}\}$

# Validity of erotetic inferences of the second kind

- We introduce the concept of erotetic implication.
- Erotetic implication is a ternary relation between a question, a (possibly empty) set of d-wffs, and a question.
- Erotetic implication is defined in such a way that both transmission of soundness/truth into soundness and open-minded cognitive usefulness are secured *semantically*.
- Validity of erotetic inferences of the second kind is then defined in terms of erotetic implication.

# Remarks on the logical basis of IEL

- At the syntactic level IEL operates on e-formulas, that is, questions of formal languages.
- What we need is a formal language with questions build according to the "enrich with" approach. A language of this kind has both d-wffs and e-formulas/questions among its well-formed expressions, and questions are distinct from well-formed expressions of other categories.
- Then we need some assignment of nominal principal possible answers (ppa's) to e-formulas/questions. The ppa's are supposed to be d-wffs. Due to the assignment, the set of all ppa's to an e-formula is unique.

# Remarks on the logical basis of IEL

- E-formulas (but not necessarily all of them) are formalizations of natural language questions (NLQ's). Let me recall the schema:

  (♠) *An e-formula $Q$ represents a NLQ $Q^*$* CONSTRUED IN SUCH A WAY *that possible answers to $Q^*$ having the desired semantic and/or pragmatic properties are represented by nominal ppa's to $Q$*.

- In the case of IEL the desired property of possible answers to NLQ's is *just-sufficiency*, where "just-sufficient" means "satisfying the request of a question by providing neither less nor more information than it is requested".

- So *an e-formula $Q$ represents a NLQ* READ AS *its possible just-sufficient answers are just the sentences formalized by the d-wffs in* $\mathbf{d}Q$*.*

  where $\mathbf{d}Q$ is the set of (nominal) ppa's to $Q$.

# Direct answers

- Within the IEL setting it is customary to label the nominal ppa's to e-formulas/questions of formal languages simply as *direct answers*.
- I will be following this convention from now on.
- Thus $\mathbf{d}Q$ will stand for the set of direct answers to an e-formula/question of a formal language $Q$.

# Remarks on the logical basis of IEL

- In general considerations concerning IEL it is assumed that the following conditions are satisfied by the relevant formal languages with questions:

  ($\mathbf{sc}_1$) *direct answers are sentences, i.e. d-wffs with no individual or higher-order free variables*;

  ($\mathbf{sc}_2$) *each question has at least two direct answers*.

- Sometimes the following condition is also imposed:

  ($\mathbf{sc}_3$) *each finite and at least two-element set of sentences is the set of direct answers to some question.*

# Remarks on the logical basis of IEL

- In the general setting we use the conceptual apparatus of Minimal Erotetic Semantics. (See the previous lecture.) We suppose that the class of admissible partitions of a language is determined and this, as we have seen, enables us to define – and operate with – the remaining concepts.
- Needles to say, the conceptual apparatus of MiES is general enough to allow both Classical Logic and a non-classical logic to be the logic of d-wffs. IEL is neutral with respect to the issue of what “The Logic” of declaratives is.

# Multiple-conclusion entailment

Let me recall:

- Let $\mathcal{L}$ be a language of the considered kind, and let $X$ and $Y$ be sets of d-wffs of $\mathcal{L}$. The relation $||\!\!=_{\mathcal{L}}$ of *multiple-conclusion entailment in $\mathcal{L}$* is defined as follows:

**Definition (*Multiple-conclusion entailment*)**

$X \; ||\!\!=_{\mathcal{L}} Y$ iff there is no admissible partition $\mathsf{P} = \langle \mathsf{T}_{\mathsf{P}}, \mathsf{U}_{\mathsf{P}} \rangle$ of $\mathcal{L}$ such that $X \subset \mathsf{T}_{\mathsf{P}}$ and $Y \subset \mathsf{U}_{\mathsf{P}}$.

- We consider an arbitrary but fixed language of the analysed kind, so the specifications "of $\mathcal{L}$" and "in $\mathcal{L}$" will be omitted.
- $\models$ will stand for ("single-conclusion") entailment. Of course, $X \models A$ iff $X \; ||\!\!= \{A\}$.

# Evocation of questions

**Definition (*Evocation of questions*)**

A set of d-wffs $X$ evokes a question $Q$ (in symbols: $\mathbf{E}(X, Q)$) iff

(1) $X \mid\!\models \mathbf{d}Q$, and

(2) for each $A \in \mathbf{d}Q : X \mid\!\not\models \{A\}$.

- Clause (*2*) is formulated in terms of mc-entailment for uniformity only, since $X$ does not mc-entail $\{A\}$ if, and only if $X$ does not entail $A$.

**Definition (*Validity I*)**

An erotetic inference of the first kind, $\langle X, Q\rangle$, is valid iff $\mathbf{E}(X, Q)$.

# Evocation and validity

- The clause (*1*) $X \,||\!\!= \mathbf{d}Q$ amounts to:
- For each admissible partition P of the language: if $X \subset \mathbf{T}_{\mathsf{P}}$, then $Q$ is sound in P.
- Compare it with:

  ($\mathbf{C}_1$) (TRANSMISSION OF TRUTH INTO SOUNDNESS). *If the premises are all true, then the question which is the conclusion must be sound.*

- Thus "must" is explicated by "for each admissible partition".

# Evocation and validity

- Now compare:

  ($\mathbf{C}_2$) (INFORMATIVENESS). *A question which is the conclusion must be informative relative to the premises.*

  with the clause:
- (*2*) for each $A \in \mathbf{d}Q : X \,\|\!\not\models \{A\}$.
- Informativeness is explicated in terms of lack of entailment.
- The degree of idealization is high.
- However, nobody claims that each inference which is not valid is substantially faulty.

# Examples of evocation in $\mathcal{L}^{+}_{CPC}$

$$\mathbf{E}(p \vee \neg p, ?\, p) \tag{1}$$

$$\mathbf{E}(p \vee q, ?\, \{p, q\}) \tag{2}$$

$$\mathbf{E}(p \vee q, ?\, (p \wedge q)) \tag{3}$$

$$\mathbf{E}(p \rightarrow q \vee r, ?\, \{p \rightarrow q, p \rightarrow r\}) \tag{4}$$

$$\mathbf{E}(p \wedge q \rightarrow r, ?\, \{p \rightarrow r, q \rightarrow r\}) \tag{5}$$

$$\mathbf{E}(p \wedge q \rightarrow r, \neg r, ?\, \{\neg p, \neg q\}) \tag{6}$$

# Examples of evocation in $\mathcal{L}^{+}_{mFOL}$

[Distinct metalanguage variables are supposed to represent distinct object-level language entities.]

$$\mathbf{E}(Pc_1 \wedge Rc_1, \ldots, Pc_n \wedge Rc_n, ?\,\forall x(Px \rightarrow Rx)) \tag{7}$$

$$\mathbf{E}(Pc_1 \wedge Rc_1, \ldots, Pc_n \wedge Rc_n, Pc_{n+1}, ?\ Rc_{n+1}) \tag{8}$$

$$\mathbf{E}(\exists x(Px \wedge (x = c_1 \vee \ldots \vee x = c_n)), ?\,\mathbf{S}(Px)), \tag{9}$$

where $n > 1$.

$$\mathbf{E}(\neg\forall x(x = c_1 \vee \ldots \vee x = c_n \rightarrow Px), ?\,\mathbf{S}(\neg Px)), \tag{10}$$

where $n > 1$.

$$\mathbf{E}(\forall x(Px \leftrightarrow x = c_1 \vee \ldots \vee x = c_n) \vee \forall x(Px \leftrightarrow x = c^*_1 \vee \ldots \vee x = c^*_k)?\ \mathbf{U}(Px)), \tag{11}$$

$$\mathbf{E}(\exists xPx, ?\,\mathbf{S}(Px)) \tag{12}$$

# Some corollaries

**Definition (*Normal question*)**

A question $Q$ is normal iff $\mathbf{Pres}Q \neq \emptyset$ and $\mathbf{Pres}Q \mid\!\models \mathbf{d}Q$.

**Corollary**

*Let $Q$ be a normal question. Then $\mathbf{E}(X, Q)$ iff $X \models B$ for each $B \in \mathbf{Pres}Q$, and $X \not\models A$ for each $A \in \mathbf{d}Q$.*

# Some corollaries

**Definition (*Regular question*)**

A question $Q$ is regular iff there exists $B \in \mathbf{Pres}Q$ such that $B \mid\models \mathbf{d}Q$.

**Corollary**

*Let $Q$ be a regular question. Then $\mathbf{E}(X, Q)$ iff $X \models B$ for some $B \in \mathbf{PPres}Q$, and $X \not\models A$ for each $A \in \mathbf{d}Q$.*

# Some corollaries

**Definition (*Self-rhetorical question*)**

A question $Q$ is self-rhetorical iff $\mathbf{Pres}Q \models A$ for some $A \in \mathbf{d}Q$.

**Definition (*Proper question*)**

A question $Q$ is proper iff $Q$ is normal, but not self-rhetorical.

**Corollary**

*A question $Q$ is proper iff $\mathbf{Pres}Q \neq \emptyset$ and $\mathbf{E}(\mathbf{Pres}Q, Q)$.*

# Definition of erotetic implication

Let me now define the concept of *erotetic implication*

**Definition (*Erotetic implication*)**

A question $Q$ implies a question $Q_1$ on the basis of a set of d-wffs $X$ (in symbols: $\mathbf{Im}(Q, X, Q_1)$) iff:

(1) for each $A \in \mathbf{d}Q : X \cup \{A\} \mid\models \mathbf{d}Q_1$, and

(2) for each $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset $Y$ of $\mathbf{d}Q$ such that $X \cup \{B\} \mid\models Y$.

# Some examples

$$\mathbf{Im}(?\,\{A, B \vee C\}, ?\,\{A, B, C\})$$

$$Y_A = \{A\}$$

$$Y_B = \{B \vee C\}$$

$$Y_C = \{B \vee C\}$$

$$\mathbf{Im}(?\,\{A, B, C\}, ?\,\{A, B \vee C\})$$

$$Y_A = \{A\}$$

$$Y_{B \vee C} = \{B, C\}$$

# Some examples

$$\mathbf{Im}(?\pm|A,B|,?A)$$

$$Y_A = \{A \wedge B, A \wedge \neg B\}$$

$$Y_{\neg A} = \{\neg A \wedge B, \neg A \wedge \neg B\}$$

$$\mathbf{Im}(?A, A \leftrightarrow B, ?B)$$

$$Y_B = \{A\}$$

$$Y_{\neg B} = \{\neg A\}$$

# Some examples

$$\mathbf{Im}(?\,\{A_1, \ldots, A_n\}, A_1 \vee \ldots \vee A_n, ?\,A_i)$$

$$Y_{A_i} = \{A_i\}$$

$$Y_{\neg A_i} = \{A_1, \ldots, A_n\} \setminus \{A_i\}$$

$$\mathbf{Im}(?\,\mathbf{S}(Ax), \exists x Ax, ?\,A(x/c))$$

$$Y_{A(x/c)} = \{A(x/c)\}$$

$$Y_{\neg A(x/c)} = \mathbf{S}(Ax) \setminus \{A(x/c)\}$$

# Some examples

$$\mathbf{Im}(?\,\mathbf{S}(Ax), \exists x(Bx), \forall x(Bx \rightarrow Ax), ?\,\mathbf{S}(Bx))$$

$$Y_{B(x/c)} = \{A(x/c)\}$$

# Partial answering and elimination

- The second clause of the definition of erotetic implication can be fulfilled for quite familiar reasons.
- Let us consider the following clauses (in both cases it is assumed that $B \in \mathbf{d}Q_1$):

(**nd**) *there exists a non-empty proper subset* $Y$ *of* $\mathbf{d}Q$ *such that* $X \cup \{B\} \mid\models Y$.

(**el**) $X \cup \{B\}$ *eliminates a certain direct answer to* $Q$.

- (**el**) yields (**nd**) given that $X \mid\models \mathbf{d}Q$ holds.
- (**nd**) yields (**el**) assuming that for any $A, C \in \mathbf{d}Q$, where $A \neq C$: $A$ eliminates $C$.

# Partial answering and elimination

- If the following condition holds (again, we assume that $B \in \mathbf{d}Q_1$):

  (**dvp**) $X \cup \{B\}$ *entails a direct or partial answer to* $Q$

  then (**nd**) holds.

- On the other hand, (**dvp**) yields (**nd**) if mc-entailment in the language is compact, the language includes disjunction $\vee$, and the following condition holds:

  ($\triangledown$) for each admissible partition $\mathsf{P} = \langle \mathsf{T_P}, \mathsf{U_P} \rangle$ of the language:

  $$\{A_1, \ldots, A_n\} \cap \mathsf{T_P} \neq \emptyset \text{ \textit{iff} } \ulcorner A_1 \vee \ldots \vee A_n \urcorner \in \mathsf{T_P}.$$

- As it happens in CL!

# Erotetic implication and validity of erotetic inferences

## Corollary

*The condition:*

(1) *for each* $A \in \mathbf{d}Q : X \cup \{A\} \mid\!\models \mathbf{d}Q_1$

*is fulfilled iff the following condition holds:*

(1') *for each admissible partition* $\mathsf{P} = \langle \mathsf{T}_{\mathsf{P}}, \mathsf{U}_{\mathsf{P}} \rangle$ *of the language: if* $Q$ *is sound in* $\mathsf{P}$ *and* $X \subset \mathsf{T}_{\mathsf{P}}$, *then* $Q_1$ *is sound in* $\mathsf{P}$.

- Thus (1) mirrors:

  ($\mathbf{C}_3$) (TRANSMISSION OF SOUNDNESS/TRUTH INTO SOUNDNESS). *If the initial question is sound and all the declarative premises are true, then the question which is the conclusion must be sound.*

## Erotetic implication and validity of erotetic inferences

### Corollary

*The condition:*

(2) *for each* $B \in \mathbf{d}Q_1$ *there exists a non-empty proper subset* $Y$ *of* $\mathbf{d}Q$ *such that* $X \cup \{B\} \mid\!\models Y$

*is satisfied iff the following condition is fulfilled:*

(2') *for each* $B \in \mathbf{d}Q_1$ *there exists a non-empty proper subset* $Y$ *of* $\mathbf{d}Q$ *such that for each admissible partition* $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ *of the language: if* $X \cup \{B\} \subset \mathsf{T}_\mathsf{P}$*, then* $A \in \mathsf{T}_\mathsf{P}$ *for some* $A \in Y$*.*

- Hence (2) explicates:

($\mathbf{C}_4$) (OPEN-MINDED COGNITIVE USEFULNESS). *For each direct answer B to the question which is the conclusion there exists a non-empty proper subset Y of the set of direct answers to the initial question such that the following condition holds*:

(♣) *if B is true and all the declarative premises are true, then at least one direct answer* $A \in Y$ *to the initial question must be true.*

# Erotetic implication and validity of erotetic inferences

- So the following is no suprise:

**Definition (*Validity II*)**

An erotetic inference of the second kind, $\langle Q, X, Q_1 \rangle$, is valid iff $\mathbf{Im}(Q, X, Q_1)$.

# Some surprises

- One can easily prove:

**Corollary**

*Let* $\mathbf{Im}(Q, X, Q_1)$*. Then* $X \Vert\!\!= \mathbf{d}Q$ *iff* $X \Vert\!\!= \mathbf{d}Q_1$*.*

- Hence the following is true:

**Corollary**

*Let* $\mathbf{Im}(Q, X, Q_1)$ *and let* $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ *be an admissible partition of the language such that* $X \subset \mathsf{T}_\mathsf{P}$*. Then* $Q_1$ *is sound in* $\mathsf{P}$ *iff* $Q$ *is sound in* $\mathsf{P}$*.*

# Some surprises

- Take a look at:

Table: *From implying question to implied question*

| $Q$ | $X$ | $Q_1$ |
|---|---|---|
| *sound in* P | $X \subset \mathsf{T_P}$ | *sound in* P |
| *unsound in* P | $X \subset \mathsf{T_P}$ | *unsound in* P |
| *sound in* P | $X \not\subset \mathsf{T_P}$ | *sound in* P *or unsound in* P |
| *unsound in* P | $X \not\subset \mathsf{T_P}$ | *sound in* P *or unsound in* P |

- The second row is suprising. It follows that *a valid inference based on true declarative premises, but unsound erotetic premise always leads to an unsound "erotetic" conclusion, that is, to an unsound question.*

# Some surprises

- Now take a look at:

Table: *From implied question to implying question*

| $Q_1$ | $X$ | $Q$ |
|---|---|---|
| *sound in* P | $X \subset \mathsf{T}_{\mathsf{P}}$ | *sound in* P |
| *unsound in* P | $X \subset \mathsf{T}_{\mathsf{P}}$ | *unsound in* P |
| *sound in* P | $X \not\subset \mathsf{T}_{\mathsf{P}}$ | *sound in* P *or unsound in* P |
| *unsound in* P | $X \not\subset \mathsf{T}_{\mathsf{P}}$ | *sound in* P *or unsound in* P |

- The first row shows, in turn, that a valid inference with a sound conclusion, based on true declarative premises, always has a sound erotetic premise.

# The transitivity issue

- Erotetic implication is not "transitive" in the sense that there are cases in which $\mathbf{Im}(Q, X, Q_1)$ and $\mathbf{Im}(Q_1, X, Q_2)$ hold, but $\mathbf{Im}(Q, X, Q_2)$ does not hold.
- Here is a simple (counter)example taken from language $\mathcal{L}^{+}_{CPC}$. We have:

$$\mathbf{Im}(?\, p, \emptyset, ?\, \{p \wedge q, p \wedge \neg q, \neg p\}) \tag{13}$$

$$\mathbf{Im}(?\, \{p \wedge q, p \wedge \neg q, \neg p\}, \emptyset, ?\, q) \tag{14}$$

but we do not have $\mathbf{Im}(?\, p, \emptyset, ?\, q)$.

# The transitivity issue. Regular erotetic implication

However, regular erotetic implication is "transitive".

**Definition (*Regular erotetic implication*)**

A question $Q$ regularly implies a question $Q_1$ on the basis of a set of d-wffs $X$ iff

(1) for each $A \in \mathbf{d}Q : X \cup \{A\} \Vert\!\models \mathbf{d}Q_1$, and

(2) for each $B \in \mathbf{d}Q_1$ there exists $C \in \mathbf{d}Q$ such that $X \cup \{B\} \models C$.

# No cuts!

- One cannot pass, in a valid erotetic inference, from:

$$? \, A \tag{15}$$

on the basis of:

$$B \rightarrow A \tag{16}$$

to:

$$? \, B \tag{17}$$

- However, one can reach $? \, B$ in two valid steps. The following hold:

$$\mathbf{Im}(? \, A, B \rightarrow A, ? \, \{A, \neg A, B\})$$

$$\mathbf{Im}(? \, \{A, \neg A, B\}, ? \, B)$$

- So an erotetic reasoning gives more than an erotetic inference.

# Evocation vs. erotetic implication

- Although the definitions of validity are diverse, the analysis of validity of erotetic inferences proposed by IEL is based on a certain general idea: *the conclusion of a valid erotetic inference arises from the premises.*
- The proposed definitions of evocation and erotetic implication are explications of the relevant notions of question raising (cf. Wiśniewski [Wiś95], Chapter 1). By defining the semantic concept "a set of d-wffs $X$ evokes a question $Q$" we explicate the concept "a question $Q$ arises from a set of declarative sentences $X$". The definition of "a question $Q$ implies a question $Q_1$ on the basis of a set of d-wffs $X$" provides an explication of the notion "a question $Q_1$ arises from a question $Q$ and a set of declarative sentences $X$".

# Evocation vs. erotetic implication

- From a purely formal point of view evocation of questions and erotetic implication are interrelated in many ways.
- A question $Q$ is *non-factual* iff for each $A \in \mathbf{d}Q$ and any admissible partition $\mathsf{P}$ of the language we have: $A \in \mathsf{T}_{\mathsf{P}}$.

## Corollary

*Assume that $X$ and $Q$ are expressions of a language in which non-factual question(s) occur. Then $\mathbf{E}(X, Q)$ iff*

- *$Q$ is informative relative to $X$, and*
- *for each non-factual question $Q^{\star}$ of the language: $\mathbf{Im}(Q^{\star}, X, Q)$.*

# Answering evoked questions by means of answers to implied questions

- Let $\Xi$ be a non-empty set of questions. By a $\Xi$-*answer set* we mean a set of d-wffs that comprises only direct answers to questions of $\Xi$ and such that the set includes exactly one direct answer to each question of $\Xi$.
- For instance, if $\Xi = \{?\, p, ?\, q\}$, any of the following is a $\Xi$-answer set: $\{p, q\}, \{\neg p, q\}, \{p, \neg q\}, \{\neg p, \neg q\}$.
- By a *binary question* we mean a question which has exactly two direct answers.
- We say that entailment in $\mathcal{L}$, $\models_{\mathcal{L}}$, is *compact* iff $X \models_{\mathcal{L}} A$ yields $X^{\star} \models_{\mathcal{L}} A$ for some finite subset $X^{\star}$ of $X$.
- By a *valid* d-wff we mean a d-wff which is true in each admissible partition of the language.
- As for strong erotetic implication, we simply require, in addition, that $X \,||\!\!\not\models\, Y$ (for any $Y$ associated with a direct answer to $Q_1$).

# Answering evoked questions by means of answers to implied questions

## Theorem

*Let $\mathcal{L}$ be a language of the considered kind such that:*

(a) *entailment in $\mathcal{L}$ is compact,*

(b) *for each d-wff $A$ of $\mathcal{L}$ there exists a d-wff, $\overline{A}$, of the language such that $\overline{A}$ eliminates $A$ in $\mathcal{L}$ and $\emptyset \mid\models_{\mathcal{L}} \{A, \overline{A}\}$,*

(c) *at least one d-wff of $\mathcal{L}$ is valid.*

*If $\mathbf{E}_{\mathcal{L}}(X, Q)$, then there exists a non-empty finite set, $\Xi$, of binary questions of $\mathcal{L}$ that fulfils the following conditions:*

1. *each question of $\Xi$ is evoked in $\mathcal{L}$ by $X$,*
2. *each question of $\Xi$ is strongly implied in $\mathcal{L}$ by $Q$ on the basis of $X$, and*
3. *for each $\Xi$-answer set $Z$: $X \cup Z$ entails in $\mathcal{L}$ a direct answer to $Q$.*

# Answering evoked questions by means of answers to implied questions

- Note that the compactness assumption is dispensable if the set of direct answers to $Q$ is finite.
- The same holds true for the assumption (c).
- Needles to say, the assumption

  (b) for each d-wff $A$ of $\mathcal{L}$ there exists a d-wff, $\overline{A}$, of the language such that $\overline{A}$ eliminates $A$ in $\mathcal{L}$ and $\emptyset \mid\models_{\mathcal{L}} \{A, \overline{A}\}$,

  is fulfilled if the language contains negation classically construed.

# Questions as actions of issue management

- The above idea was proposed in van Benthem & Minică [vBcM09] and developed in Minică [cM11] as well as van Benthem & Minică [vBcM12].
- An issue is understood as a partition of the set of all possibilities, where partition blocks represent the relevant classes of alternatives.
- An *epistemic issue model* is a structure:

$$M = \langle W, \approx, \sim, V \rangle$$

where:

- $W$ is a set of possible worlds (intuitively: *epistemic alternatives*),
- $\approx$ is an equivalence relation on $W$ (*the abstract issue relation*),
- $\sim$ is an equivalence relation on $W$ (*epistemic indistinguishability*),
- $V$ is a valuation for atomic propositions (into $\wp(W)$).

# Questions as actions of issue management

- Intuitively, $A?$ represents the action of asking yes-no question with answers $A$ and $\neg A$. Such an action is executed in an epistemic issue model.
- Executing action $A?$ in an epistemic issue model:

$$M = \langle W, \approx, \sim, V \rangle$$

results in:

$$M_{A?} = \langle W_{A?}, \approx_{A?}, \sim_{A?}, V_{A?} \rangle \tag{18}$$

where $W_{A?} = W$, $V_{A?} = V$, $\sim_{A?} = \sim$, and

$$\approx_{A?} = \approx \cap \equiv^A_M$$

$$\equiv^A_M \ =_{df} \ \{\langle w, v \rangle : ||A||^M_w = ||A||^M_v\}$$

- Thus asking a yes-no question has the effect that the blocks of the partition induced by $\approx$ split; note that $\approx_{A?}$ is still an equivalence and (18) is an epistemic issue model.

# Questions as actions of issue management

- Formulas which "correspond" to question asking are introduced into a language (of dynamic epistemic logic with a stock of modalities and formulas that "correspond" to announcements).
- For instance, formulas of the form:

$$[A?]B \tag{19}$$

- The semantic clause for (19) is:

$$M \models_w [A?]B \textit{ iff } M_{A?} \models_w B$$

- Here are examples of validities:

$$[A?]p \leftrightarrow p$$
$$[A?]\neg B \leftrightarrow \neg[A?]B$$
$$[A?](B \wedge C) \leftrightarrow [A?]B \wedge [A?]C$$
$$[A?]\mathbf{K}B \leftrightarrow \mathbf{K}[A?]B$$

# The challenge

- Is IEL relevant to a logic of issue management?
- Can we describe knowledge dynamics in terms of IEL?

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