E-scenarios

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Outline

1. Erotetic Decomposition Principle
2. E-derivations
3. E-scenarios
4. E-scenarios and problem-solving

Problem decomposition

- One of the crucial principles which govern effective problem solving is the following:

  (**DP**) (***Decomposition Principle***): *Decompose a principal problem (PP) into simpler sub-problems (SPs) in such a way that solutions to SPs can be assembled into an overall solution to PP.*

- When we consider a problem definite enough to be adequately expressed by a question, its decomposition amounts, generally speaking, to finding an appropriate collection of auxiliary questions.
- A decomposition can be *static*, that is, resulting in finding a set of mutually independent auxiliary questions such that once *all* of them are answered, the initial problem is resolved.
- Yet, a more interesting case is that of *dynamic* decomposition that comes in *stages*: the consecutive auxiliary questions (which constitute the sub-goals of the next stage) depend on how the previous requests for information have been fulfilled.

Erotetic decomposition principle

- The main goal, determined by the initial problem, remains unchanged, but sub-goals are processed in a goal-directed way.
- Moreover, the erotetic decomposition principle:

  (EDP) (***Erotetic Decomposition Principle***): *Transform a principal question into auxiliary questions in such a way that: (a) consecutive auxiliary questions are dependent upon previous questions and, possibly, answers to previous auxiliary questions, and (b) once auxiliary questions are resolved, the principal question is resolved as well.*

  is obeyed until the initial problem becomes solved.
- IEL gives an account of this kind of problem decomposition.
- Yet, before I turn to it, let's take a look how problem decomposition is addressed in Hintikka's Interrogative Model of Inquiry (hereafter: IMI).

Interrogative Model of Inquiry (IMI)

- IMI was developed by Jaakko Hintikka in a series of papers published in the 1980's and 1990's.[1]
- The concept of *interrogative game* is a central concept of IMI. An interrogative game is played by two parties: an Inquirer and an external source of information, called Nature or Oracle.
- In the simplest case the aim of a game is to prove a predetermined conclusion, which is an answer to the principal question.
- In a slightly more complicated case the aim is to prove at least one among previously specified sentences, which are regarded as possible answers to the principal question. Such a game is conceived as consisting of separate games, which are simple games for consecutive answers.
- Sometimes the aim of an interrogative game is to prove the desideratum of the principal question; a desideratum of a question is, roughly speaking, a proposition which specifies the cognitive state of affairs which the Inquirer wants to be brought about.

[1] The papers are collected in [Hin99]. For IMI see also, e.g., [HHM02], [Hin07].

IMI

- In each case it is assumed that the Inquirer has at his/her disposal some initial premises.
- The Inquirer can perform moves of the following kinds:
  1. *deductive moves*, in which conclusions are drawn from what has already been established;
  2. *interrogative moves*, in which auxiliary questions are addressed to a source of information (the answers received are added to the premises and thus can be used in further deductive moves);
  3. *definitory moves*, in which new concepts are introduced by explicit definitions;
  4. *assertoric moves*, in which the conclusion to be proved is strengthened.
- Moves of the third and fourth kind occur only in more sophisticated games.
- The only restriction imposed on questions which may occur in interrogative moves is that the presuppositions of these questions have to be established, i.e. must be conclusions of some earlier deductive move(s) or belong to the set of initial premises.

IMI

- The Inquirer is free to choose between a deductive move and an interrogative move: he/she can either use the (already obtained) presupposition of a question as a premise in a deductive move or can ask the corresponding question and (possibly) receive new information, which may be used in further derivation(s).
- The choice between moves as well as the choice between admissible questions is a matter of strategy; interrogative games are called games not in order to use the mathematical results of game theory, but to do justice to the importance of research strategies, modelled in IMI by different questioning strategies.
- The deductive moves are the only inferential moves of an interrogative game, and both premises and conclusions of the inferences are declarative sentences/d-wffs.
- Questions do not perform the roles of premises and conclusions. They are devices by means of which new relevant information comes into play (of course, with the exception of the principal question, which specifies the aim of the game).

Some comments

- As for IMI, the only inferential moves are those which have premises and conclusions being declarative sentences/d-wffs.
- IEL provides an account of inferences which have questions as premises and conclusions.
- So, in particular, transitions from questions to questions can be modelled as inferences.
- The concept of *Erotetic Search Scenario*, defined in terms of IEL, can be used in modelling the phenomenon of goal-directed processing of sub-goals, and dynamic problem decomposition in general.
- The proposed solution transcends the common schema of “production of a sequence of questions and affirmations”; the fact that information requests can be satisfied in one way or another is treated seriously.

An example

Let the principal question be:

- *Where did Andrew leave for: Paris, London, or Rome?*

Assume that it is known that, int.al., the following hold:

- *Andrew left for Paris, London or Rome.*
- *If Andrew flew by Air France, then he left for Paris.*
- *If Andrew did not fly by Air France, then he did not leave for Rome.*
- *Andrew left for London if and only if he flew by BA or Rynair.*

The problem is: how to decompose the principal question such that the following would hold:

- consecutive auxiliary questions are dependent upon previous questions and, possibly, answers to previous auxiliary questions, and
- once auxiliary questions are resolved, the principal question is resolved as well.

An option is: let us build an **erotetic search scenario**.

“Smart” Decomposition

- We use the conceptual apparatus of IEL, in particular *erotetic implication*.
- Let me recall:

Definition (*Erotetic implication*)

A question Q implies a question Q_1 on the basis of a set of d-wffs X (in symbols: $\mathbf{Im}(Q, X, Q_1)$) iff:

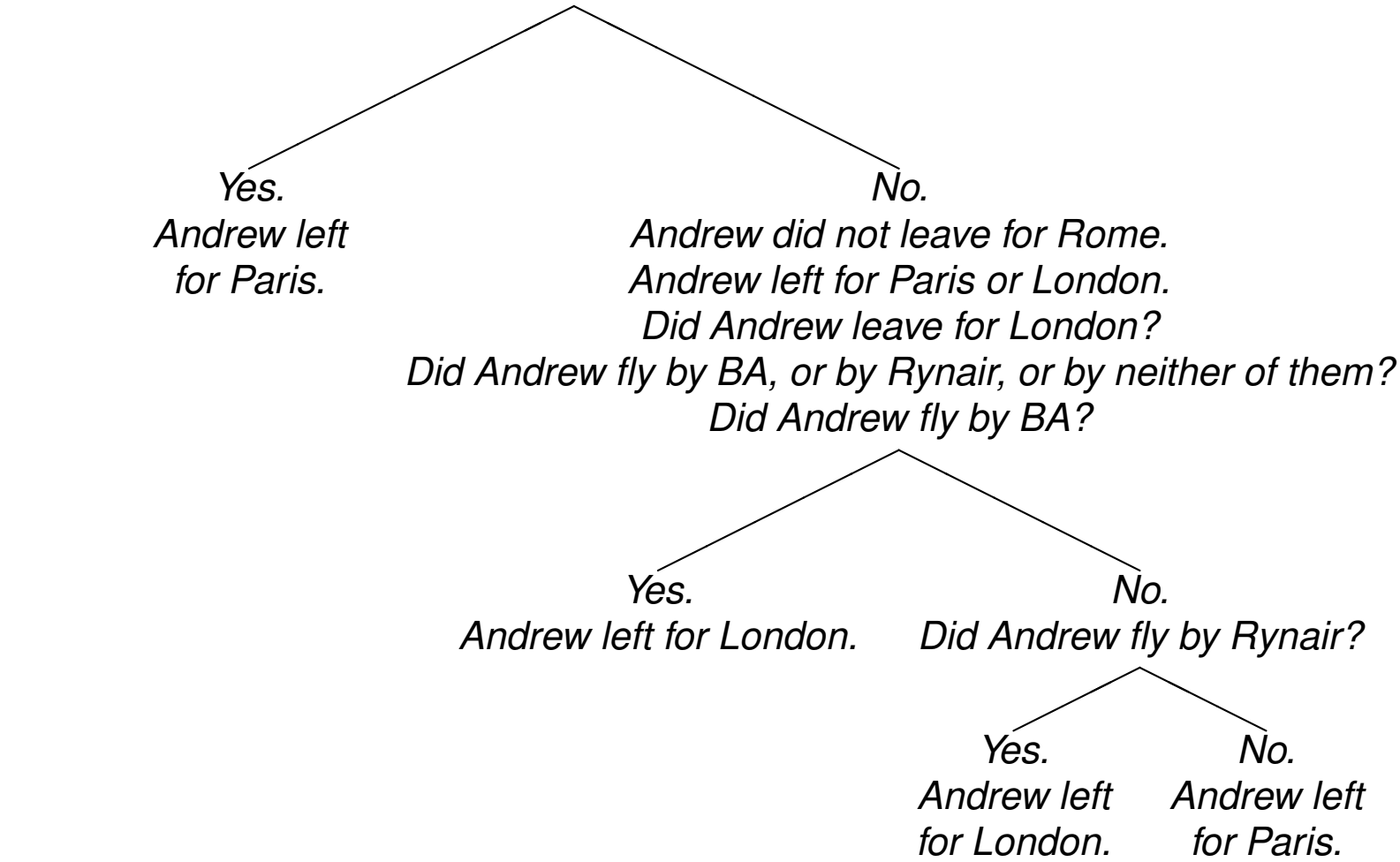
(1) for each $A \in \mathbf{d}Q : X \cup \{A\} \mid\models \mathbf{d}Q_1$, and

(2) for each $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \mid\models Y$.

The story again

- Let the principal question be:
  - *Where did Andrew leave for: Paris, London, or Rome?*
- Assume that it is known that, int.al., the following hold:
  - *Andrew left for Paris, London or Rome.*
  - *If Andrew flew by Air France, then he left for Paris.*
  - *If Andrew did not fly by Air France, then he did not leave for Rome.*
  - *Andrew left for London if and only if he flew by BA or Rynair.*
- An appropriate erotetic search scenario is displayed in the next slide.

Where did Andrew leave for: Paris, London, or Rome?
Andrew left for Paris, London or Rome.
If Andrew flew by Air France, then he left for Paris.
If Andrew did not fly by Air France, then he did not leave for Rome.
Andrew left for London if and only if he flew by BA or Rynair.
Did Andrew fly by Air France?



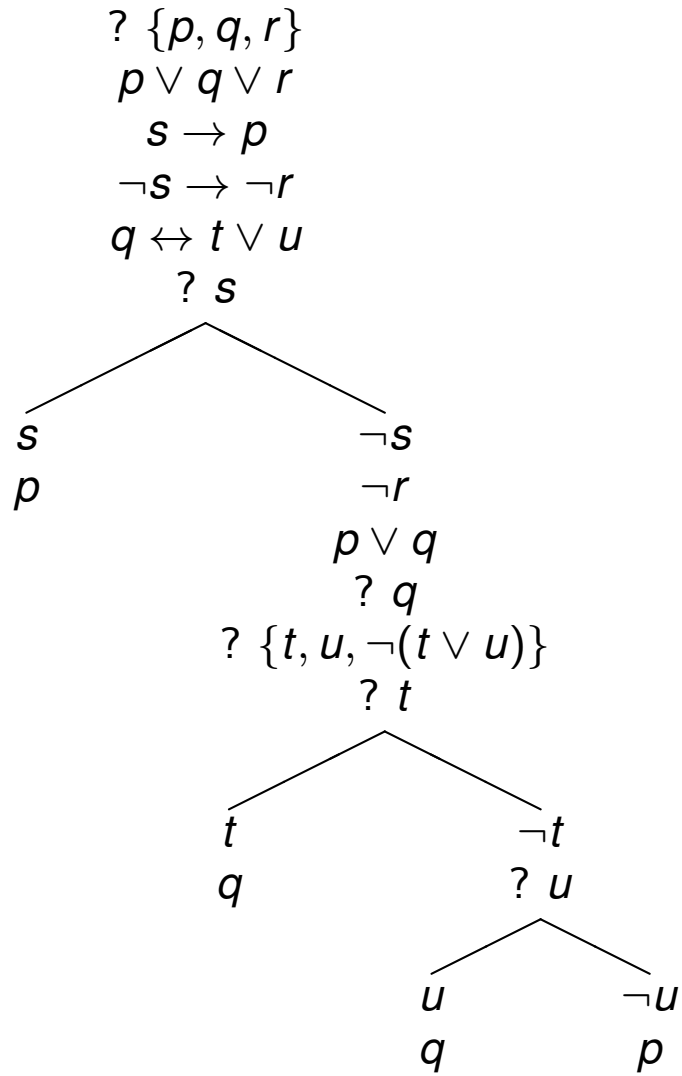


Figure: The logical structure of the scenario considered.

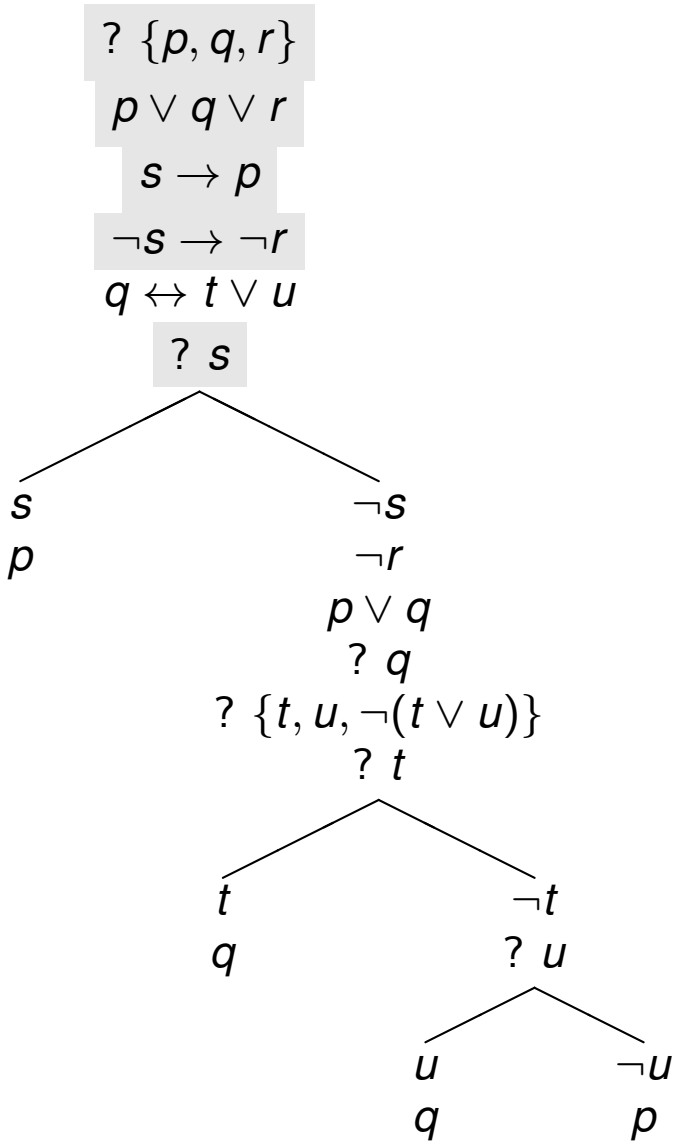


Figure: Erotetic implications used.

- $\mathbf{Im}(?\ \{p, q, r\}, p \vee q \vee r, s \to p, \neg s \to \neg r, ?\ s)$
- $Y_s = \{p\}$
- $Y_{\neg s} = \{p, q\}$

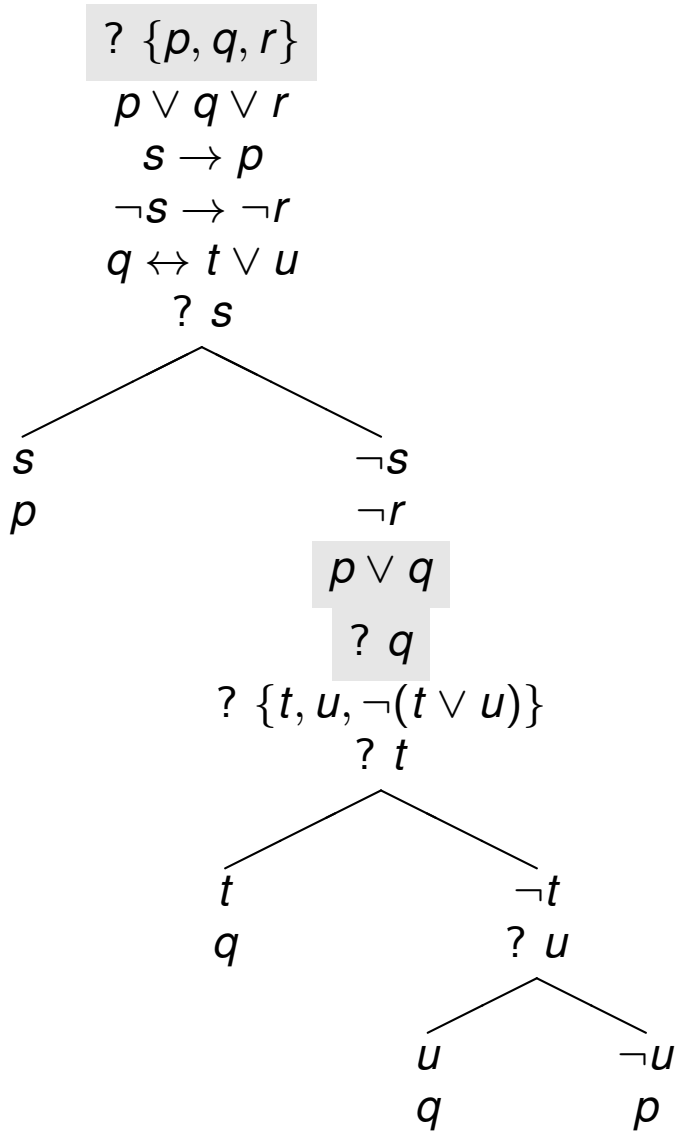


Figure: Erotetic implications used.

- $\mathbf{Im}(?\ \{p, q, r\}, p \vee q, ?\ q)$
- $Y_q = \{q\}$
- $Y_{\neg q} = \{p\}$

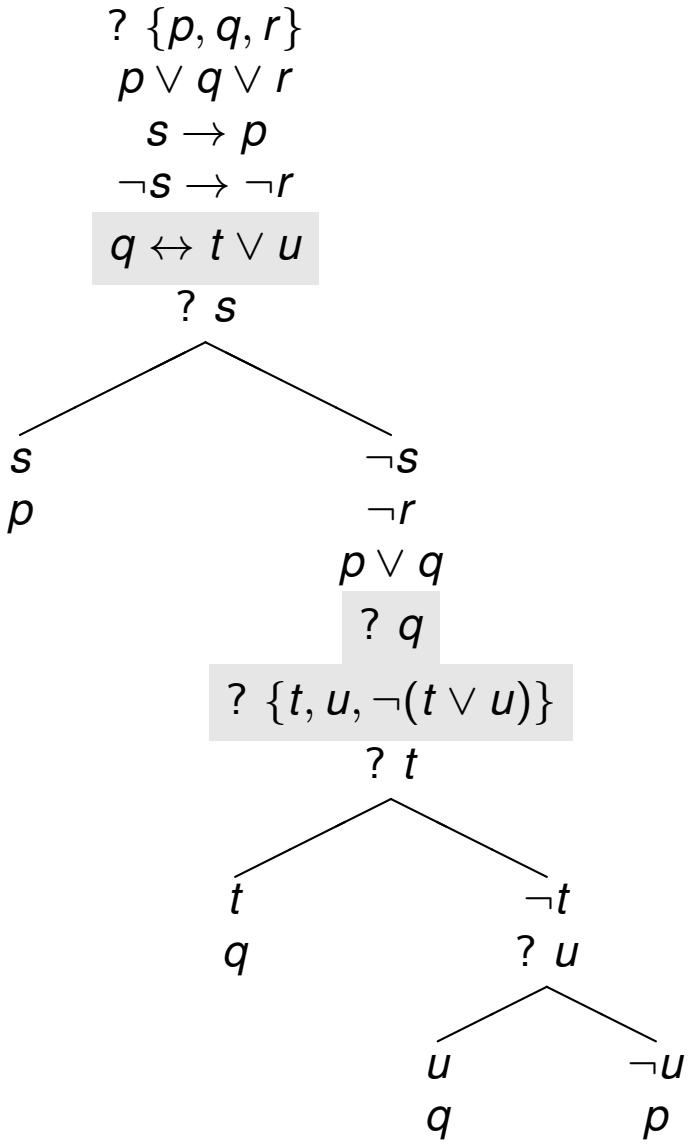


Figure: Erotetic implications used.

- $\mathbf{Im}(?\ q, q \leftrightarrow t \vee u, ?\ \{t, u, \neg(t \vee u)\})$
- $Y_t = \{q\}$
- $Y_u = \{q\}$
- $Y_{\neg(t \vee u)} = \{\neg q\}$

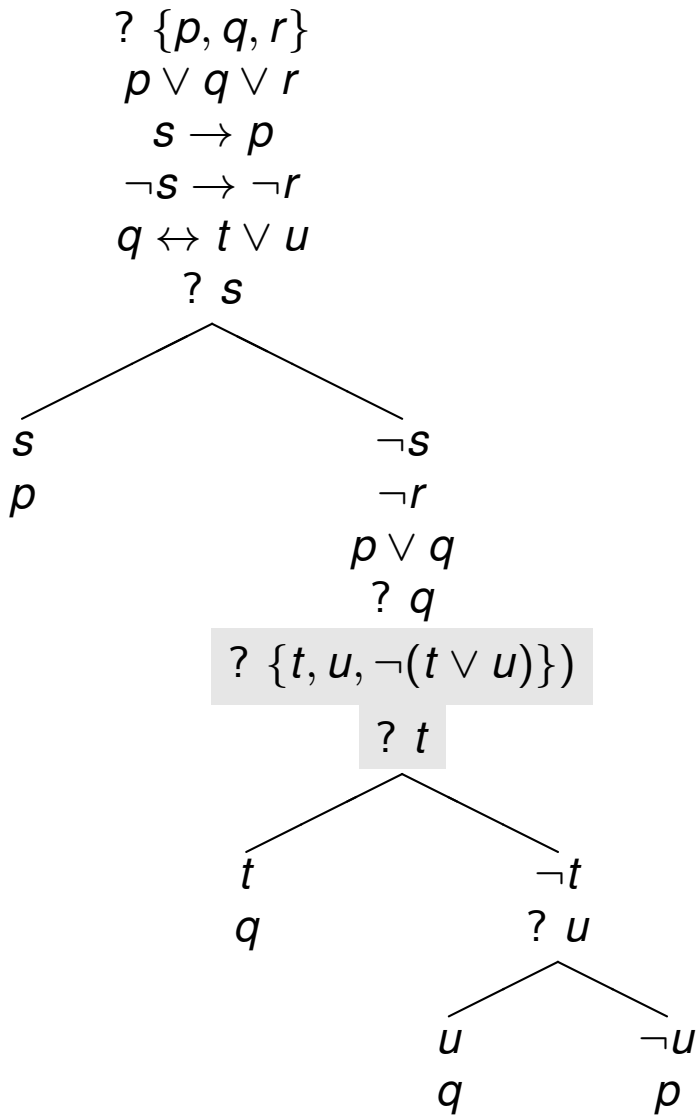


Figure: Erotetic implications used.

- $\mathbf{Im}(?\ \{t, u, \neg(t \vee u)\}, ?\ t)$
- $Y_t = \{t\}$
- $Y_{\neg t} = \{u, \neg(t \vee u)\}$

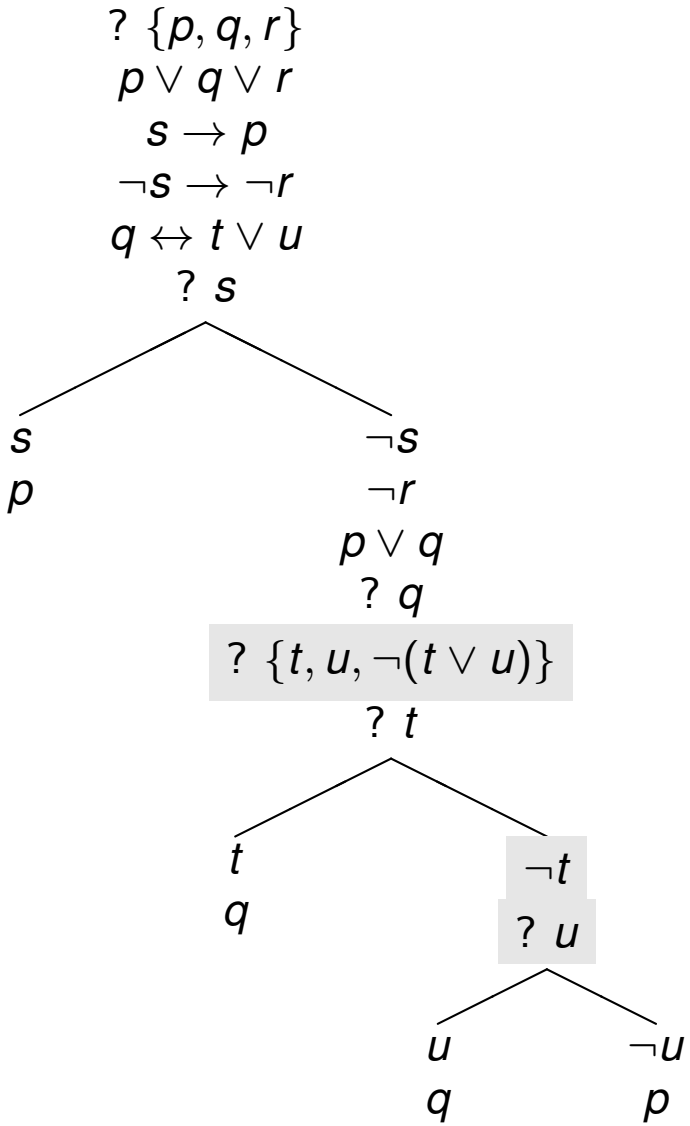


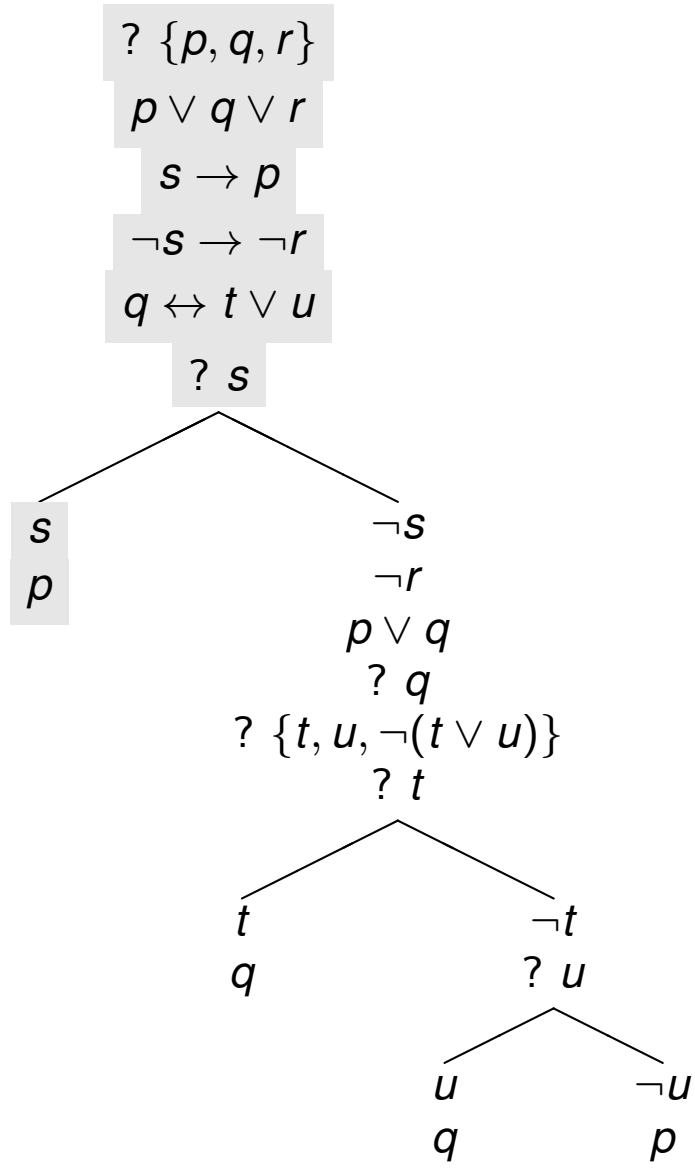
Figure: Erotetic implications used.

- $\mathbf{Im}(? \{t, u, \neg(t \vee u)\}, \neg t, ? u)$
- $Y_u = \{u\}$
- $Y_{\neg u} = \{\neg(t \vee u)\}$

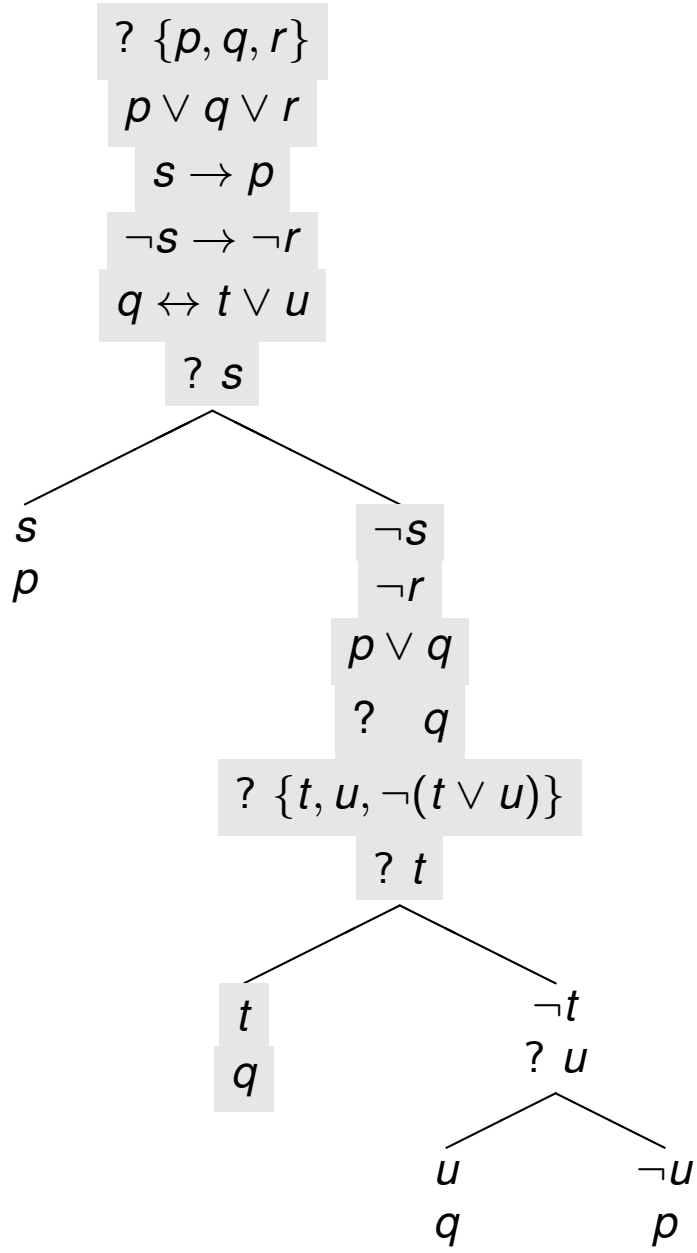
Facts

- All the questions (with the exception of the first one) are erotetically implied.
- Some implied questions are immediately succeeded by direct answers to them.
- However, there are questions that are immediately succeeded with questions.
- The scenario comprises four *paths*.

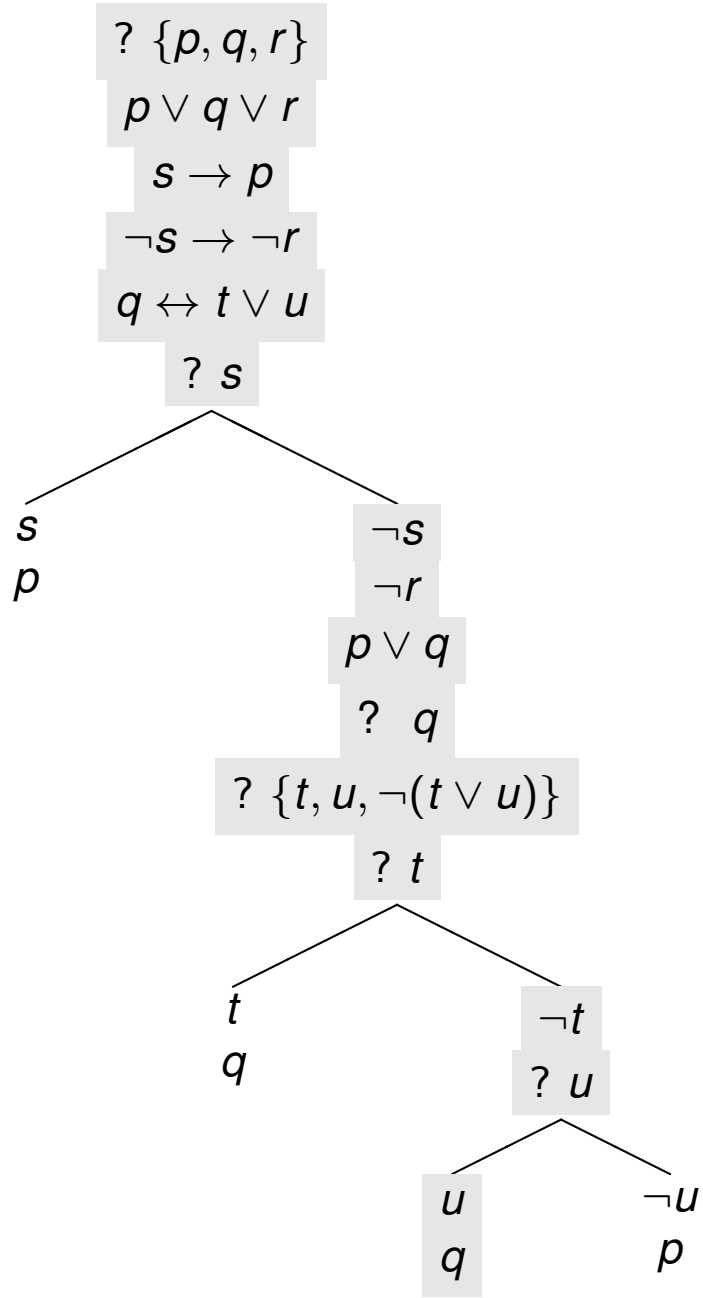
First path


? $\{p, q, r\}$
$p \vee q \vee r$
$s \rightarrow p$
$\neg s \rightarrow \neg r$
$q \leftrightarrow t \vee u$
? s
s
p
$\neg s$
$\neg r$
$p \vee q$
? q
? $\{t, u, \neg(t \vee u)\}$
? t
t
q
$\neg t$
? u
u
q
$\neg u$
p

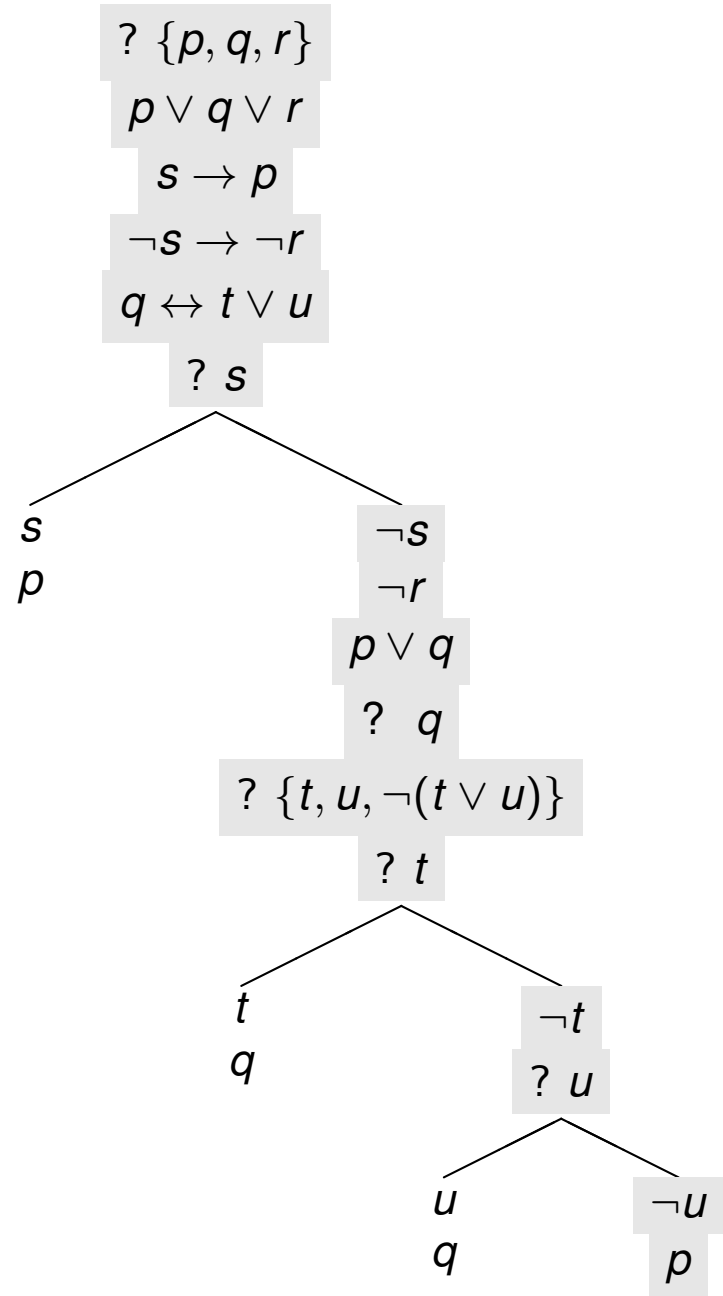
Second path



Third path


? $\{p, q, r\}$
$p \vee q \vee r$
$s \rightarrow p$
$\neg s \rightarrow \neg r$
$q \leftrightarrow t \vee u$
? s
s
p
$\neg s$
$\neg r$
$p \vee q$
? q
? $\{t, u, \neg(t \vee u)\}$
? t
t
q
$\neg t$
? u
u
q
$\neg u$
p

Fourth path


? $\{p, q, r\}$
$p \vee q \vee r$
$s \rightarrow p$
$\neg s \rightarrow \neg r$
$q \leftrightarrow t \vee u$
? s
s
p
$\neg s$
$\neg r$
$p \vee q$
? q
? $\{t, u, \neg(t \vee u)\}$
? t
t
q
$\neg t$
? u
u
q
$\neg u$
p

Facts

- Each path leads to (i.e. ends with) a direct answer to the principal question.
- All the d-wffs which occur on a path that are neither initial premises nor direct answers to questions (that immediately precede them on the path) are entailed by some d-wffs which occur earlier on the path.
- **Terminology**: *wffs* are d-wffs and e-wffs/questions.

$s \to p, s \models p$

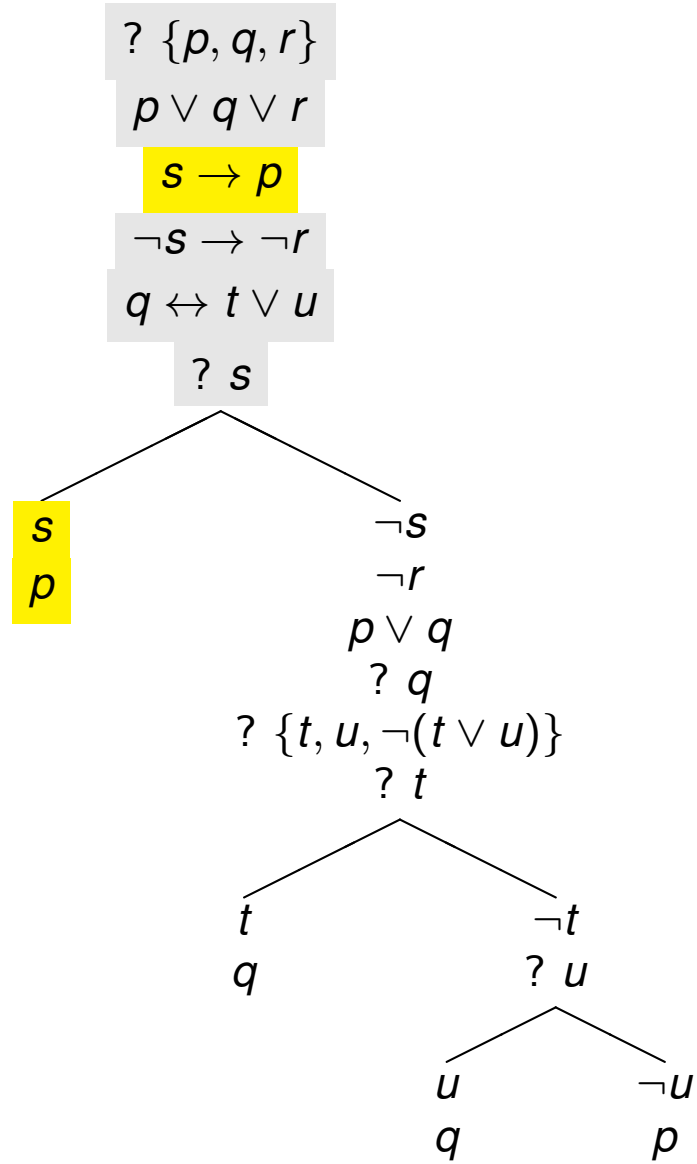


Figure: Entailment: first path.

$\neg s \to \neg r, \neg s \models \neg r$

$p \vee q \vee r, \neg r \models p \vee q$

$q \leftrightarrow t \vee u, t \models q$

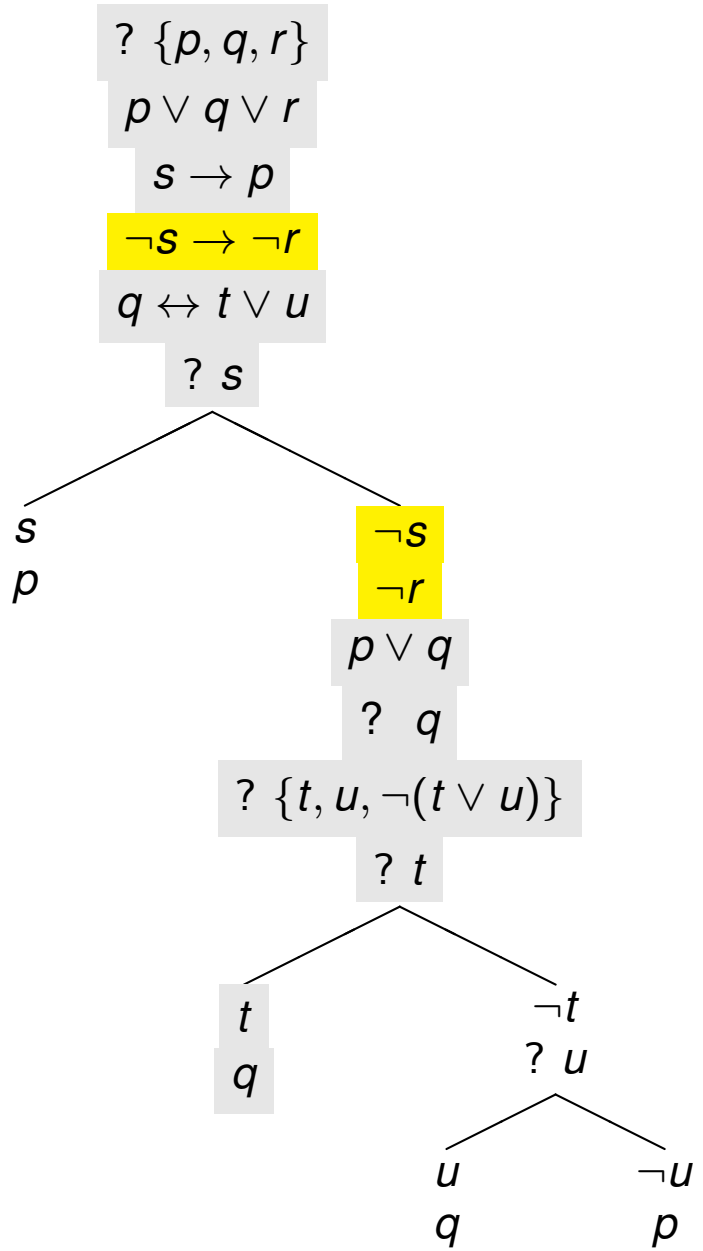


Figure: Entailments: second path

$\neg s \to \neg r, \neg s \models \neg r$

$p \vee q \vee r, \neg r \models p \vee q$

$q \leftrightarrow t \vee u, t \models q$

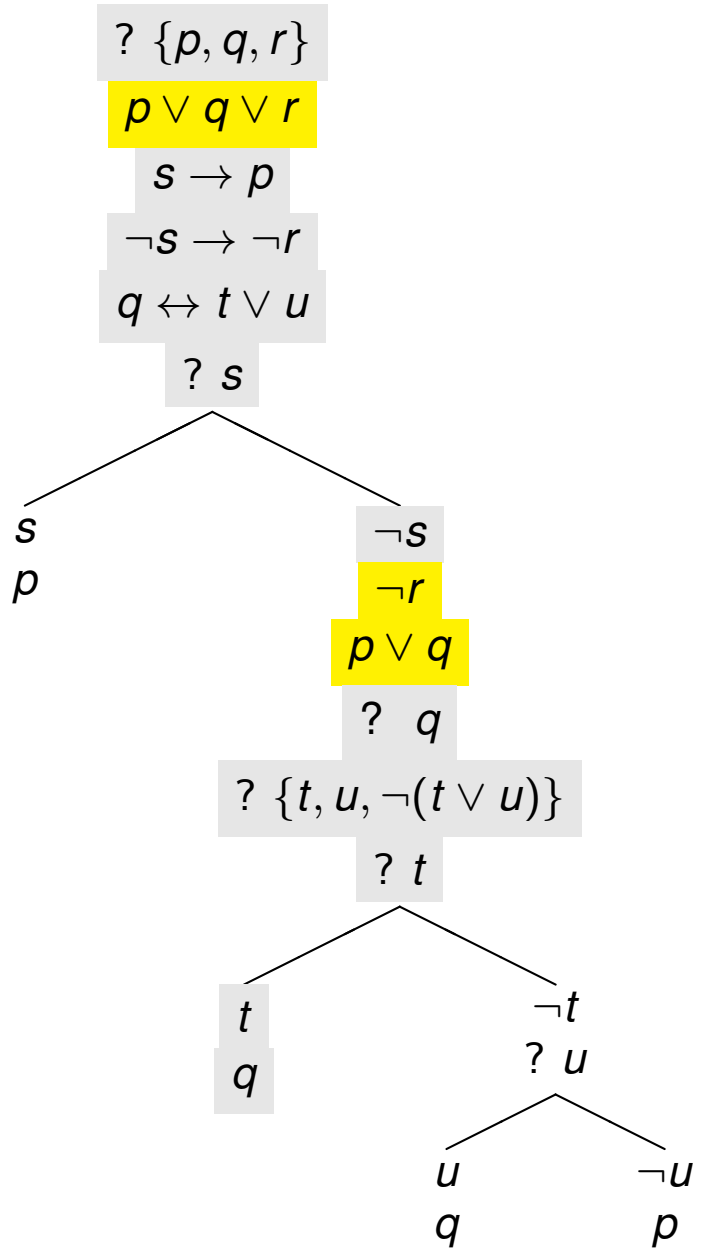


Figure: Entailments: second path

$\neg s \to \neg r, \neg s \models \neg r$

$p \vee q \vee r, \neg r \models p \vee q$

$q \leftrightarrow t \vee u, t \models q$

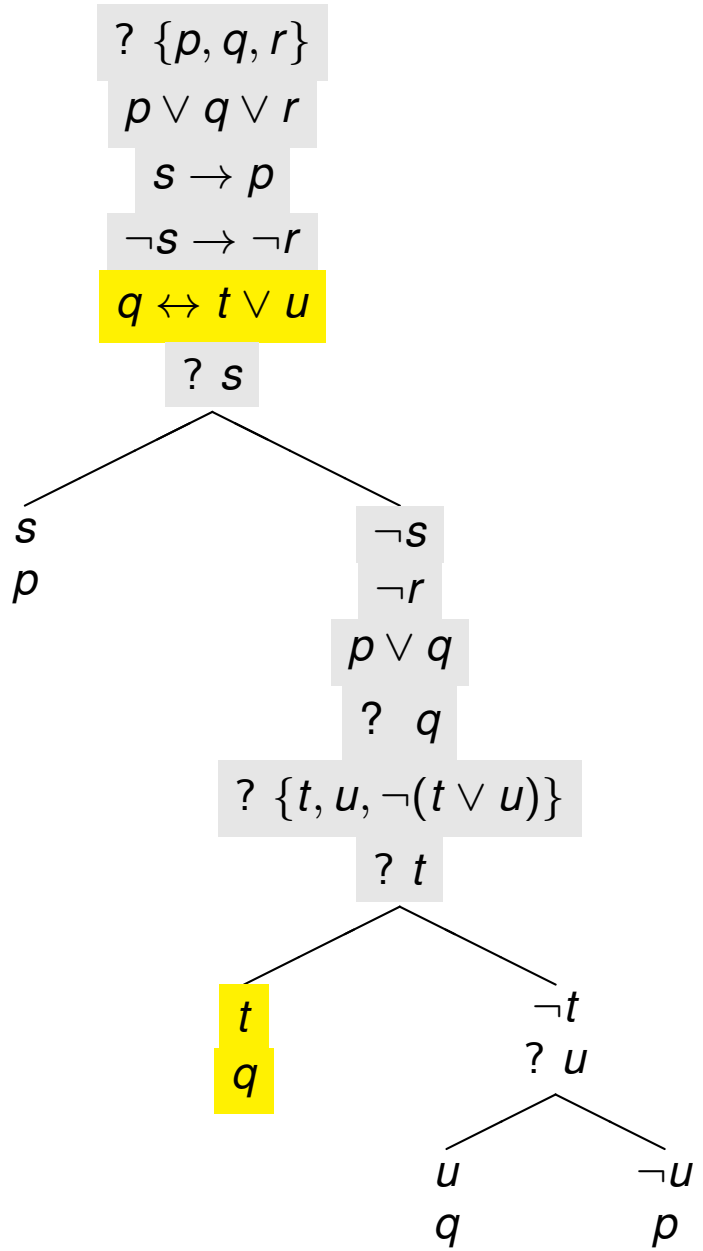


Figure: Entailments: second path

$\neg s \to \neg r, \neg s \models \neg r$

$p \vee q \vee r, \neg r \models p \vee q$

$q \leftrightarrow t \vee u, u \models q$

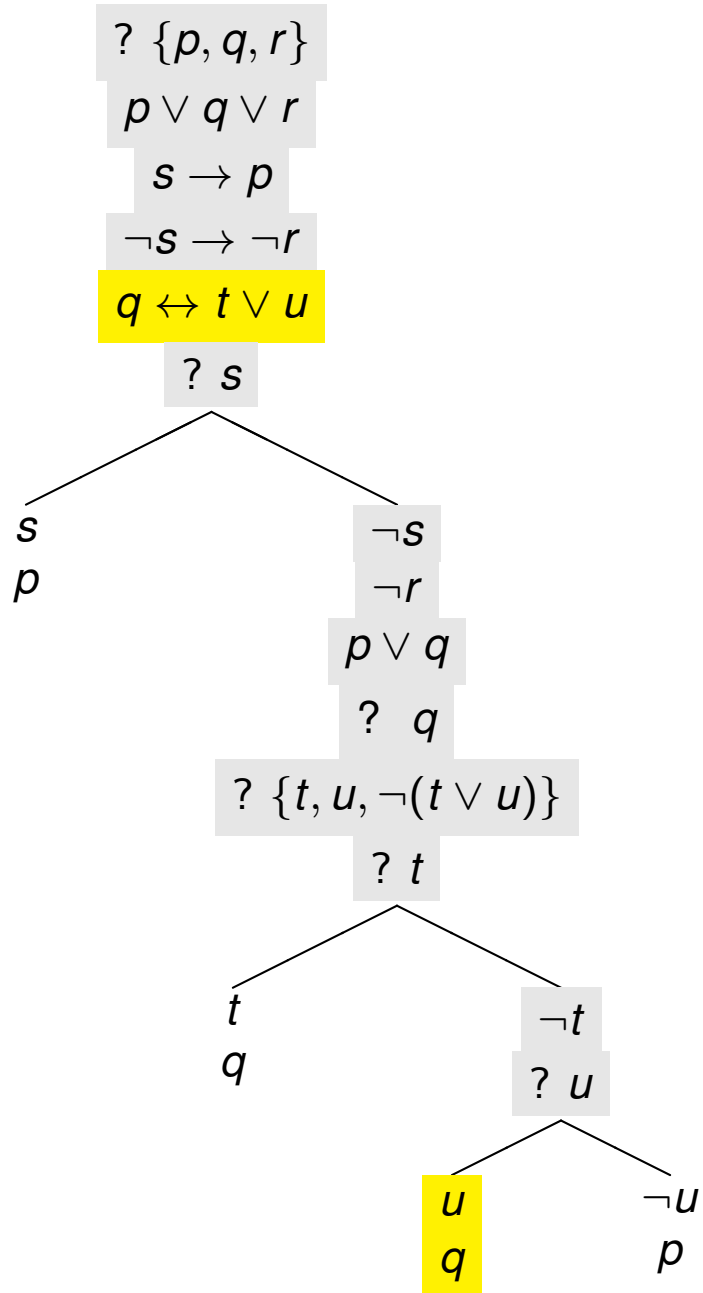


Figure: Entailments: third path.

$\neg s \to \neg r, \neg s \models \neg r$

$p \vee q \vee r, \neg r \models p \vee q$

$q \leftrightarrow t \vee u, p \vee q, \neg t, \neg u \models p$

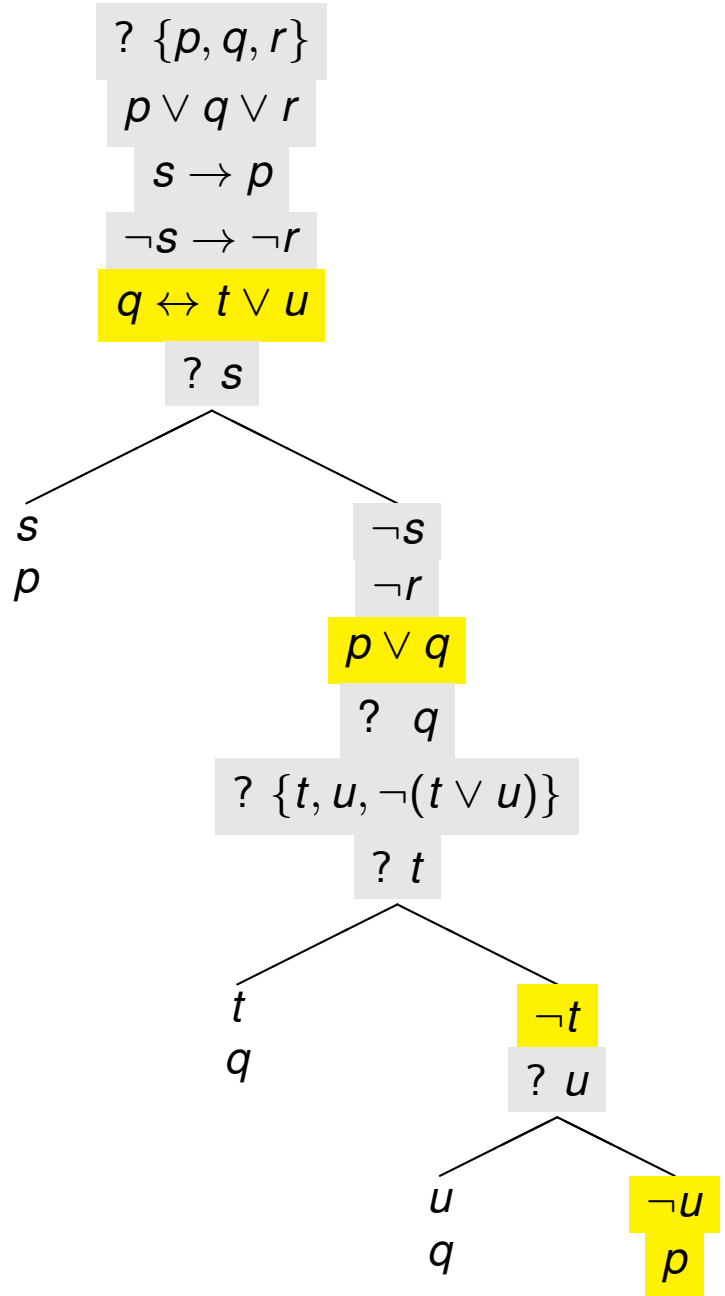


Figure: Entailments: fourth path.

An auxiliary concept: e-derivation

Definition (*E-derivation*)

A finite sequence $\mathbf{s} = \mathrm{s}_1, \ldots, \mathrm{s}_n$ of wffs is an erotetic derivation (e-derivation for short) of a direct answer A to question Q on the basis of a set of d-wffs X iff $\mathrm{s}_1 = Q$, $\mathrm{s}_n = A$, and the following conditions hold:

(1) for each question s_k of Σ such that $k > 1$:

- (a) $\mathbf{d}\mathrm{s}_k \neq \mathbf{d}Q$,
- (b) s_k is implied by a certain question s_j which precedes s_k in s on the basis of the empty set, or on the basis of a set of d-wffs such that each element of this set precedes s_k in s, and
- (c) s_{k+1} is either a direct answer to s_k or a question;

The definition continued. Queries

Definition

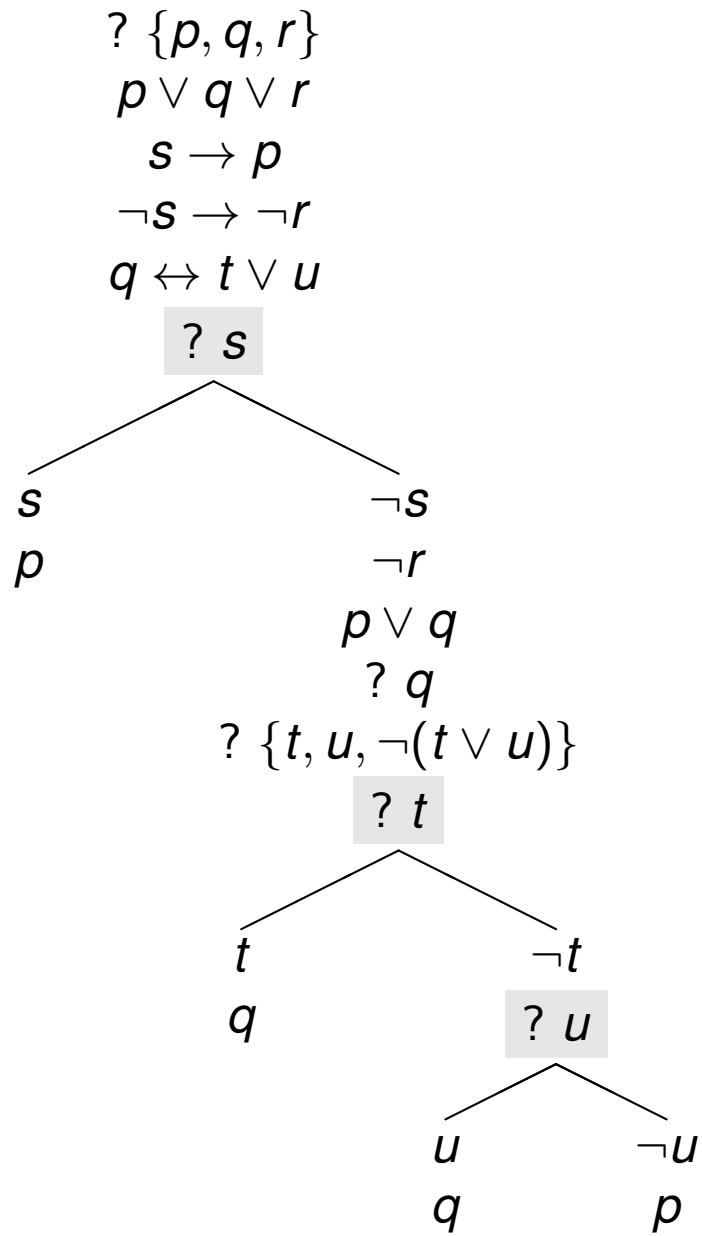
(2) for each d-wff s_i of **s**:

- (a) $s_i \in X$, or
- (b) s_i is a direct answer to s_{i-1}, where $s_{i-1} \neq Q$, or
- (c) s_i is entailed by a certain set of d-wffs such that each element of this set precedes s_i in **s**;

Definition (*Query of e-derivation*)

A term s_k (where $1 < k < n$) of an e-derivation $\mathbf{s} = s_1, \ldots, s_n$ is a query of **s** if s_k is a question and s_{k+1} is a direct answer to s_k.

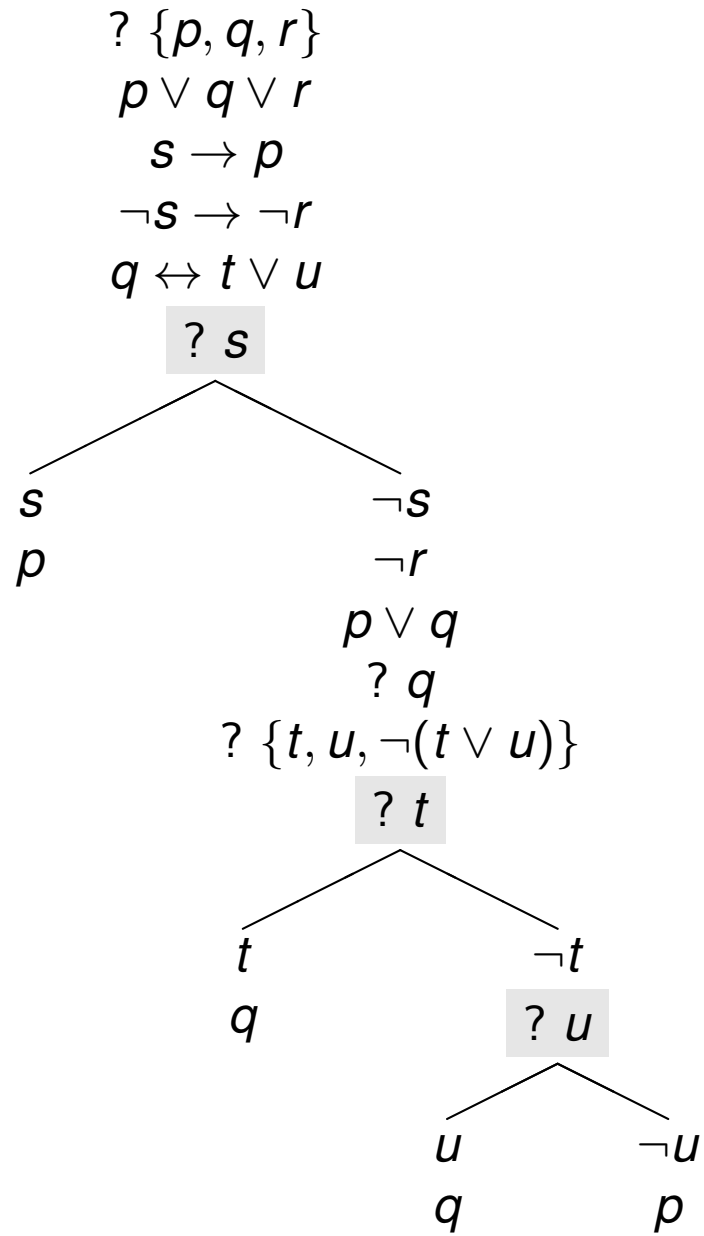
Queries



Towards a definition of e-scenarios

- An e-scenario for a question Q relative to a set of d-wffs X is a set of e-derivations of direct answers to Q on the basis of X.
- However, not *any* set of this kind!
- The following intuitive conditions have to be fulfilled:
  1. if a question is "answered" within an e-scenario in one way, then this question is "answered" in all the other possible ways (on related paths; the "possible ways" are determined by the set of direct answers to the question);

     $\Longrightarrow$ NO DEAD ENDS!
  2. only questions can perform the function of "branching points/nodes" of e-scenarios.


? $\{p, q, r\}$
$p \vee q \vee r$
$s \to p$
$\neg s \to \neg r$
$q \leftrightarrow t \vee u$
? s
s
p
$\neg s$
$\neg r$
$p \vee q$
? q
? $\{t, u, \neg(t \vee u)\}$
? t
t
q
$\neg t$
? u
u
q
$\neg u$
p

Definition of e-scenarios

Definition (*E-scenario*)

A finite family Σ of sequences of wffs is an erotetic search scenario (e-scenario for short) for a question Q relative to a set of d-wffs X iff each element of Ψ is an e-derivation of a direct answer to Q on the basis of X and the following conditions hold:

(1) $\mathbf{d}Q \cap X = \emptyset$;

(2) Σ contains at least two elements;

The definition continued

Definition

(3) for each element $\mathbf{s} = \mathrm{s}_1, \ldots, \mathrm{s}_n$ of Σ, for each index k, where $1 \leq k < n$:

- (a) if s_k is a question and s_{k+1} is a direct answer to s_k, then for each direct answer B to s_k: the family Σ contains a certain e-derivation $\mathbf{s}^* = \mathrm{s}_1^*, \mathrm{s}_2^*, \ldots, \mathrm{s}_m^*$ such that $\mathrm{s}_j = \mathrm{s}_j^*$ for $j = 1, \ldots, k$, and $\mathrm{s}_{k+1}^* = B$;
- (b) if s_k is a d-wff, or s_k is a question and s_{k+1} is not a direct answer to s_k, then for each e-derivation $\mathbf{s}^* = \mathrm{s}_1^*, \mathrm{s}_2^*, \ldots, \mathrm{s}_m^*$ in Σ such that $\mathrm{s}_j = \mathrm{s}_j^*$ for $j = 1, \ldots, k$ we have $\mathrm{s}_{k+1} = \mathrm{s}_{k+1}^*$.

Queries and non-queries of e-scenarios

Definition (*Query of e-scenario*)

A query of an e-scenario is a query of a path of the e-scenario.

- A quick look at definitions gives the following: a query of an e-scenario is simply the first element of a question-answer pair that occurs on a path of the e-scenario, where the question is an auxiliary one and the answer immediately succeeds the question. Thus each query is a question, but e-scenarios can involve auxiliary questions that are not queries. If such questions occur, they are indispensable, but, roughly, “are not supposed to be asked and answered”.
- Auxiliary questions that are not queries enable IEL-based introduction of queries.

The most useful property

Theorem (*Golden Path Theorem*)

Let Σ be an e-scenario for a question Q relative to a set of d-wffs X. Assume that Q is sound in an admissible partition P*, and all the d-wffs in X are true in* P*. The e-scenario Σ contains at least one path* **s** *such that:*

(1) *each d-wff of* **s** *is true in* P*,*

(2) *each question of* **s** *is sound in* P*, and*

(3) **s** *leads to a direct answer to Q which is true in* P*.*

A pragmatic account of e-scenarios

- E-scenarios are abstract entities defined in terms of IEL. But, looking from the pragmatic point of view, an e-scenario provides *conditional instructions* which tell what auxiliary questions should be asked and when they should be asked.
- Queries of e-scenarios can be viewed as requests for information. An e-scenario shows what is the next advisable query if the information request of the previous query has been satisfied in such–and–such way.
- What is important, an e-scenario does this with regard to every possible way of satisfying the request, where the ways are determined by direct answers to the question which functions as a query.
- Moreover, an e-scenario behaves in this manner in the case of every query of the e-scenario.

E-scenarios and problem-solving

- When faced with a problem-solving task, it is ***advisable*** to build a PRELIMINARY E-SCENARIO for the question that expresses the problem just considered.
- Items of information which are supposed to be relevant to the case can be used as the background X, and declarative premises (but not necessarily answers to queries) are successively taken from X if/when needed.
- In practice, it is wise to start with a relatively simple initial e-scenario.

E-scenarios and problem-solving

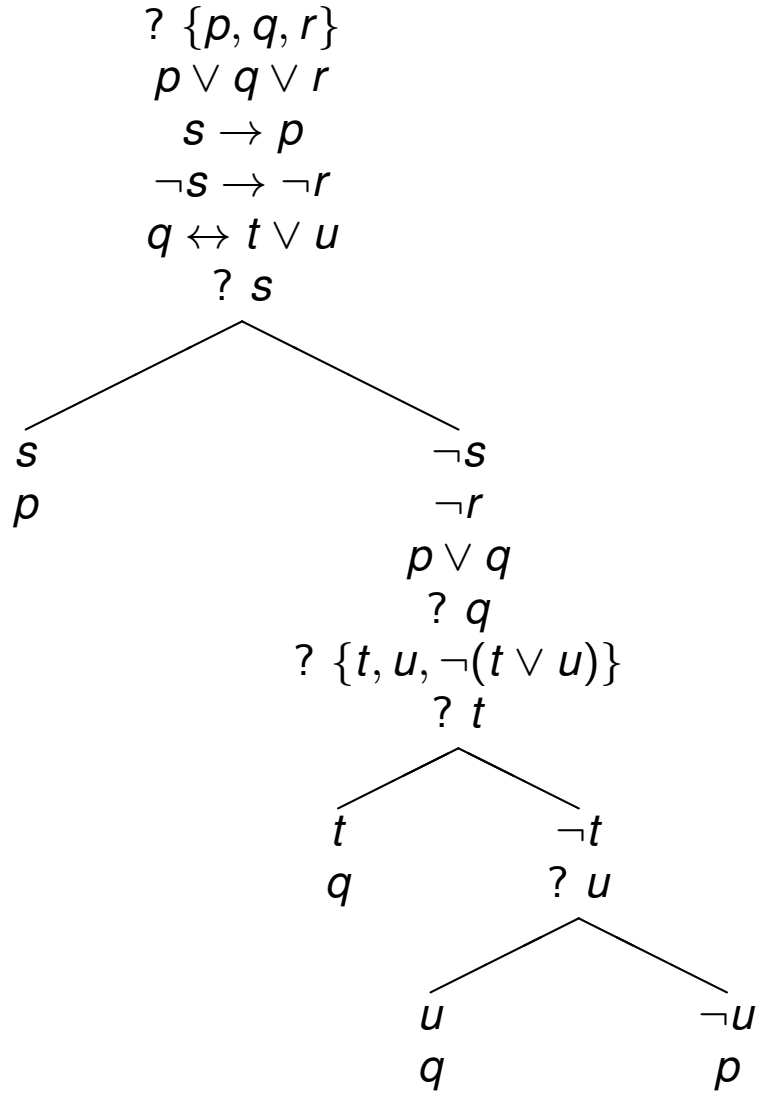
- Questions differ as to “costs” of obtaining answers, where the costs are co-determined by such factors as the amounts of effort and/or time needed for obtaining an answer, data costs and/or charges, etc. It is advisable to use as queries only questions which are less “costly” than the principal one.
- There must be good reasons to believe that answers to queries are available by accessible means.
- A preliminary e-scenario is, in a sense, superfluous. The execution of the scenario is supposed to proceed from top to bottom: one attempts to resolve the first query and then, depending on the answer received, moves to the query recommended by the e-scenario as the next one, and so forth.
- Instructions based on answers different from those which have been actually received (or hypothetically assumed) will not be activated.

E-scenarios and problem-solving

- A success in resolving a query amounts to *contraction* of the e-scenario just executed (contraction by the answer to the query obtained).
- But a query hoped to be resolvable at a reasonable cost may occur to be unanswerable by available means. When this happens, an advisable way out is to use the mechanism of *embedding*.
- Both contraction and embedding can be defined in exact terms. I skip the definitions, however.
- Let me illustrate the above claims by examples.

Contraction

- Consider the exemplary e-scenario:

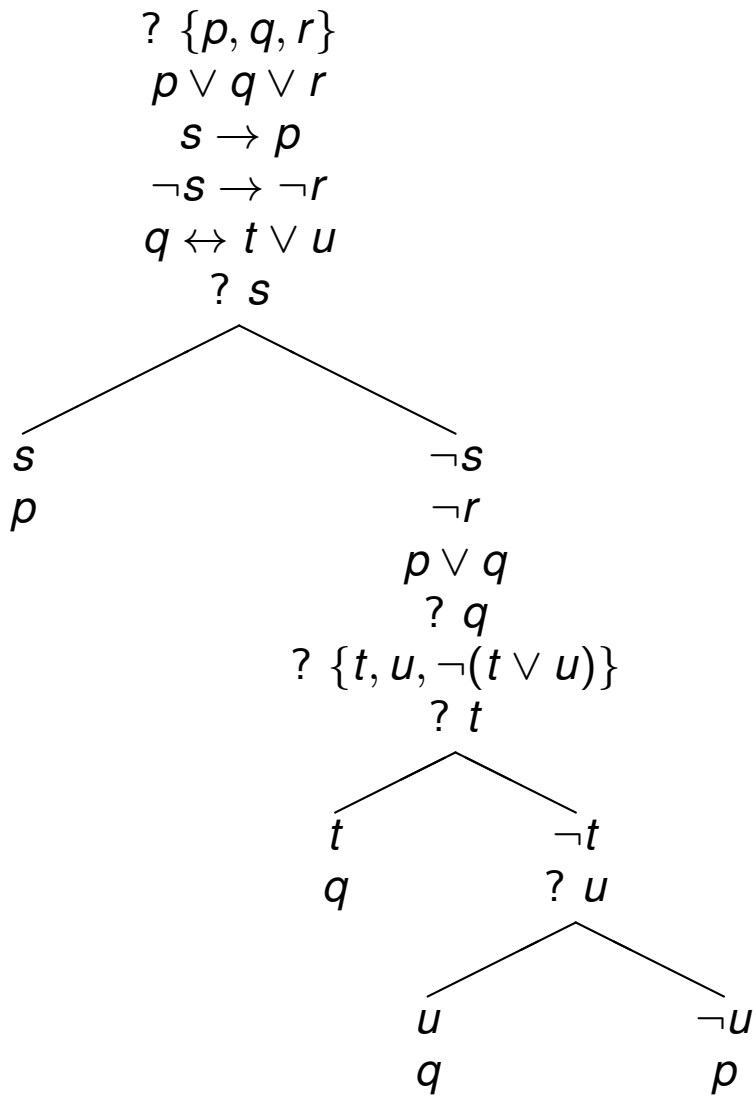


Contraction

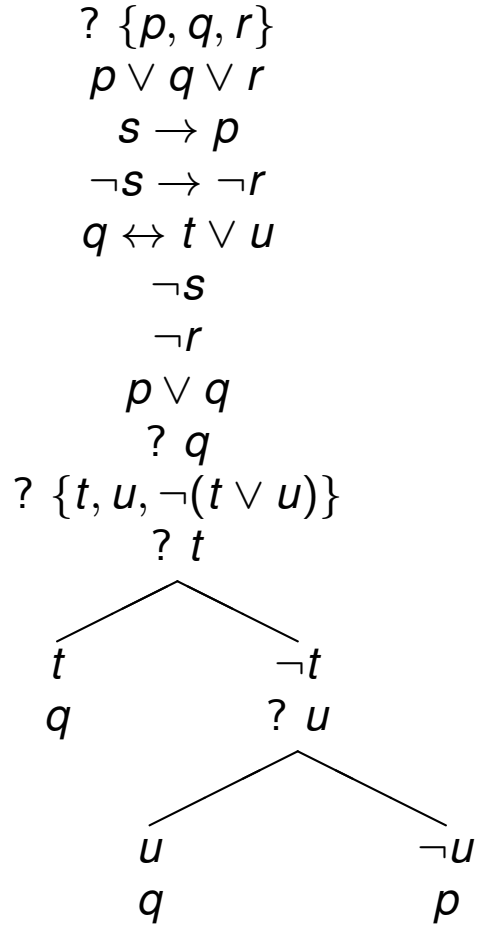
- Suppose that the answer s to the first query $?\ s$ has occurred to be the case. The e-scenario contracts to:

$$
\begin{gathered}
?\ \{p, q, r\} \\
p \vee q \vee r \\
s \rightarrow p \\
\neg s \rightarrow \neg r \\
q \leftrightarrow t \vee u \\
s \\
p
\end{gathered}
$$

- What we have got is an e-derivation of the answer p to the principal question. No further querying is needed.
- Now suppose that the answer $\neg s$ to the query $?\ s$ has been obtained.



We contract by the answer $\neg s$ to question/query ? s. We get:



Contraction Theorem

- We contact by a (direct) answer to a query of a path.
- The result need not be an e-scenario.

Theorem (Contraction Theorem)

Let Σ be an e-scenario for a question Q relative to a set of d-wffs X, let Q^* be a query of Σ and A be a direct answer to Q^*. The result of contraction of Σ by A is an e-scenario for Q relative to $X \cup \{A\}$ if

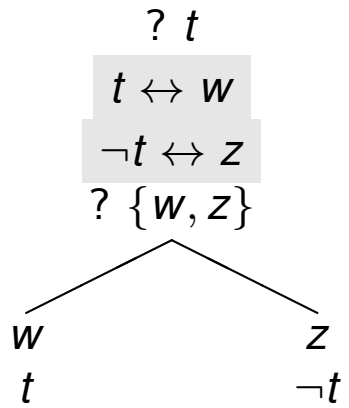
(1) *$A \notin \mathbf{d}Q$ and*

(2) *it still involves at least one query.*

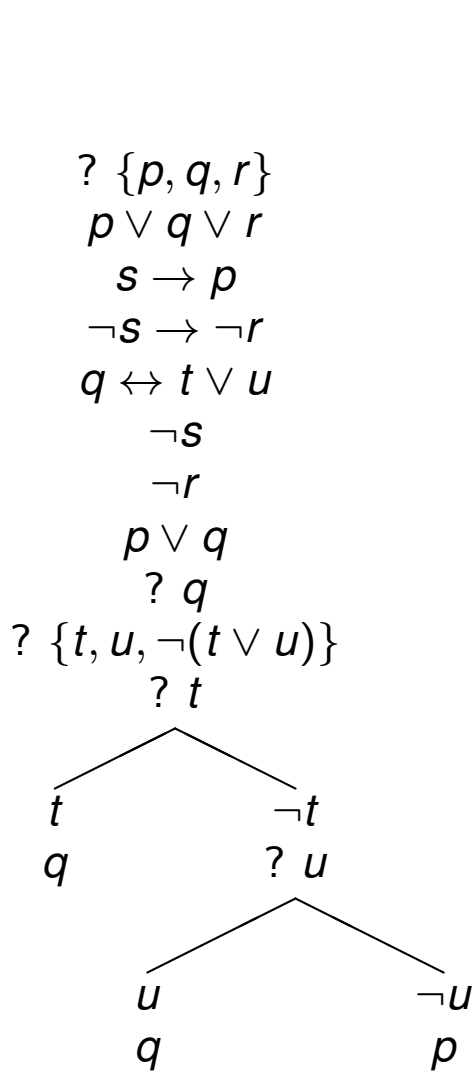
- If the result is an e-scenario, we execute it starting from its first query; otherwise either an answer to the principal question is "found" or we arrive at an e-derivation of an answer to the question.

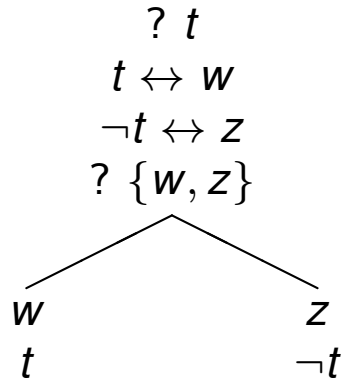
Embedding

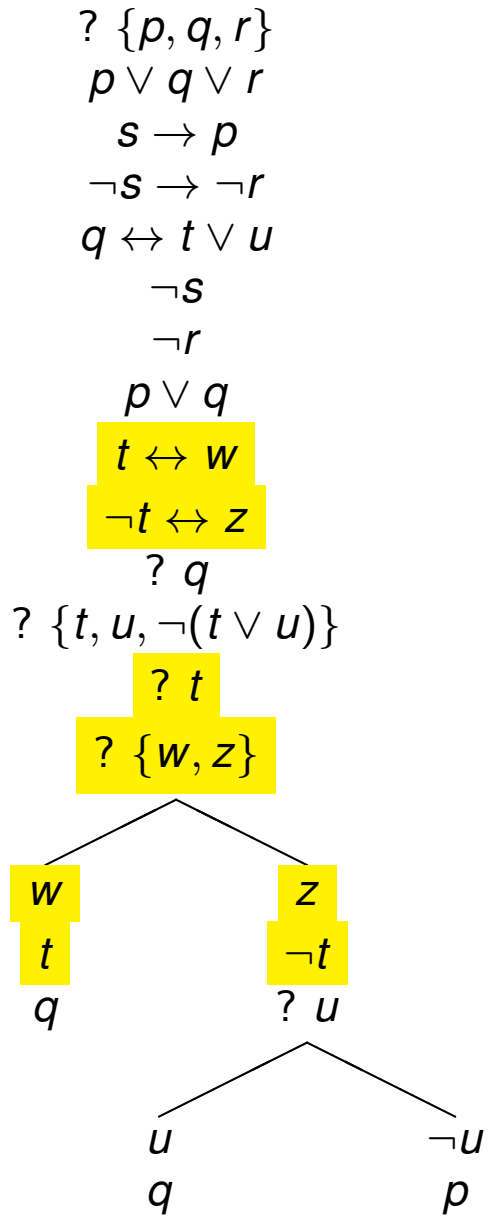
- When one encounters problems with resolving a query, a possible rescue option is to *embed* into the e-scenario just executed a complete e-scenario for the troublemaking query.
- An e-scenario is complete iff its leaves are labelled by all the direct answers to the principal question (there is no direct answer which does not label any leaf).
- For example, suppose that the query ? t is a troublemaker. Suppose further that there are good reasons to believe that $t \leftrightarrow w$ and $\neg t \leftrightarrow z$ hold.
- The following is a complete e-scenario for the question/query ? t relative to the set $\{t \leftrightarrow w, \neg t \leftrightarrow z\}$:



An example of embedding







Embedding

- We embed with respect to (an occurrence of) a query.
- What is embedded is a *complete* e-scenario for the question which is the query.
- In this particular example the embedded e-scenario has only one query. This is not a rule, however.
- If the embedded e-scenario has a non-empty initial declarative segment, this segment is placed either just before the query considered, or – when the query is preceded by a sequence of question – just before the first question of the sequence.
- If the initial declarative segment of the embedded e-scenario is empty, the above complication does not arise.
- Embedding is an operation that can be defined in general terms.
- However, there is no enough time for presenting the definition.

Embedding Theorem

- Embedding produces a new e-scenario only if some conditions are met.
- "EMB($\Delta/\mathbf{s}, \mathrm{s}_k, \Sigma$)" reads "the result of embedding Δ into Σ with respect to query s_k of path $\mathbf{s}$ of Σ".

Theorem

Let Σ be an e-scenario for a question Q relative to a set of d-wffs X, and let s_k be a query of a path $\mathbf{s}$ of Σ. Let Δ be a complete e-scenario for question s_k relative to a set of d-wffs Y. EMB($\Delta/\mathbf{s}, \mathrm{s}_k, \Sigma$) *is an e-scenario for Q relative to $X \cup Y$ if the following conditions hold:*

1. $Y \cap \mathrm{d}Q = \emptyset$, *and*
2. *for each question Q^* of Δ : $\mathrm{d}Q^* \neq \mathrm{d}Q$.*

Remarks

- Embedding can help, but there is no warranty of a success. What if some of the “new” queries remain unresolved?
- There are two rescue options possible.
- The first is to *backtrack* the already performed embedding and then embed another e-scenario for the troublemaking query.
- The second amounts to performing further embedding(s) without backtracking.
- Let me add that persistent failures in resolving a query need not be tantamount to a complete failure. One can contract by an *only hypothetically accepted* answer to a troublemaking query and then try to proceed further.
- If one successfully proceeds with the consecutive queries recommended, the outcome carries information of the following kind: *A*, the endpoint, provides a right solution to the initial problem *on condition that* the hypothetically accepted answers to the troublemaking queries or query are right.
- The added value of such outcome lies in an identification of knowledge gaps.

Remarks

- At each stage of the process sketched above, with the exception of the last one, an e-scenario is executed.
- The consecutive e-scenario is dependent upon the result of execution of the previous one.
- Note that it is the preliminary e-scenario that is being transformed. As a consequence, the following desirable property is retained: *each path of an intermediate scenario leads to an answer to the principal question.*
- Thus the process as a whole is goal-directed, and the sub-goals are processed in a goal-directed way.
- Recall that any e-scenario has the golden path property and describes a search plan with no “dead ends”: the plan copes with any direct answer to a query.

Remarks

- A warning is in order.
- In the last stage of the process a solution to the initial problem emerges as the endpoint of the e-derivation arrived at.
- The solution is either a direct answer to the only query of the derivation or is entailed by some preceding d-wffs of the derivation.
- But it cannot be said that once the process is successfully completed, a “true” or “right” solution is already found.
- Mere entailment is not enough: there must be good reasons to believe that the premises involved and/or the answer to the query are true.

Remarks

- A preliminary e-scenario can be transformed in reaction to a success/failure in resolving a query.
- However, both embedding and contraction are formal operations which can be performed on a preliminary e-scenario prior to its execution.
- It is a rational strategy to estimate in advance both the chances of answering queries and costs of answering them.
- If the former are low and/or the latter are high, there is a possibility of "fine-tuning" the preliminary e-scenario by embedding.
- But neither fine-tuning nor mere transformations of e-scenarios are sufficient to solve a problem: we need answers to queries.

A note on applications

- The concept of e-scenario was introduced (cf. Wiśniewski [Wiś03], [Wiś01]) order to model some aspects of effective problem solving.
- It's applicability, however, has occurred to be wider.
- E-scenarios are useful tools in the area of cooperative answering, in a modelling of interrogator's hidden agenda, and in an analysis of the Turing Test (cf. Łupkowski [Łu10a], Urbański & Łupkowski [UL10], Łupkowski [Łu10b], [Łu11])
- Some aspects of question answering can be modelled by means of e-scenarios as well (cf. Łupkowski [Łu12], Łupkowski [Łu13], Wiśniewski [Wiś13])
- There exist proof methods based on e-scenarios (cf. Urbański [Urb01a], [Urb01b], [Urb02a], [Urb02b], and Wiśniewski [Wiś04]).

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