Socratic Proofs

Andrzej Wiśniewski

Department of Logic and Cognitive Science
Institute of Psychology
Adam Mickiewicz University
Poznań, Poland
Andrzej.Wisniewski@amu.edu.pl

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- Is it possible to solve a (logical) problem by pure questioning? IEL says "Yes".
- We build ***erotetic calculi***.
- Rules of these calculi operate on questions only, transforming a question into a further question.
- A *Socratic transformation* of a question *Q* about entailment/ derivability/ theoremhood is a sequence of questions that begins with *Q*. Each consecutive term results by applying a rule of the calculi to its immediate predecessor.
- There are successful and unsuccessful transformations; a successful transformation ends with a question of a required 'final' form, in a sense a rhetorical one (the details depend on the logic under consideration). A successful transformation is a *Socratic proof*.

- The rules are designed in such a way that once a successful transformation is accomplished, the affirmative answer to the initial question is found and *there is no need for performing any further deductive moves.*
- Moreover, each step of a Socratic transformation is a valid erotetic inference, in the sense of **IEL**.
- So far erotetic calculi have been developed for Classical Logic, some paraconsistent propositional logics and normal modal propositional logics.
- In each case completeness theorems are proven; in some cases decision procedures are described.

We will illustrate this by the simplest possible example: an erotetic calculus for CPC, $\boldsymbol{E}^{\mathrm{CPC}}$.

An erotetic sequent language: syntax

We take the language of CPC as the point of departure. For simplicity, we assume that the biconditional $\leftrightarrow$ is not a primary connective of the language.

The vocabulary of the new language needed, $\boldsymbol{L_3}$, comprises the vocabulary of the language of CPC, and the following signs: $\vdash$, ?, ng ($\boldsymbol{L_3}$-negation), & ($\boldsymbol{L_3}$-conjunction), as well as the comma ',' .

By a CPC-*sequent* we mean an expression of the form:

$$S \vdash A$$

where A is a (single!) CPC-wff, and S is a finite sequence of CPC-wffs (the empty sequence included).

Comment: Let us stress that we consider single-conclusioned sequents only. This is intended; in what follows we will show why.

The CPC-sequents perform the role of *atomic d-wffs* of $\boldsymbol{L_3}$.

An erotetic sequent language: syntax

We use Greek lower-case letters ϕ, φ, with subscripts if needed, as metalinguistic variables for atomic d-wffs of $\boldsymbol{L_3}$.

Compound d-wffs of $\boldsymbol{L_3}$ are built up from atomic d-wffs by means of & and/or ng; the construction is standard. Observe that & and ng never occur inside atomic d-wffs; one should not confuse them with $\wedge$ and $\neg$.

Questions of $\boldsymbol{L_3}$ have the form:

$$? \, (\Phi)$$

where Φ is a non-empty and finite sequence of atomic d-wffs of $\boldsymbol{L_3}$.

Let $\Phi = \langle \phi_1, \ldots, \phi_n \rangle$, and let $Q = ? \, (\Phi)$. The following:

$$\phi_1 \,\&\, (\phi_2 \,\&\, \ldots \,\&\, (\phi_{n-1} \,\&\, \phi_n) \ldots)$$

$$\underline{\text{ng}}(\phi_1 \,\&\, (\phi_2 \,\&\, \ldots \,\&\, (\phi_{n-1} \,\&\, \phi_n) \ldots))$$

are the *affirmative answer* to Q and the *negative answer* to Q, respectively. The set of direct answers to a question of $\boldsymbol{L_3}$ is made up of the affirmative answer and the negative answer, exclusively.

A (CPC) sequent, $S \vdash A$, is CPC-*valid* iff A is true under each CPC-valuation under which all the terms of the sequence S are true. Intuitively, a question $\boldsymbol{L_3}$ of asks about *joint validity* of all of the sequents that occur in the question; in the present case CPC-validity.

$$? \ (A_{1_1},\ldots, A_{1_n} \vdash B_1;\ A_{2_1},\ldots, A_{2_m} \vdash B_2;\ \ldots\ ;\ A_{t_1},\ldots, A_{t_k} \vdash B_t)$$

Is it the case that: $A_{1_1},\ldots, A_{1_n} \vdash B_1$ *is valid* *and* $A_{2_1},\ldots, A_{2_m} \vdash B_2$ *is valid and ... and* $A_{t_1},\ldots, A_{t_k} \vdash B_t$ *is valid ?*

Due to the completeness theorem for CPC, “CPC-valid” can also be replaced by “CPC-derivable”.

When $t = 1$, we get:

Is it the case that $A_1, \ldots, A_n \vdash B$ *is valid?*

and when $n = 0$, we have:

Is it the case that B *is valid?*

Notation

We adopt the Smullyan's α, β–notation, but with small adjustments.

α	α_1	α_2		β	β_1	β_2	β_1^*
$B \wedge C$	B	C		$\neg(B \wedge C)$	$\neg B$	$\neg C$	B
$\neg(B \vee C)$	$\neg B$	$\neg C$		$B \vee C$	B	C	$\neg B$
$\neg(B \to C)$	B	$\neg C$		$B \to C$	$\neg B$	C	B

Moreover:

- S, W, Z, V stand for finite sequences of CPC-wffs
- $S\,{'}A$ stands for the concatenation of S and $\langle A \rangle$
- Φ, Ψ stand for finite sequences of atomic d-wffs of $\boldsymbol{L_3}$
- $\Phi; S \vdash A$ stands for the concatenation of Φ and $\langle S \vdash A \rangle$

Definition: A partition $\boldsymbol{P} = \langle \boldsymbol{T_P}, \boldsymbol{U_P} \rangle$ of $\boldsymbol{L_3}$ is *admissible* iff the following conditions hold:

(i) "$S \vdash \alpha$" $\in \boldsymbol{T_P}$ *iff* "$S \vdash \alpha_1$" $\in \boldsymbol{T_P}$ and "$S \vdash \alpha_2$" $\in \boldsymbol{T_P}$,

(ii) "$S\ 'W \vdash \beta$" $\in \boldsymbol{T_P}$ iff "$S\ '\beta_1^*\ 'W \vdash \beta_2$" $\in \boldsymbol{T_P}$,

(iii) "$S\ '\alpha\ 'W \vdash C$" $\in \boldsymbol{T_P}$ iff "$S\ '\alpha_1\ '\alpha_2\ 'W \vdash C$" $\in \boldsymbol{T_P}$,

(iv) "$S\ '\beta\ 'W \vdash C$" $\in \boldsymbol{T_P}$ iff "$S\ '\beta_1\ 'W \vdash C$" $\in \boldsymbol{T_P}$ and "$S\ '\beta_2\ 'W \vdash C$" $\in \boldsymbol{T_P}$,

(v) "$S \vdash \neg\neg A$" $\in \boldsymbol{T_P}$ iff "$S \vdash A$" $\in \boldsymbol{T_P}$,

(vi) "$S\ '\neg\neg A\ 'W \vdash B$" $\in \boldsymbol{T_P}$ iff "$S\ 'A\ 'W \vdash B$" $\in \boldsymbol{T_P}$,

(vii) "$\rho\ \&\ \sigma$" $\in \boldsymbol{T_P}$ iff $\rho \in \boldsymbol{T_P}$ and $\sigma \in \boldsymbol{T_P}$,

(viii) if $\rho \notin \boldsymbol{T_P}$, then "$\underline{\text{ng}}\rho$" $\in \boldsymbol{T_P}$,

(ix) if $\rho \in \boldsymbol{T_P}$, then "$\underline{\text{ng}}\rho$" $\notin \boldsymbol{T_P}$.

The concept of admissible partition is crucial; we are now ready to introduce the remaining semantic concepts for $\boldsymbol{L_3}$. In particular, we have:

Definition: Let ρ and σ be d-wffs of $\boldsymbol{L_3}$. We say that ρ *entails* σ *in* $\boldsymbol{L_3}$ iff there is no admissible partition $\boldsymbol{P} = \langle \boldsymbol{T_P}, \boldsymbol{U_P} \rangle$ of $\boldsymbol{L}_3$ such that $\rho \in \boldsymbol{T_P}$ and $\sigma \in \boldsymbol{U_P}$.

The following additional concept will be needed.

Definition: Let Q and Q^* be questions of $\boldsymbol{L_3}$. Q^* is *positively equipollent* to Q iff the affirmative answers to Q^* and Q entail each other, and the negative answers to Q^* and Q entail each other.

It is easily seen that the following is true:

Lemma: *If Q* is positively equipollent to Q, then* **Im**(*Q, Q**), *that is, Q* is (erotetically) implied by Q.*

$\boldsymbol{E}^{\text{CPC}}$: *an erotetic calculus for CPC*

Let me recall the notational conventions:

α	α_1	α_2		β	β_1	β_2	$\beta_1{}^*$
$B \wedge C$	B	C		$\neg(B \wedge C)$	$\neg B$	$\neg C$	B
$\neg(B \vee C)$	$\neg B$	$\neg C$		$B \vee C$	B	C	$\neg B$
$\neg(B \rightarrow C)$	B	$\neg C$		$B \rightarrow C$	$\neg B$	C	B

S, *W, Z, V* stand for finite sequences of *CPC*-wffs

S'A stands for the concatenation of *S* and <*A*>

Φ, Ψ stand for finite sequences of atomic d-wffs

$\Phi; S \vdash A$ stands for the concatenation of Φ and $\langle S \vdash A \rangle$

$\boldsymbol{E}^{\mathrm{CPC}}$ has only rules that operate on questions:

L$_\alpha$: $$\frac{?\,(\Phi;\ S\,\text{'}\,\alpha\,\text{'}\,W \vdash A;\ \Psi)}{?\,(\Phi;\ S\,\text{'}\,\alpha_1\,\text{'}\,\alpha_2\,\text{'}\,W \vdash A;\ \Psi)}$$

R$_\alpha$: $$\frac{?\,(\Phi;\ S \vdash \alpha;\ \Psi)}{?\,(\Phi;\ S \vdash \alpha_1;\ S \vdash \alpha_2;\ \Psi)}$$

L$_\beta$: $$\frac{?\,(\Phi;\ S\,\text{'}\,\beta\,\text{'}\,W \vdash A;\ \Psi)}{?\,(\Phi;\ S\,\text{'}\,\beta_1\,\text{'}\,W \vdash A;\ S\,\text{'}\,\beta_2\,\text{'}\,W \vdash A;\ \Psi)}$$

R$_\beta$: $$\frac{?\,(\Phi;\ S \vdash \beta;\ \Psi)}{?\,(\Phi;\ S\,\text{'}\,\beta_1{}^{*} \vdash \beta_2;\ \Psi)}$$

L$_{\neg\neg}$: $$\frac{?\,(\Phi;\ S\,\text{'}\,\neg\neg B\,\text{'}\,W \vdash A;\ \Psi)}{?\,(\Phi;\ S\,\text{'}\,B\,\text{'}\,W \vdash A;\ \Psi)}$$

R$_{\neg\neg}$: $$\frac{?\,(\Phi;\ S \vdash \neg\neg A;\ \Psi)}{?\,(\Phi;\ S \vdash A;\ \Psi)}$$

$\boldsymbol{E}^{\mathrm{CPC}}$: *Socratic transformations*

In what follows, unless otherwise stated, by ‘questions’ we will mean questions of $\boldsymbol{L_3}$, and by ‘rules’ we will mean the rules of $\boldsymbol{E}^{\mathrm{CPC}}$.

One can easily prove:

Lemma: *If Q* results from Q by a rule, then Q* is positively equipollent to Q and hence* **Im**(*Q, Q**) *holds*.

Thus the erotetic inference from *Q* to *Q** is valid (in the sense of **IEL**).

Now let us introduce:

Definition: A sequence of questions <Q_1, Q_2, ... > is a *Socratic transformation* of a question *Q* iff $Q_1 = Q$, and Q_{i+1} results from Q_i ($i \geq 1$) by applying a rule.

Thus ***each step in a Socratic transformation is a valid erotetic inference*.**

Terminology: A *constituent* of a question (of $\boldsymbol{L}_3$) is any (CPC)-sequent which occurs in the question.

The rules of $\boldsymbol{E}^{\text{CPC}}$ are designed in such a way that the following is true:

Lemma: *If Q* results from Q by a rule, then each constituent of Q is (CPC) valid if and only if each constituent of Q* is (CPC) valid.*

In other words, rules of the calculus ***preserve joint validity of sequents in both directions.***

Socratic proofs can be defined in terms of Socratic transformations.

Definition: A *Socratic proof* of $S \vdash A$ is a finite Socratic transformation of $?(S \vdash A)$ such that for each constituent ϕ of the last question of the transformation:

(a) ϕ is of the form $W\,'B\,'Z \vdash B$, *or*
(b) ϕ is of the form $W\,'B\,'Z\,'\neg B\,'V \vdash C$, *or*
(c) ϕ is of the form $W\,'\neg B\,'Z\,'B\,'V \vdash C$.

Fact: Each sequent of any of the forms (a), (b), or (c) specified above is CPC-valid.

Thus when one starts a Socratic transformation with a question based on a CPC-valid sequent and ends the transformation with a question whose all constituents are of the forms (a), (b), or (c), the initial sequent must be CPC-valid.

$\boldsymbol{E}^{\mathrm{CPC}}$: *Socratic proofs*

So once a Socratic proof is accomplished, there is no need for any further deductive moves. Rules preserve joint validity of sequents in both directions, and the constituents of the last question are all valid.

One can prove the following soundness and completeness theorem for $\boldsymbol{E}^{\mathrm{CPC}}$.

Theorem [Wiśniewski 2004]:

$S \vdash A$ is CPC-valid iff $S \vdash A$ has a Socratic proof.

Notation: *p*, *q*, *r*, ... are propositional variables.

Example 1: $p \to q \vdash \neg q \to \neg p$

? $(p \to q \vdash \neg q \to \neg p)$ **(R$_\to$)**

? $(p \to q, \neg q \vdash \neg p)$ **(L$_\to$)**

? $(\neg p, \neg q \vdash \neg p; \; q, \neg q \vdash \neg p)$

Example 2: $\vdash (p \to q) \to (\neg q \to \neg p)$

? $(\vdash (p \to q) \to (\neg q \to \neg p))$ **(R$_\to$)**

? $(p \to q \vdash \neg q \to \neg p)$ **(R$_\to$)**

? $(p \to q, \neg q \vdash \neg p)$ **(L$_\to$)**

? $(\neg p, \neg q \vdash \neg p; \; q, \neg q \vdash \neg p)$

Example 3: $p \to (q \to r) \vdash p \wedge q \to r$

? $(p \to (q \to r) \vdash p \wedge q \to r)$ $(\mathbf{R}_{\to})$

? $(p \to (q \to r),\ p \wedge q \vdash r)$ $(\mathbf{L}_{\wedge})$

? $(p \to (q \to r),\ p,\ q \vdash r)$ $(\mathbf{L}_{\to})$

? $(\neg p,\ p,\ q \vdash r;\ (q \to r),\ p,\ q \vdash r)$ $(\mathbf{L}_{\to})$

? $(\neg p,\ p,\ q \vdash r; \neg q,\ p,\ q \vdash r;\ r,\ p,\ q \vdash r)$

Let us observe that the first constituent of the question in the last line is the same as the first constituent of the question in the preceding line; moreover, this constituent has the “final” form. For brevity, we can put the sign - “ - instead of repeating the constituent; of course, we only put this sign if no further analysis is necessary. Thus Example 3 can be rewritten as:

Examples

Example 3*: $p \to (q \to r) \vdash p \land q \to r$

? $(p \to (q \to r) \vdash p \land q \to r)$ **($R_{\to}$)**

? $(p \to (q \to r),\ p \land q \vdash r)$ **($L_{\land}$)**

? $(p \to (q \to r),\ p,\ q \vdash r)$ **($L_{\to}$)**

? $(\neg p,\ p,\ q \vdash r;\ (q \to r),\ p,\ q \vdash r)$ **($L_{\to}$)**

? (- “ - $;\ \neg q,\ p,\ q \vdash r;\ r,\ p,\ q \vdash r)$

Example 4: $p \to (q \to r),\ p \to q \vdash p \to r$

? $(p \to (q \to r),\ p \to q \vdash p \to r)$ **($R_{\to}$)**

? $(p \to (q \to r),\ p \to q,\ p \vdash r)$ **($L_{\to}$)**

? $(\neg p,\ p \to q,\ p \vdash r;\ q \to r,\ p \to q,\ p \vdash r)$ **($L_{\to}$)**

? (- “ - $;\ q \to r, \neg p,\ p \vdash r;\ q \to r,\ q,\ p \vdash r)$ **($L_{\to}$)**

? (- “ - ; - “ - $;\ \neg q,\ q,\ p \vdash r;\ r,\ q,\ p \vdash r)$

Note that the constituents of the last question of Example 4 involve (CPC) wffs which are not literals, i.e. propositional variables or their negations. The situation is different, however, in the case of the previous examples. Thus Example 4 shows that there are cases in which a proof is accomplished, but some CPC-wffs that occur in constituents of the last question have not been analyzed to the level of literals. This remark pertains both to wffs that stand left of the turnstile and right of the turnstile, viz.:

Example 5: $\neg(p \wedge q) \rightarrow \neg s \vdash s \rightarrow p \wedge q$

$? (\neg(p \wedge q) \rightarrow \neg s \vdash s \rightarrow p \wedge q)$ $(\mathbf{R}_{\rightarrow})$

$? (\neg(p \wedge q) \rightarrow \neg s, s \vdash p \wedge q)$ $(\mathbf{L}_{\rightarrow})$

$? (\neg\neg(p \wedge q), s \vdash p \wedge q; \neg s, s \vdash p \wedge q)$ $(\mathbf{L}_{\neg\neg})$

$? (p \wedge q, s \vdash p \wedge q$; - " -)

There are shorter and longer Socratic proofs of the same fact, viz.:

Example 6: $\neg(p \wedge q) \rightarrow \neg s \vdash s \rightarrow p \wedge q$

$? (\neg(p \wedge q) \rightarrow \neg s \vdash s \rightarrow p \wedge q)$	$(\mathbf{L}_{\rightarrow})$
$? (\neg\neg(p \wedge q) \vdash s \rightarrow p \wedge q; \neg s \vdash s \rightarrow p \wedge q)$	$(\mathbf{L}_{\neg\neg})$
$? (p \wedge q \vdash s \rightarrow p \wedge q; \neg s \vdash s \rightarrow p \wedge q)$	$(\mathbf{R}_{\rightarrow})$
$? (p \wedge q, s \vdash p \wedge q; \neg s \vdash s \rightarrow p \wedge q)$	$(\mathbf{R}_{\rightarrow})$
? (- " - ; $\neg s, s \vdash p \wedge q$)	

Now let us consider a case in which the affirmative answer is not so transparent at first sight.

Example 7: $p, \neg r, p \wedge q \to r \vdash s \vee \neg q$

$? \; (p, \neg r, p \wedge q \to r \vdash s \vee \neg q)$ (**R**$_\vee$)

$? \; (p, \neg r, p \wedge q \to r, \neg s \vdash \neg q)$ (**L**$_\to$)

$? \; (p, \neg r, \neg(p \wedge q), \neg s \vdash \neg q \quad ; \; p, \neg r, r, \neg s \vdash \neg q)$ (**L**$_{\neg\wedge}$)

$? \; (p, \neg r, \neg p, \neg s \vdash \neg q; \; p, \neg r, \neg q, \neg s \vdash \neg q;$ - " -)

The rule(s) $\mathbf{R}_\beta$ require(s) that when we have a β-wff right of the turnstile, it is always β_1* that should be "moved" to the left. This, however, does not affect the generality of the construction, as the following example shows:

Example 8: $p, \neg r, \ p \wedge q \rightarrow r \vdash \neg q \vee s$

$?\ (p, \neg r, \ p \wedge q \rightarrow r \vdash \neg q \vee s)$ **(R$_\vee$)**

$?\ (p, \neg r, \ p \wedge q \rightarrow r, \neg\neg q \vdash s)$ **(L$_{\neg\neg}$)**

$?\ (p, \neg r, \ p \wedge q \rightarrow r, q \vdash s)$ **(L$_\rightarrow$)**

$?\ (p, \neg r, \neg(p \wedge q), q \vdash s \ ; p, \neg r, \ r, \ q \vdash s)$ **(L$_{\neg\wedge}$)**

$?\ (p, \neg r, \neg p, \ q \vdash s; \ p, \neg r, \neg q, q \vdash s;$ - " - $)$

Finally, let us present Socratic proofs of some CPC-valid wffs.

Example 9: $\vdash (p \to q) \land (\neg p \to q) \to q$

? ($\vdash (p \to q) \land (\neg p \to q) \to q$) **(R$_\to$)**

? ($(p \to q) \land (\neg p \to q) \vdash q$) **(L$_\land$)**

? ($p \to q, \neg p \to q \vdash q$) **(L$_\to$)**

? ($\neg p, \neg p \to q \vdash q$; $q, \neg p \to q \vdash q$) **(L$_\to$)**

? ($\neg p, \neg\neg p \vdash q$; $\neg p, q \vdash q$; - " -)

Example 10: $\vdash (p \lor q) \land \neg p \to q$

? ($\vdash (p \lor q) \land \neg p \to q$) **(R$_\to$)**

? ($(p \lor q) \land \neg p \vdash q$) **(L$_\land$)**

? ($p \lor q, \neg p \vdash q$) **(L$_\lor$)**

? ($p, \neg p \vdash q$; $q, \neg p \vdash q$)

Example 11: $\vdash \neg(p \wedge \neg p)$

? $(\vdash \neg(p \wedge \neg p))$ $(\mathbf{R}_{\neg\wedge})$

? $(p \vdash \neg\neg p)$ $(\mathbf{R}_{\neg\neg})$

? $(p \vdash p)$

Example 12: $\vdash p \vee \neg p$

? $(\vdash p \vee \neg p)$ $(\mathbf{R}_{\vee})$

? $(\neg p \vdash \neg p)$

$\boldsymbol{E}^{\mathrm{CPC}}$ gives rise to a ‘parallel’ sequent calculus. The ***rules*** are:

$$\frac{S\,'\alpha_1\,'\alpha_2\,'T \vdash A}{S\,'\alpha\,'T \vdash A} \qquad \frac{S \vdash \alpha_1 \qquad S \vdash \alpha_2}{S \vdash \alpha}$$

$$\frac{S\,'\beta_1\,'T \vdash A \qquad S\,'\beta_2\,'T \vdash A}{S\,'\beta\,'T \vdash A} \qquad \frac{S\,'\beta_1^{*} \vdash \beta_2}{S \vdash \beta}$$

$$\frac{S\,'B\,'T \vdash A}{S\,'\neg\neg B\,'T \vdash A} \qquad \frac{S \vdash B}{S \vdash \neg\neg B}$$

Basic sequents:

$$W\text{’}B\text{’}Z \vdash B,$$
$$W\text{’}B\text{’}Z\text{’}\neg B\text{’}V \vdash C,$$
$$W\text{’}\neg B\text{’}Z\text{’}B\text{’}V \vdash C.$$

We get a sound and complete sequent calculus for Classical (Propositional) Logic; the calculus operates on single-conclusioned sequents ! Moreover, the calculus has no primary structural rules.

A Socratic proof can be transformed into a proof in the calculus.

$? (p \to q \vdash \neg q \to \neg p)$

$? (p \to q, \neg q \vdash \neg p)$

$? (\neg p, \neg q \vdash \neg p; \; q, \neg q \vdash \neg p)$

$$\dfrac{\dfrac{\neg p, \neg q \vdash \neg p \qquad q, \neg q \vdash \neg p}{p \to q, \neg q \vdash \neg p}}{p \to q \vdash \neg q \to \neg p}$$

- So far erotetic calculi have been developed for Classical Logic, (both propositional and quantificational), some paraconsistent propositional logics, normal modal propositional logics, and Intuitionistic Propositional Logic. In each case completeness theorems are proven; in some cases decision procedures are described.
- Erotetic calculi for logics different from CPC have the basic properties pointed at when speaking about $\boldsymbol{E}^{\mathrm{CPC}}$, although their syntax is usually more complex. In particular, we need some labelling technique when dealing with modal logics.

- Wiśniewski, A. , ‘Socratic proofs’, *Journal of Philosophical Logic* **33**, 2004, pp. 299-326.
- Wiśniewski, A., Shangin, V., ‘Socratic proofs for quantifiers’, *Journal of Philosophical Logic* **35**, 2006, pp. 147-178.
- Wiśniewski, A., ‘A Right-sided Socratic calculus for Classical Logic. Research report’, 2006.

- Leszczyńska, D., ‘Socratic proofs for some normal modal propositional logics’, *Logique et Analyse* **185-188,** 2004, pp. 259-285
- Leszczyńska, D., *The Method of Socratic Proofs for Normal Modal Propositional Logics,* AMU University Press, Poznań 2007, 97 p.
- Leszczyńska-Jasion, D, ‘The method of Socratic proofs for modal propositional logics: K5, S4.2, S4.3, S4M, S4F, S4R and G', *Studia Logica* **89**, No.3, 2008, pp. 365-399.
- Leszczyńska-Jasion, D., ‘A loop-free decision procedure for modal propositional logics K4, S4 and S5’, *Journal of Philosophical Logic* **38**, 2009, pp.151-177.

Erotetic calculi: other

- Wiśniewski, A., Vanackere, G., Leszczyńska, D., 'Socratic proofs and paraconsistency: A case study', *Studia Logica* **80**, 2005, pp. 431-466

Metatheory

- D. Leszczyńska, M. Urbański, A. Wiśniewski, 'Socratic Trees', *Studia Logica*, DOI: 10.1007/s11225-012-9404-0
- Wiśniewski, A., Shangin, V., 'Некоторые допустимые правила для системы сократического вывода', *Вестник Московского университета. Серия Философия*, No. 5, 2007, pp. 77-88.