An Introduction to Erotetic Search Scenarios

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Outline

Erotetic Search Scenarios

- The concept of *Erotetic Search Scenario* (e-scenario for short) was introduced in Inferential Erotetic Logic (IEL) in order to model problem-solving subjected to the so-called erotetic decomposition principle (EDP). ▸ more on IEL ▸ more on EDP
- The area of applicability of the concept, however, has soon appeared to be wider.
- In particular, e-scenarios are useful tools in the areas of dialogue modelling, question answering, and problem solving in general. E-scenarios also gave rise to some proof-theoretic developments.
- The earliest full-length paper on e-scenarios is [5]. ▸ go to references
- A detailed exposition of e-scenarios and their logical properties can be found in Part III of the book [6]. ▸ go to references
- For recent developments see, e.g., [3], [2], and [1]. ▸ go to references

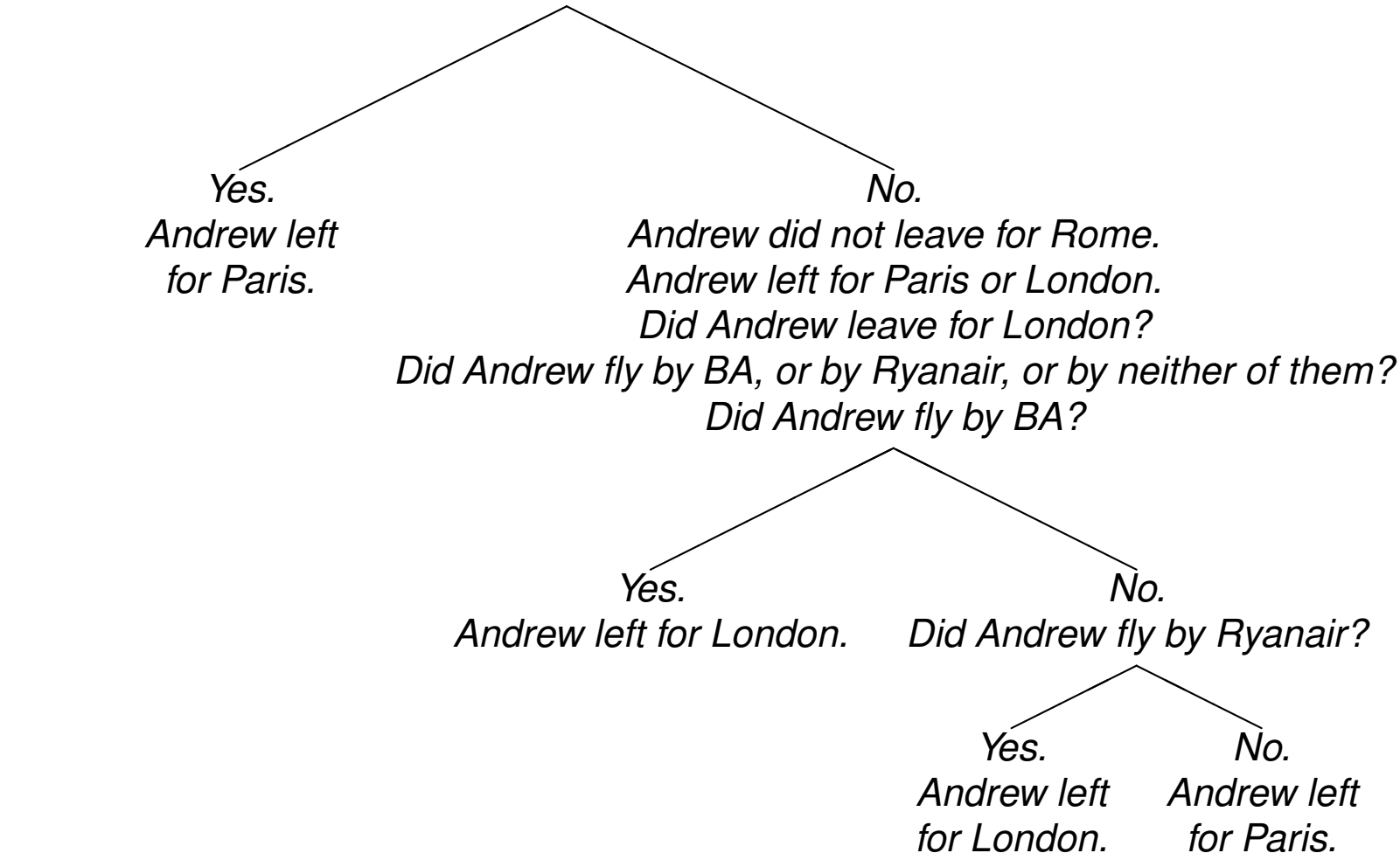
An example

Remark. An e-scenario is always *for* a principal question, as is *relative to* a (possibly empty) set of declarative sentences/formulas.

- Let the principal question be:
  - *Where did Andrew leave for: Paris, London, or Rome?*
- Assume that there are good reasons to believe that the following hold:
  - *Andrew left for Paris, London or Rome.*
  - *If Andrew flew by Air France, then he left for Paris.*
  - *If Andrew did not fly by Air France, then he did not leave for Rome.*
  - *Andrew left for London iff*[1] *he flew by BA or Ryanair.*
- An e-scenario pertaining to the above case is displayed on the next slide.

[1] Here and below, 'iff' abbreviates 'if and only if.'

Where did Andrew leave for: Paris, London, or Rome?
Andrew left for Paris, London or Rome.
If Andrew flew by Air France, then he left for Paris.
If Andrew did not fly by Air France, then he did not leave for Rome.
Andrew left for London iff he flew by BA or Ryanair.
Did Andrew fly by Air France?



Here is the schema of the above e-scenario written down in a propositional language enriched with questions:

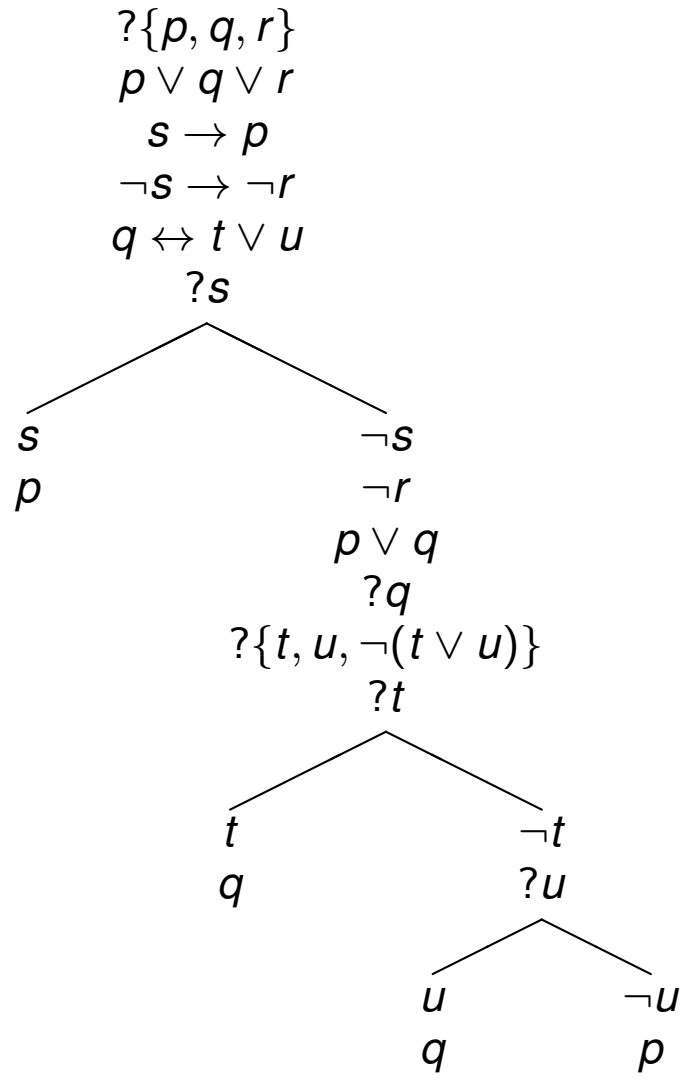


Figure: The logical structure of the scenario considered

A propositional language enriched with questions

The formal language used above can be briefly characterized as follows.

- We enrich the language of Classical Propositional Logic (CPL) with a new category of well-formed formulas (wffs), namely questions. More precisely, we add the following signs to the vocabulary: ?, {, }, and the comma. A *question* is an expression of the form:

  $$?\{A_1, \ldots, A_n\}$$

  where $n > 1$ and $A_1, \ldots, A_n$ are nonequiform, that is, pairwise syntactically distinct CPL-wffs.

- If $?\{A_1, \ldots, A_n\}$ is a question, then each of the CPL-wffs $A_1, \ldots, A_n$ is a *direct answer* to the question, and these are the only direct answers.

- For conciseness, we abbreviate '$?\{A, \neg A\}$' as '$?A$'.

- In general, a question of the form $?\{A_1, \ldots, A_n\}$ can be read:

  $$\textit{Is it the case that } A_1, \textit{ or } \ldots, \textit{or is it the case that } A_n?$$

- In some special cases, however, a different reading can be recommended. For instance, one can read "$?A$" as: "*Is it the case that A?*".

- At the metalanguage level, the set of direct answers to a question Q will be denoted by $\mathbf{d}Q$.

A propositional language enriched with questions

- A *Boolean valuation* is a function v that assigns a truth value, **1** or **0**, to each propositional variable and is extended to all CPL-wffs in the standard manner by using the Boolean functions corresponding to the connectives.
- *Entailment*, $\models$, is a relation between a set of CPL-wffs and a CPL-wff.

Definition (Entailment)

$X \models A$ iff $v(A) = \mathbf{1}$ for every Boolean valuation v such that $v(B) = \mathbf{1}$ for all $B \in X$.

- *Multiple-conclusion entailment*, $\|\!\models$, is a relation between sets of CPL-wffs.

Definition (Multiple-conclusion entailment; mc-entailment)

$X \|\!\models Y$ iff for each Boolean valuation v in which $v(B) = \mathbf{1}$ for every $B \in X$, there exists $A \in Y$ such that $v(A) = \mathbf{1}$.

Needles to say, in the case of CPL we have:

- $X \models A$ iff $X \|\!\models \{A\}$,
- $\{A_1, \ldots, A_n\} \|\!\models \{B_1, \ldots, B_k\}$ iff $A_1 \wedge \ldots \wedge A_n \models B_1 \vee \ldots \vee B_k$.

Erotetic implication

- Neither entailment nor mc-entailment pertains to questions.
- As for questions, the concept of *erotetic implication* applies.
- Erotetic implication is one of the basic conceptual tools of IEL.

Terminology. By *d-wffs* we mean declarative well-formed formulas. In the current case d-wffs are CPL-wffs.

Definition (Erotetic implication)

A question Q implies a question Q_1 on the basis of a set of d-wffs X (in symbols: $\mathbf{Im}(Q, X, Q_1)$) iff:

($\downarrow$) for each $A \in \mathbf{d}Q : X \cup \{A\} \models \mathbf{d}Q_1$, and

($\uparrow$) for each $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \not\models Y$.

▸ more on erotetic implication

Erotetic implication

In the particular case of CPL (but not in general!) we have:

Corollary

Im(Q, X, Q_1) *iff*

1. $\{\bigvee \mathbf{d}Q\} \cup X \models \bigvee \mathbf{d}Q_1$ *and*
2. *for each* $B \in \mathbf{d}Q_1 : \{B\} \cup X \models \bigvee Y$ *for some non-empty proper subset* Y *of* $\mathbf{d}Q$.

Auxiliary questions of e-scenarios and erotetic implication

Questions in e-scenarios are linked by erotetic implication.

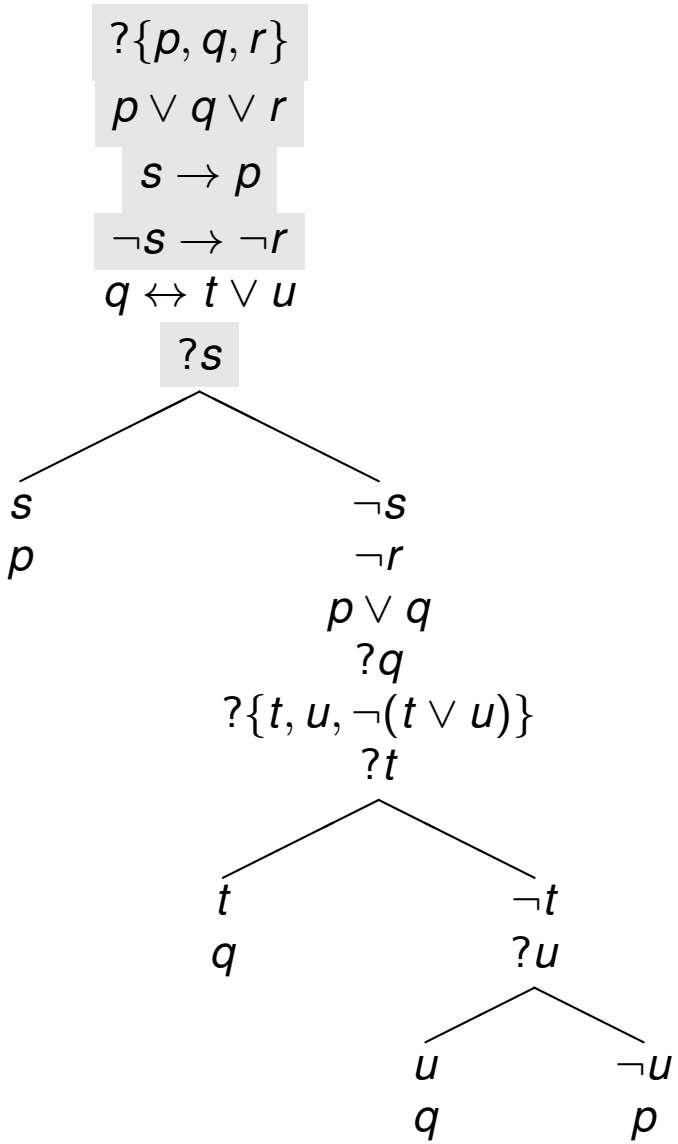


Figure: Erotetic implications used

Notation. Y_A denotes an exemplary proper subset of the set of direct answers to the implying question which is associated with the direct answer A to the implied question, i.e. which is mc-entailed by $X \cup \{A\}$.

- $\mathbf{Im}(?\{p, q, r\}, p \vee q \vee r, s \to p, \neg s \to \neg r, ?s)$
- $Y_s = \{p\}$
- $Y_{\neg s} = \{p, q\}$

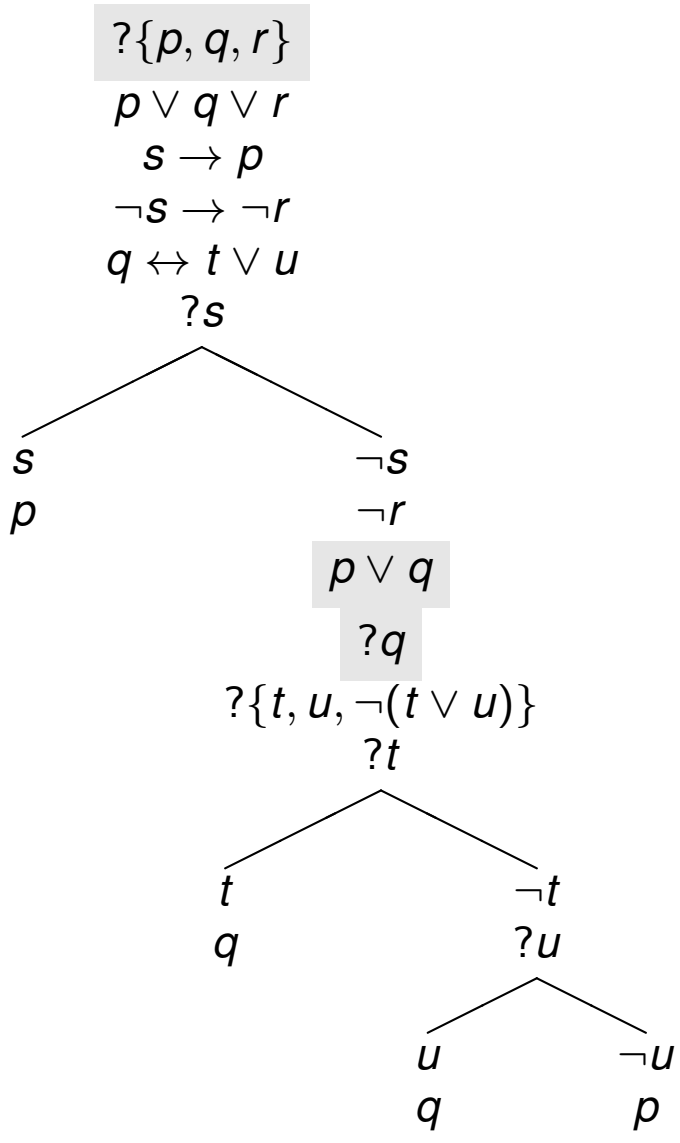


Figure: Erotetic implications used

- **Im**$(?\{p, q, r\}, p \vee q, ?q)$
- $Y_q = \{q\}$
- $Y_{\neg q} = \{p\}$

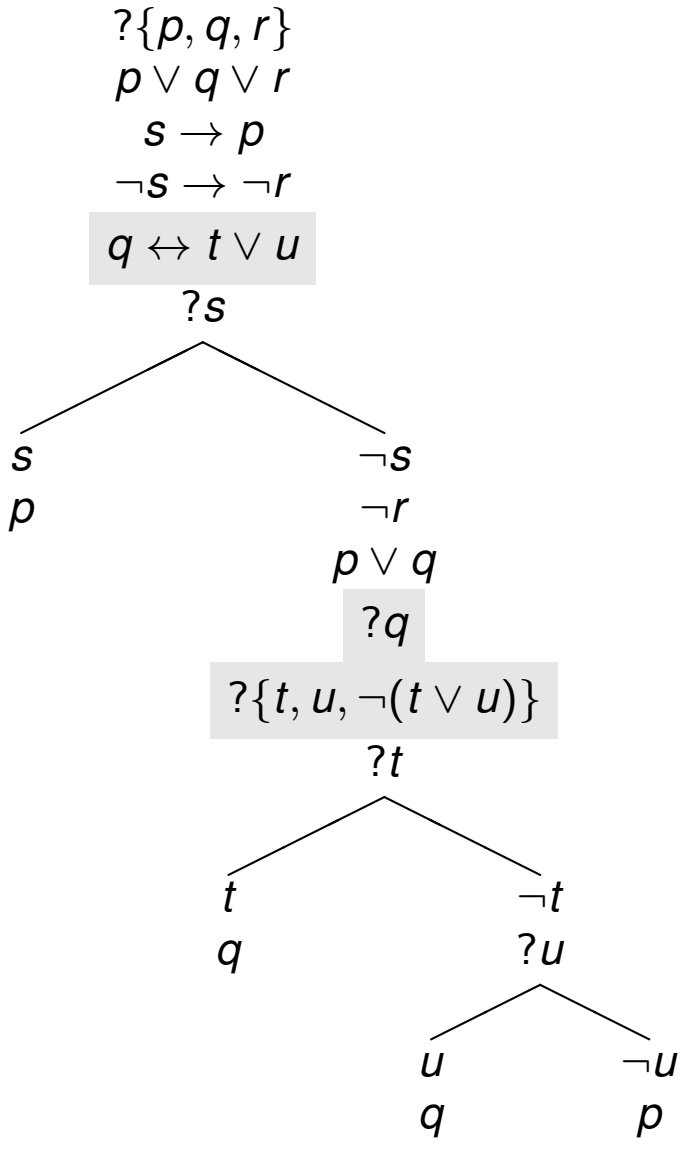


Figure: Erotetic implications used

- $\mathbf{Im}(?q, q \leftrightarrow t \vee u, ?\{t, u, \neg(t \vee u)\})$
- $Y_t = \{q\}$
- $Y_u = \{q\}$
- $Y_{\neg(t \vee u)} = \{\neg q\}$

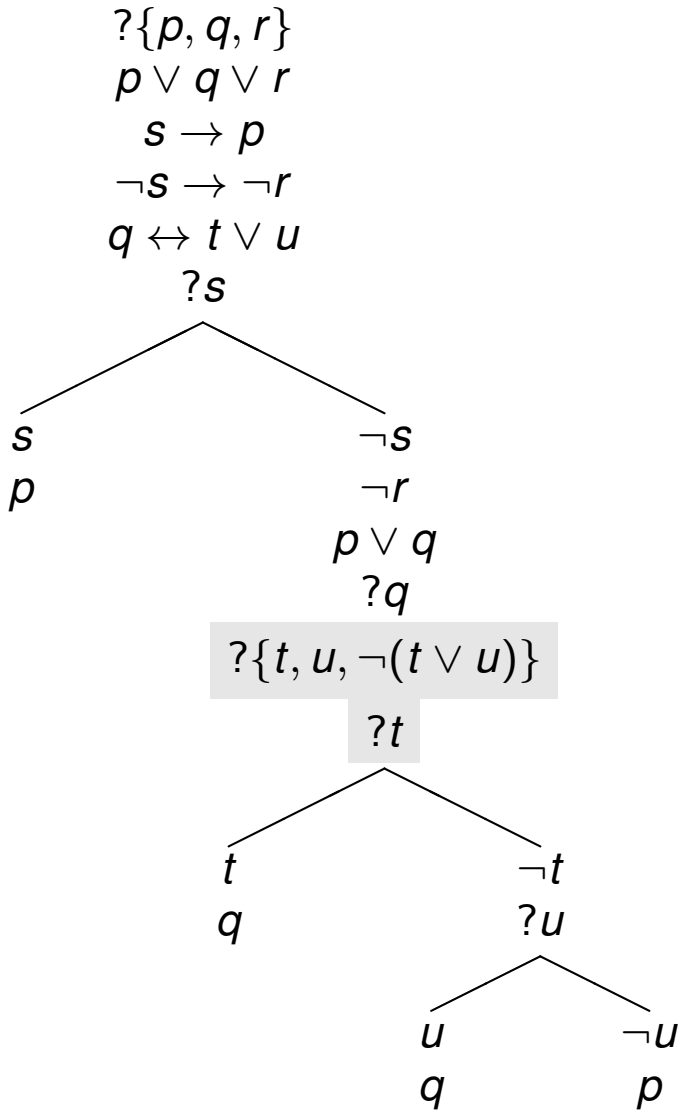


Figure: Erotetic implications used

- $\mathbf{Im}(?\{t, u, \neg(t \vee u)\}, ?t)$
- $Y_t = \{t\}$
- $Y_{\neg t} = \{u, \neg(t \vee u)\}$

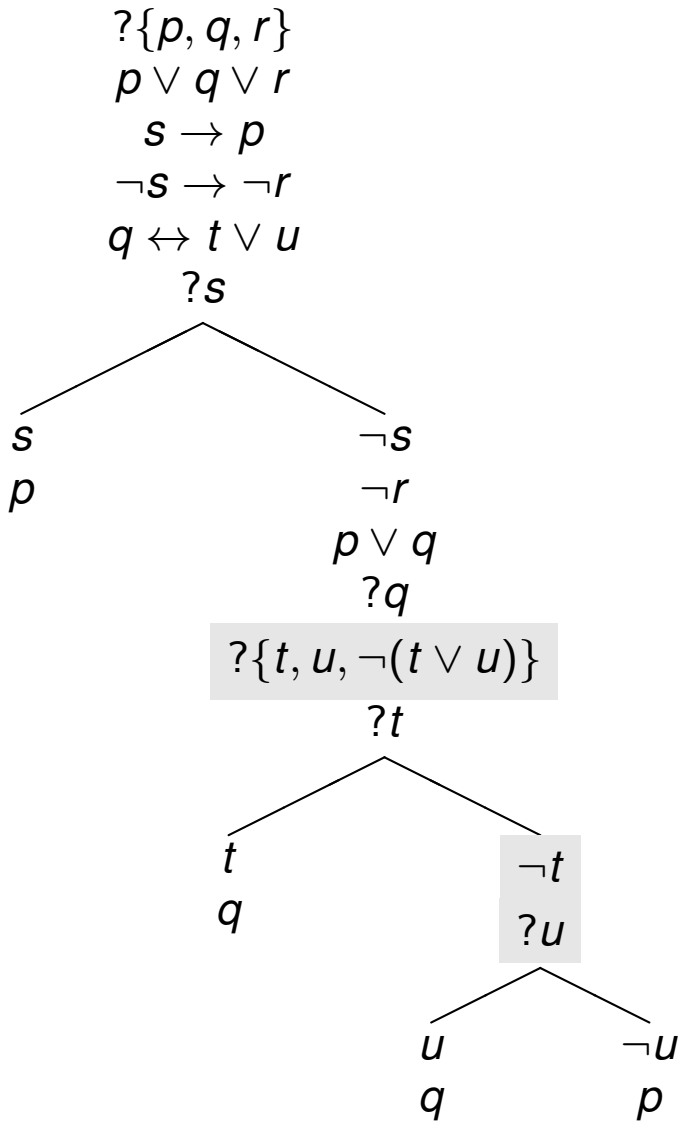


Figure: Erotetic implications used

- $\mathbf{Im}(?\{t, u, \neg(t \vee u)\}, \neg t, ?u)$
- $Y_u = \{u\}$
- $Y_{\neg u} = \{\neg(t \vee u)\}$

Observe that . . .

- All the questions (with the exception of the first one) are erotetically implied.
- Some implied questions are immediately succeeded by direct answers to them.
- However, there are questions that are immediately succeeded by questions.
- The scenario comprises four paths. A path begins with the principal question and ends with a direct answer to it.

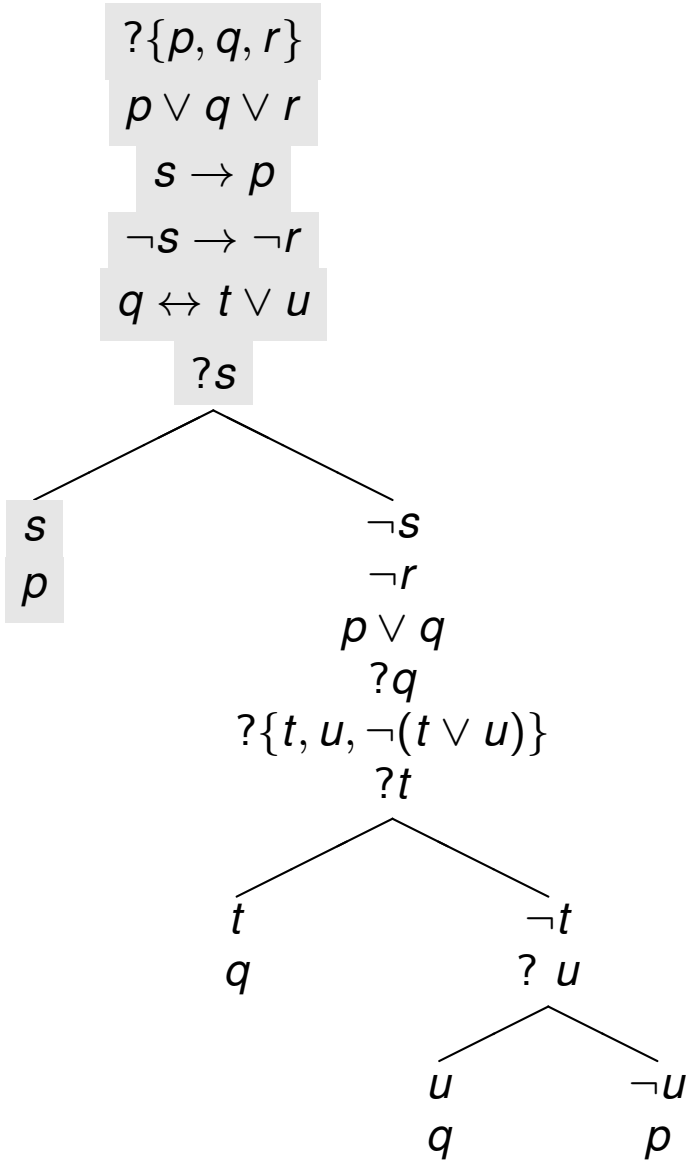


Figure: First path

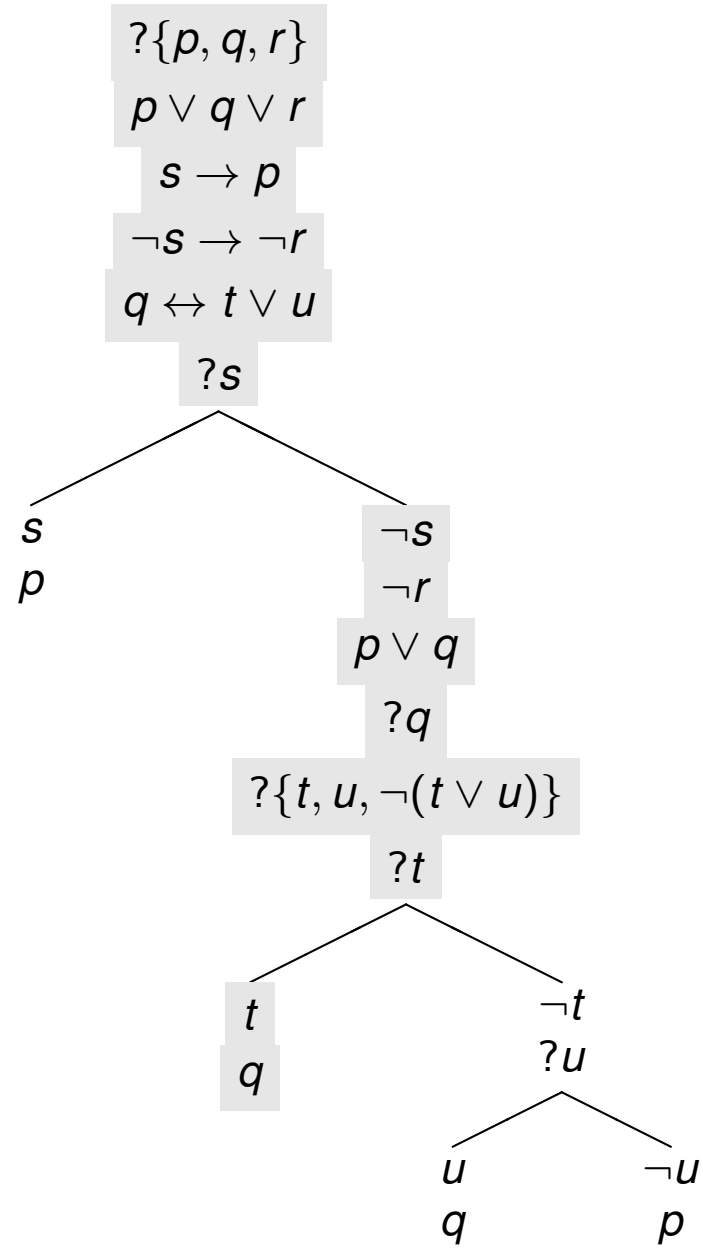


Figure: Second path

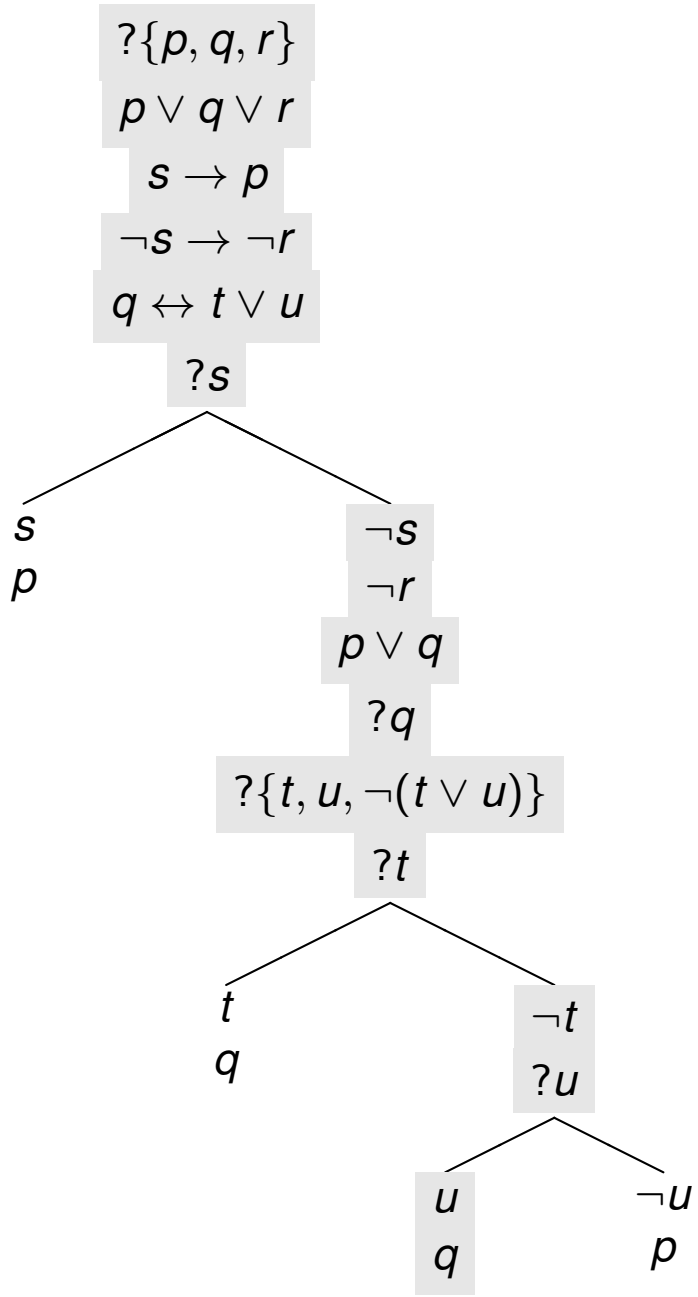


Figure: Third path

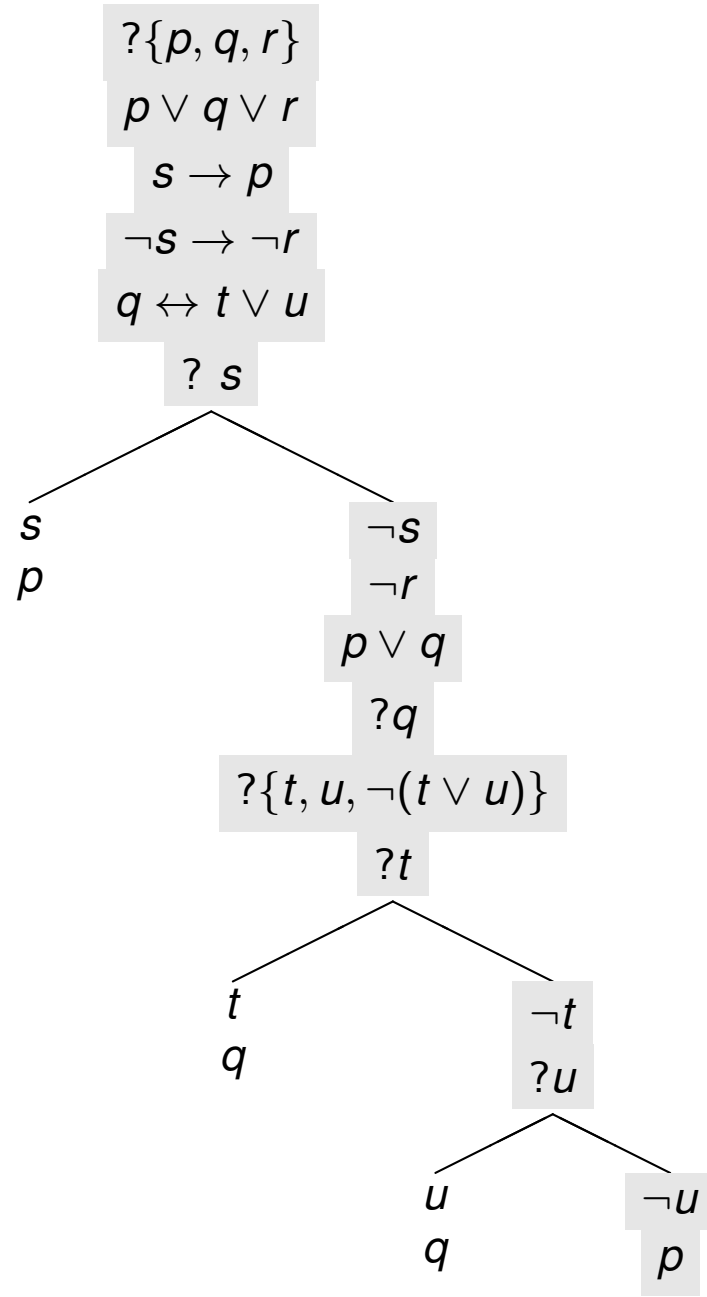


Figure: Fourth path

Observe that . . .

- Each path leads to (i.e. ends with) a direct answer to the principal question.
- Some d-wffs occur on a path because they are initial premises (i.e. belong to the set of d-wffs w.r.t. to which the e-scenario is built).
- Some d-wffs occur on a path only because they are direct answers to auxiliary questions. This holds when an auxiliary question is not immediately succeeded by another one and thus perform the role of a query.
- All the d-wffs which occur on a path that are neither initial premises nor direct answers to queries are entailed by some d-wffs which occur earlier on the path.
- This, in particular, pertains to direct answers to the principal question which terminate paths.

$s \to p, s \models p$

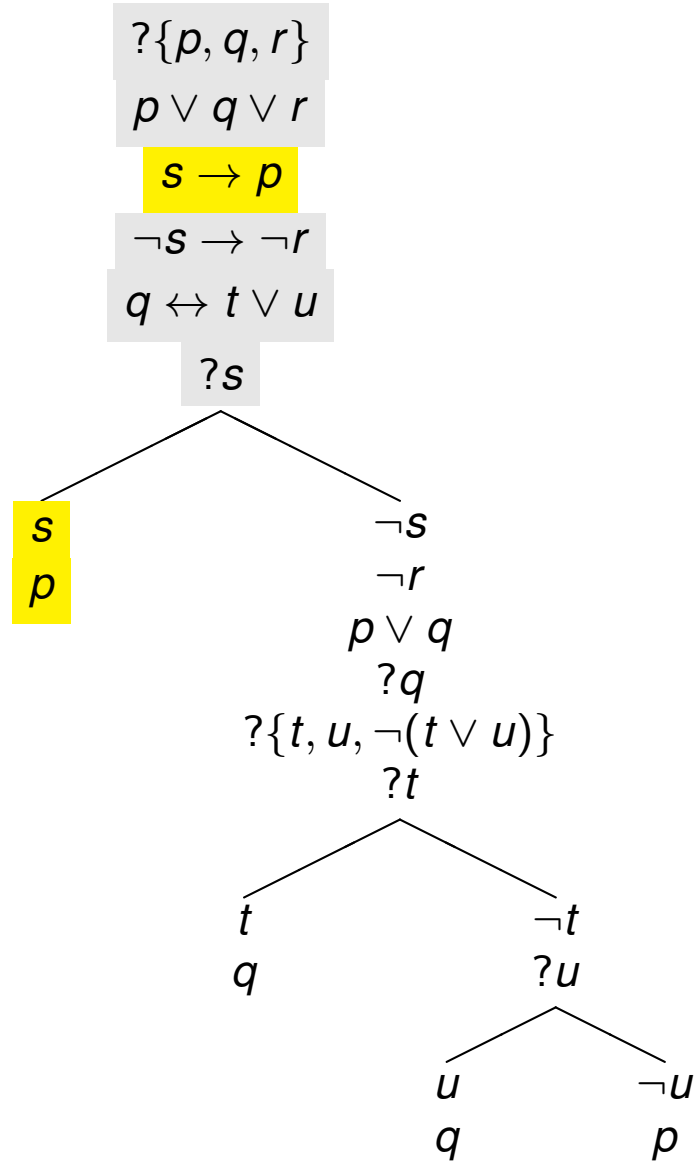


Figure: Entailment: first path

$\neg s \to \neg r, \neg s \models \neg r$

$p \vee q \vee r, \neg r \models p \vee q$

$q \leftrightarrow t \vee u, t \models q$

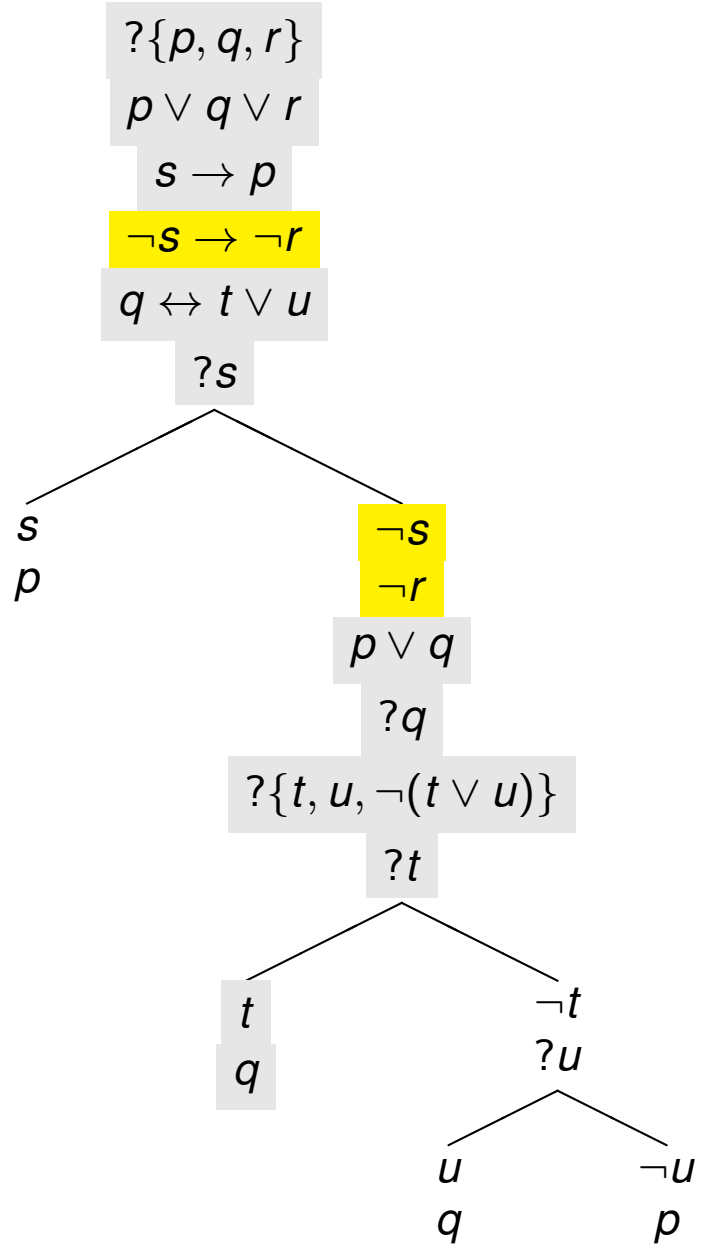


Figure: Entailments: second path

$\neg s \to \neg r, \neg s \models \neg r$

$p \vee q \vee r, \neg r \models p \vee q$

$q \leftrightarrow t \vee u, t \models q$

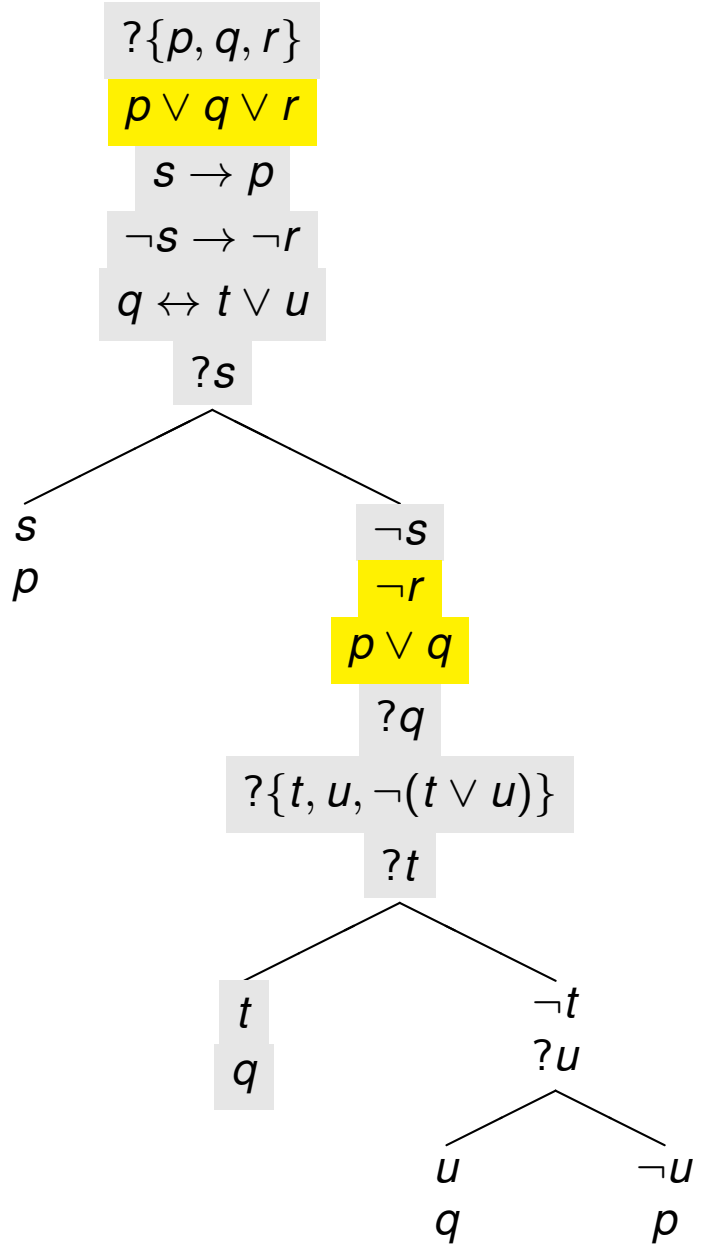


Figure: Entailments: second path

$\neg s \to \neg r, \neg s \models \neg r$

$p \vee q \vee r, \neg r \models p \vee q$

$q \leftrightarrow t \vee u, t \models q$

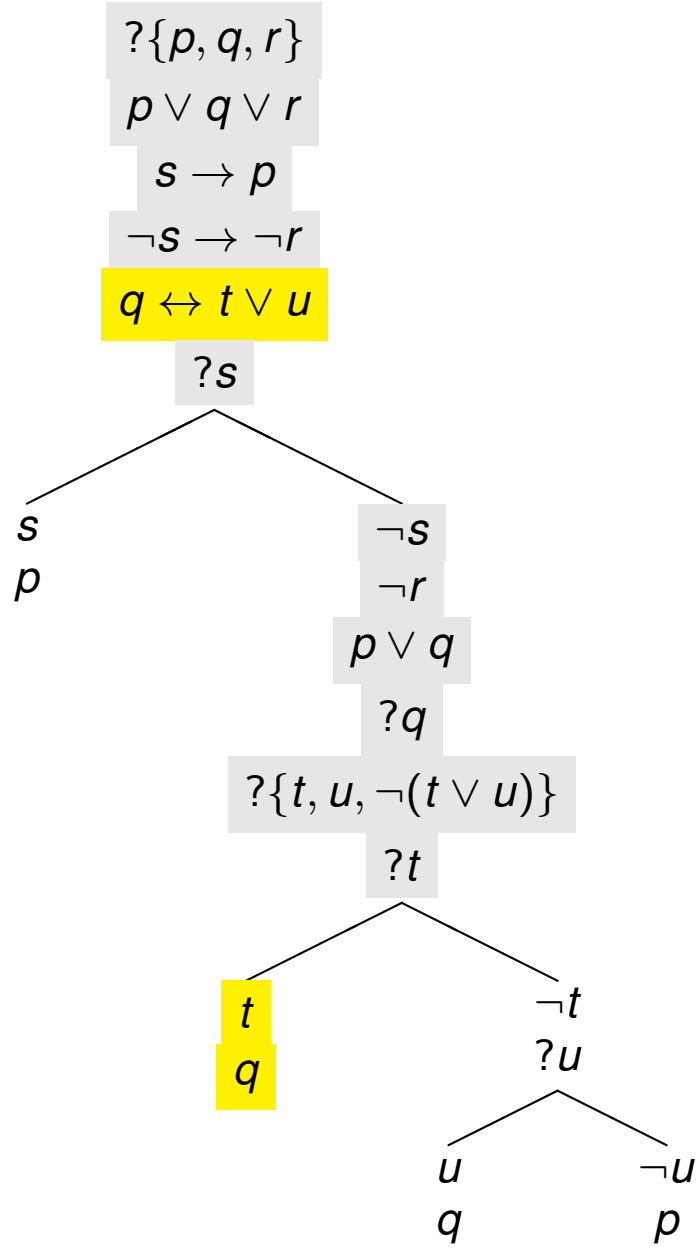


Figure: Entailments: second path

$\neg s \to \neg r, \neg s \models \neg r$

$p \vee q \vee r, \neg r \models p \vee q$

$q \leftrightarrow t \vee u, u \models q$

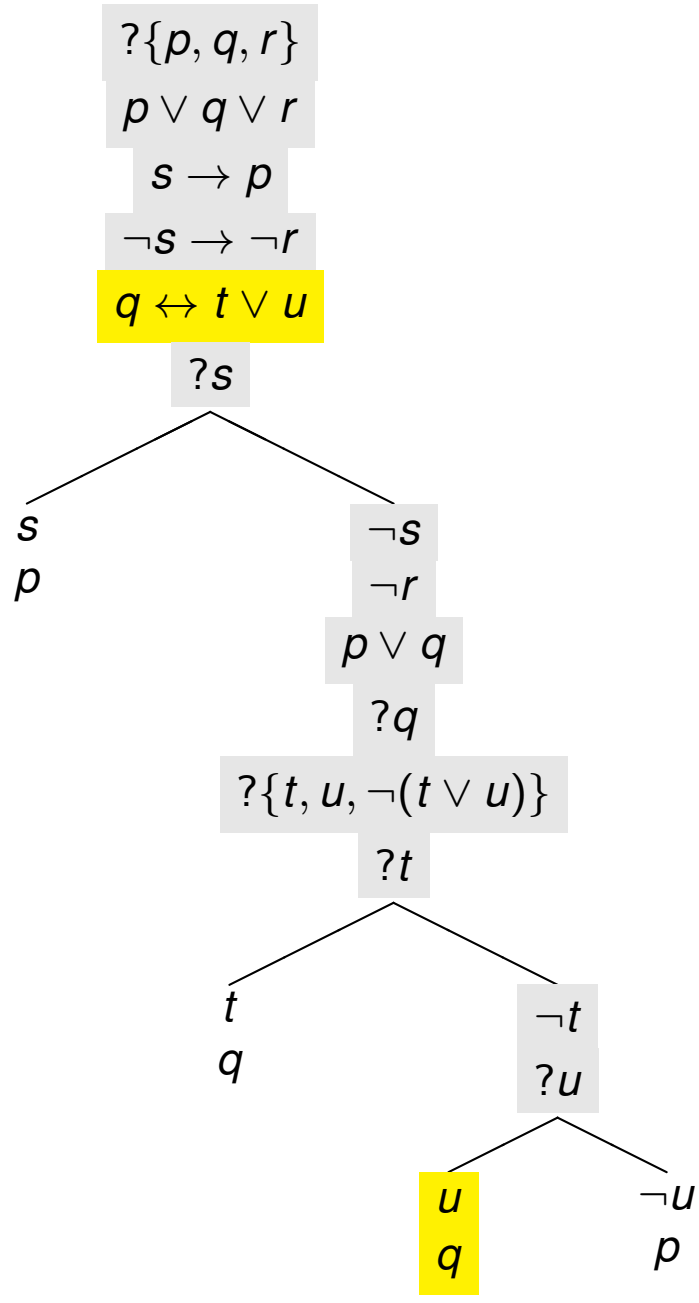


Figure: Entailments: third path

$\neg s \to \neg r, \neg s \models \neg r$

$p \vee q \vee r, \neg r \models p \vee q$

$q \leftrightarrow t \vee u, p \vee q, \neg t, \neg u \models p$

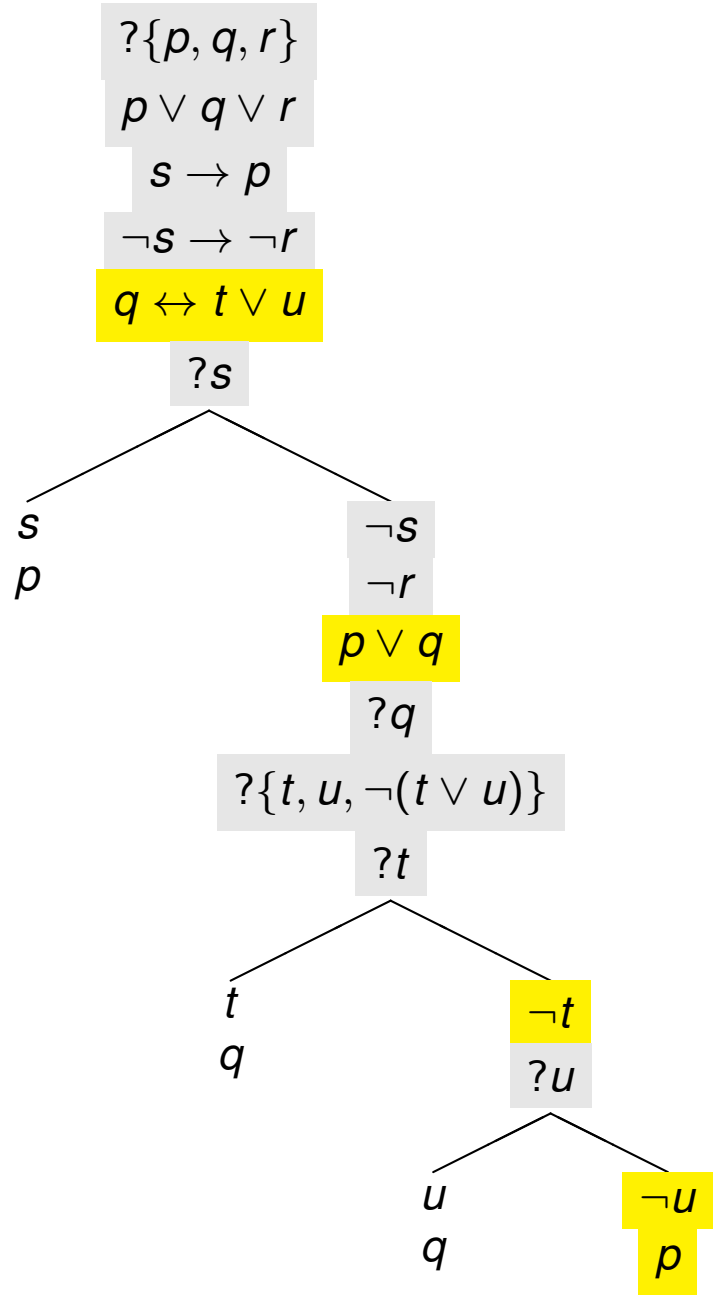


Figure: Entailments: fourth path

An auxiliary concept: e-derivation

Terminology: From now on, by *wffs* we will mean d-wffs and questions.

As above, **d***Q* refers to the set of direct answers to a question *Q*.

In order to continue we need an auxiliary concept of *erotetic derivation* (e-derivation for short). Generally speaking, paths of e-scenarios are just e-derivations.

An auxiliary concept: e-derivation

Definition (E-derivation)

A finite sequence $\mathbf{s} = \mathrm{s}_1, \ldots, \mathrm{s}_n$ of wffs is an erotetic derivation (e-derivation for short) of a direct answer A to question Q on the basis of a set of d-wffs X iff $\mathrm{s}_1 = Q$, $\mathrm{s}_n = A$, and the following conditions hold:

(1) for each question s_k of $\mathbf{s}$ such that $k > 1$:

- (a) $\mathbf{d}\mathrm{s}_k \neq \mathbf{d}Q$,
- (b) s_k is erotetically implied by a question s_j which precedes s_k in $\mathbf{s}$ on the basis of the empty set, or on the basis of a set of d-wffs such that each element of this set precedes s_k in $\mathbf{s}$, and
- (c) s_{k+1} is either a direct answer to s_k or a question;

(2) for each d-wff s_i of $\mathbf{s}$:

- (a) $\mathrm{s}_i \in X$, or
- (b) s_i is a direct answer to s_{i-1}, where $\mathrm{s}_{i-1} \neq Q$, or
- (c) s_i is entailed by a set of d-wffs such that each element of this set precedes s_i in $\mathbf{s}$.

Queries of e-derivations

Remark. The range of applicability of the concept of e-derivation is not restricted to the propositional language with questions by means of which we have illustrated our considerations. As a matter of fact, the concept applies to any formal language enriched with questions used in IEL (▸ more). The same holds for the remaining concepts we are going to introduce from now on.

An auxiliary question of an e-derivation immediately followed (in the derivation) by a direct answer to it is a *query* of the e-derivation.

Definition (Query of e-derivation)

A term s_k (where $1 < k < n$) of an e-derivation $\mathbf{s} = \mathrm{s}_1, \ldots, \mathrm{s}_n$ is a query of $\mathbf{s}$ if s_k is a question and s_{k+1} is a direct answer to s_k.

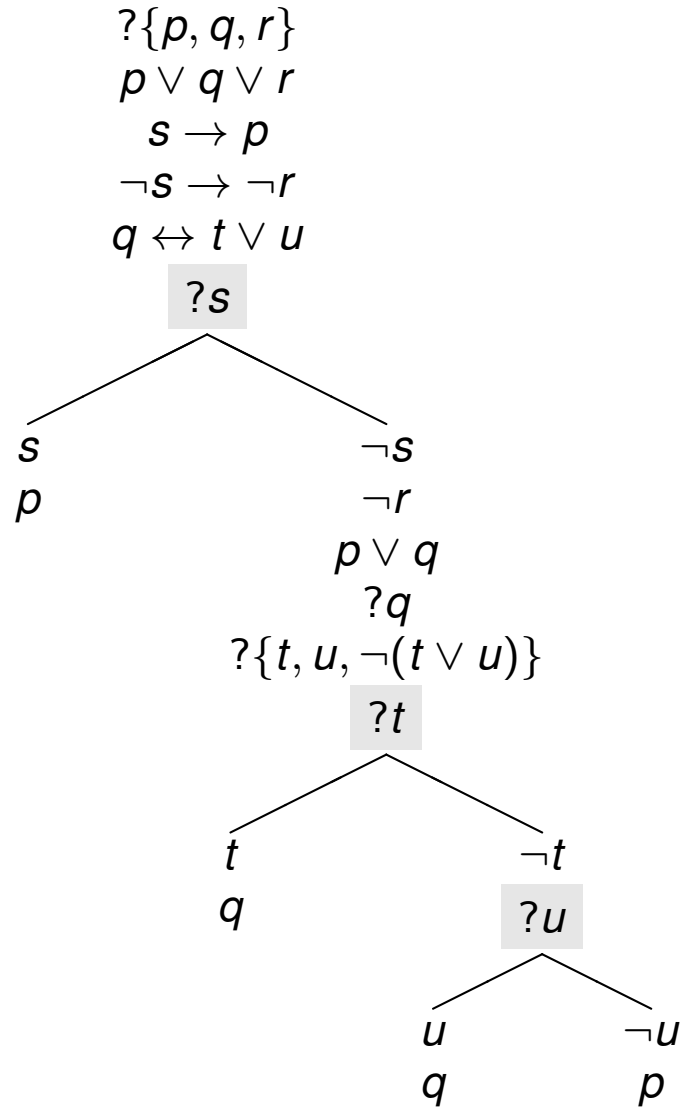
Remark. Observe that it is *not* required that the direct answer to a query (of an e-derivation) that immediately succeeds the query (in the e-derivation) has to be entailed by previous wffs (of the e-derivation). Direct answers to queries can be carriers of new information.

Towards a definition of e-scenarios

- An e-scenario for a question Q relative to a set of d-wffs X is a non-singleton set of e-derivations of direct answers to Q on the basis of X.
- However, not *any* set of this kind!
- The following intuitive conditions have to be fulfilled:
  1. the set X is disjoint with the set of direct answers to the principal question Q;
  2. if a question is "answered" within an e-scenario in one way, then this question is "answered" in all the other possible ways (on related paths; the "possible ways" are determined by the set of direct answers to the question);

     $\Longrightarrow$ NO DEAD ENDS!
  3. only these questions which are supposed to be answered (more precisely: queries of e-derivations involved) perform the function of "branching points/nodes" of e-scenarios.

Questions supposed to be answered as branching points



Definition (E-scenario)

A finite set Σ of sequences of wffs is an erotetic search scenario (e-scenario for short) for a question Q relative to a set of d-wffs X iff each element of Σ is an e-derivation of a direct answer to Q on the basis of X and the following conditions hold:

(1) $\mathbf{d}Q \cap X = \emptyset$;

(2) Σ contains at least two elements;

(3) for each element $\mathbf{s} = \mathrm{s}_1, \ldots, \mathrm{s}_n$ of Σ, for each index k, where $1 \leq k < n$:

- (a) if s_k is a question and s_{k+1} is a direct answer to s_k, then for each direct answer B to s_k: the family Σ contains an e-derivation $\mathbf{s}^* = \mathrm{s}^*_1, \mathrm{s}^*_2, \ldots, \mathrm{s}^*_m$ such that $\mathrm{s}_j = \mathrm{s}^*_j$ for $j = 1, \ldots, k$, and $\mathrm{s}^*_{k+1} = B$;
- (b) if s_k is a d-wff, or s_k is a question and s_{k+1} is not a direct answer to s_k, then for each e-derivation $\mathbf{s}^* = \mathrm{s}^*_1, \mathrm{s}^*_2, \ldots, \mathrm{s}^*_m$ in Σ such that $\mathrm{s}_j = \mathrm{s}^*_j$ for $j = 1, \ldots, k$ we have $\mathrm{s}_{k+1} = \mathrm{s}^*_{k+1}$.

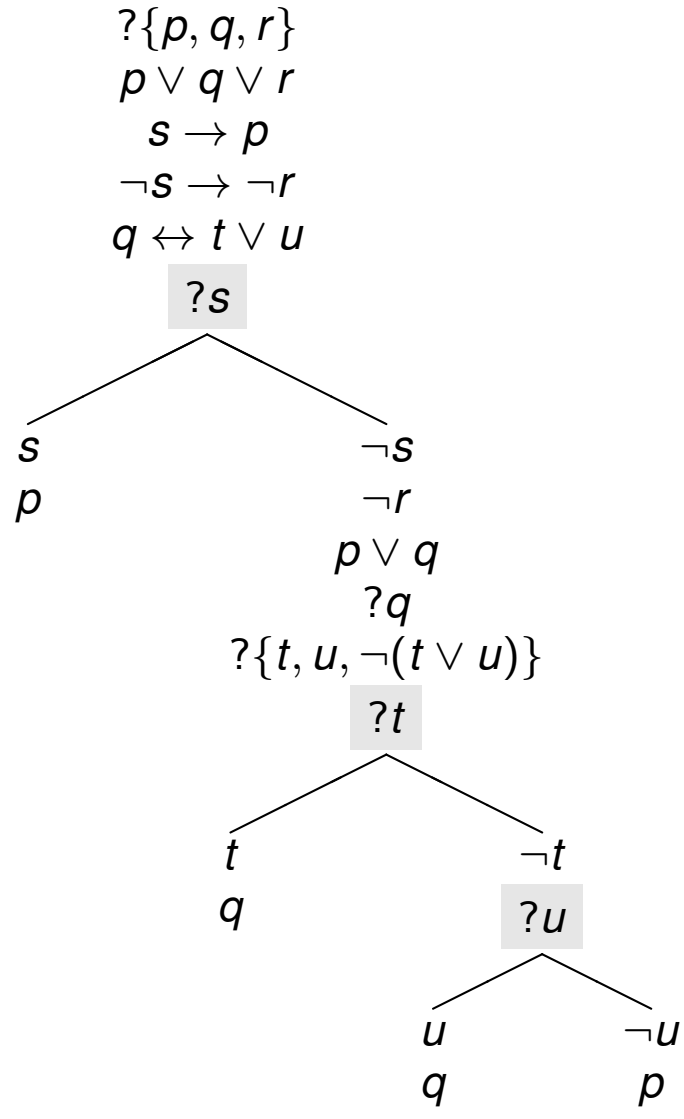
Queries and non-queries of e-scenarios

Definition (Query of e-scenario)

A query of an e-scenario is a query of an e-derivation that belongs to the e-scenario.

- A quick look at definitions gives the following: a query of an e-scenario is simply the first element of a question-answer pair that occurs on a path/e-derivation of the e-scenario, where the question is an auxiliary one and the answer immediately succeeds the question. Thus each query is a question, but e-scenarios can involve auxiliary questions that are not queries. If such questions occur, they are indispensable, but, roughly, “are not supposed to be asked and answered”.
- Auxiliary questions that are not queries enable IEL-based introduction of queries.

Queries


$?\{p, q, r\}$
$p \vee q \vee r$
$s \rightarrow p$
$\neg s \rightarrow \neg r$
$q \leftrightarrow t \vee u$
$?s$
s
p
$\neg s$
$\neg r$
$p \vee q$
$?q$
$?\{t, u, \neg(t \vee u)\}$
$?t$
t
q
$\neg t$
$?u$
u
q
$\neg u$
p

A pragmatic account of e-scenarios

- E-scenarios are abstract entities defined in terms of IEL. But, looking from the pragmatic point of view, an e-scenario provides *conditional instructions* which tell what auxiliary questions should be asked and when they should be asked.
- Queries of e-scenarios can be viewed as requests for information. An e-scenario shows what is the next advisable query if the information request of the previous query has been satisfied in such–and–such way.
- What is important, an e-scenario does this with regard to every possible way of satisfying the request, where the ways are determined by direct answers to the question which functions as a query.
- Moreover, an e-scenario behaves in this manner in the case of every query of the e-scenario.

E-scenarios as labelled trees (1/3)

E-scenarios can also be equivalently defined as labelled trees.

Definition (E-scenario as a labelled tree)

A finite labelled tree $\mathcal{T}$ is an erotetic search scenario for a question Q relative to a set of d-wffs X iff

(1) the nodes of $\mathcal{T}$ are labelled by questions and d-wffs; they are called e-nodes and d-nodes, respectively;

(2) Q labels the root of $\mathcal{T}$;

(3) each leaf of $\mathcal{T}$ is labelled by a direct answer to Q;

(4) $\mathrm{d}Q \cap X = \emptyset$;

E-scenarios as labelled trees (2/3)

Definition (Continued)

(5) for each d-node φ_δ of $\mathcal{T}$: if A is the label of φ_δ, then
   (a) $A \in X$, or
   (b) $A \in \mathbf{d}Q^*$, where $Q^* \neq Q$ and Q^* labels the immediate predecessor of φ_δ, or
   (c) $\{B_1, ..., B_n\}$ entails A, where B_i $(1 \leq i \leq n)$ labels a d-node of $\mathcal{T}$ that precedes the d-node φ_δ in $\mathcal{T}$;

(6) each d-node of $\mathcal{T}$ has at most one immediate successor;

(7) there exists at least one e-node of $\mathcal{T}$ which is different from the root;

E-scenarios as labelled trees (3/3)

Definition (continued)

(8) for each e-node φ_ε of $\mathcal{T}$ different from the root: if Q^* is the label of φ_ε, then $\mathbf{d}Q^* \neq \mathrm{d}Q$ and

- (a) $\mathbf{Im}(Q^{**}, Q^*)$ or $\mathbf{Im}(Q^{**}, B_1, ..., B_n, Q^*)$, where Q^{**} labels an e-node of $\mathcal{T}$ that precedes φ_ε in $\mathcal{T}$ and B_i $(1 \leq i \leq n)$ labels a d-node of $\mathcal{T}$ that precedes φ_ε in $\mathcal{T}$, and
- (b) an immediate successor of φ_ε is either an e-node or is a d-node labelled by a direct answer to the question that labels φ_ε; moreover
  - (i) if an immediate successor of φ_ε is an e-node, it is the only immediate successor of φ_ε,
  - (ii) if an immediate successor of φ_ε is not an e-node, then for each direct answer to the question that labels φ_ε there exists exactly one immediate successor of φ_ε labelled by the answer.

References

[1] Szymon Chlebowski, Adam Kups, and Maciej Komosinski. Automated generation of erotetic search scenarios: Classification, optimisation and knowledge extraction. *ACM Transactions on Computational Logic*, 18(2), 2017.

[2] Dorota Leszczyńska-Jasion and Paweł Łupkowski. Erotetic search scenarios and three-valued logic. *Journal of Logic, Language and Information*, 25(1):51–76, 2016.

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[6] Andrzej Wiśniewski. *Questions, Inferences, and Scenarios*. College Publications, London, 2013.

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Inferential Erotetic Logic

- By and large, Inferential Erotetic Logic (IEL for short) is a logic that analyses *erotetic inferences*, that is, inferences in which questions perform the role of conclusions and, possibly, premises. IEL proposes criteria of validity for these inferences.
- The idea of IEL originates from the late 1980s, but IEL was developed in depth in the 1990s.
- The monograph [4] summarizes results obtained until the early 1990s. The book [6] presents IEL in its current form.

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Inferences from declaratives to questions

An *erotetic inference of the first kind* leads from a set of declarative premises to a question. Here are examples:

Andrew always comes in time, but now he is late.

What has happened to him?

Mary is Peter's mother.
If Mary is Peter's mother, then John is Peter's father or George is Peter's father.

Who is Peter's father: John or George?

Inferences from questions to questions

An *erotetic inference of the second kind* has a question among its premises. The remaining premises (if there are any) are declarative sentences/formulas. The conclusion is a question.

Is Andrew lying?
Andrew lies iff he speaks very slowly.

Is Andrew speaking very slowly?

Where did Andrew leave for: Paris, London or Moscow?
Andrew left for Paris iff he departed in the morning.
Andrew left for London or Moscow iff he departed in the evening.

When did Andrew depart: in the morning, or in the evening?

Is Andrew silly and ugly?

Is Andrew ugly?

Validity

- It can be shown that erotetic inferences are subjected to patterns. Moreover, some erotetic inferences are *intuitively valid*, while others are not.
- The following can serve as a preliminary test of intuitive validity: put the expression "so the question arises:" just before the conclusion.
  If the resultant description of an erotetic inference is undoubtedly true, the inference can be regarded as intuitively valid.

Validity

Mary is Peter's mother.

If Mary is Peter's mother, then John is Peter's father or George is Peter's father.

So the question arises:

Who is Peter's father: John or George?

Is Andrew lying?

Andrew lies iff he speaks very slowly.

So the question arises:

Is Andrew speaking very slowly?

Validity

IEL proceeds as follows:

- First, some criteria of validity are proposed, separately for erotetic inferences that involve only declarative premises and for these in which an interrogative premise occurs.
- Then two semantic relations are defined:
  - *evocation* of questions by sets of declarative sentences/formulas, and
  - *erotetic implication* of a question by a question together with a set of declarative sentences/formulas.
- The above concepts are *explications* of the corresponding concepts of question raising.
- Validity of erotetic inferences of consecutive kinds is then defined in terms of question evocation and erotetic implication, respectively.

Syntax: d-wffs and questions

- We are dealing with a formal language, $\mathcal{L}$, in which at least two categories of well-formed expressions occur: *declarative well-formed formulas* (hereafther: d-wffs) and *erotetic formulas* (hereafter: e-formulas or simply *questions*).
- Generally speaking, $\mathcal{L}$ has thus (possibly among others) a "declarative part" and an "erotetic part."
- We do not assume in advance what formal language (a non-modal propositional language, a modal propositional language, a first-order language, etc.) performs the role of the declarative part of $\mathcal{L}$.

Syntax: minimal assumptions concerning the class of e-formulas

- We stay (almost) neutral concerning the form of e-formulas/ questions of $\mathcal{L}$.
- We assume, however, the existence of an assignment of *direct answers* to questions.
- More precisely, we stipulate that for each question Q of $\mathcal{L}$ there exists an at least two-element set $\mathbf{d}Q$ of d-wffs of $\mathcal{L}$, called *the set of direct answers to* Q.
- Formally, $\mathbf{d}$ is a function that assigns to a question Q a set of d-wffs.
- Intuitively, $\mathbf{d}Q$ comprises these d-wffs which are principal possible answers to Q, that is, provide neither less no more information than it is requested by Q.

Semantics

- We need a semantics for the declarative part of a language, in particular the concepts of truth and entailment for d-wffs.
- However, IEL is neutral in the controversy as to what “The Logic” of declaratives is. One can use either CL or a non-classical logic as the basis. Yet, diverse logic have diverse semantics.
- *Minimal Erotetic Semantics* (MiES) provides a general framework within which entailment relations determined by different logics can be simulated.
- For space reasons, we will not present MiES here.
- So let us only assume that the declarative part of the language considered is supplied with a semantics rich enough to define some relativized (to a valuation, a model of an appropriate kind, etc.) concept of truth for d-wffs.

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Multiple-conclusion entailment

- We need the concept of multiple-conclusion entailment (mc-entailment), being a relation between sets of d-wffs.
- For instance:

  {*There is a cat in this room. Either Andrew, or Paul, or Dorothy let the cat in.*}

  mc-entails:

  {*Andrew let the cat in. Paul let the cat in. Dorothy let the cat in.* }

- The idea is: X mc-entails Y iff the hypothetical truth of all the sentences in X warrants the existence of a true sentence in Y.
- The schema is:

  $X \Vdash Y$ iff for each $\mathcal{M} \in (\ldots)$: if all the d-wffs in X are true in $\mathcal{M}$, then at least one d-wff in Y is true in $\mathcal{M}$.

- Needless to say, single-conclusion entailment, $\models$, can be defined by: $X \models A$ iff $X \Vdash \{A\}$.

Evocation of questions

Definition

A set of d-wffs X evokes a question Q (in symbols: $\mathbf{E}(X, Q)$) iff

(1) $X \Vdash \mathbf{d}Q$, and

(2) for each $A \in \mathbf{d}Q : X \nVdash \{A\}$.

- *There is a cat in this room.*
  Where is the cat?
- *There is a cat in this room. Someone let it in.*
  Who did it?

Erotetic implication

Definition

A question Q implies a question Q_1 on the basis of a set of d-wffs X (in symbols: $\mathbf{Im}(Q, X, Q_1)$) iff:

($\downarrow$) for each $A \in \mathbf{d}Q : X \cup \{A\} \Vdash \mathbf{d}Q_1$, and

($\uparrow$) for each $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \nVdash Y$.

- *Who let the cat in: Andrew, Dorothy, or Paul?*
  Andrew let the cat in iff the cat is black.
  Dorothy or Paul let the cat in iff the cat is grey.
  Is the cat black, or is it grey?

Erotetic implication is a “forth-and-back” (this is an intended neologism!) notion:

Forth ↓

$$(\downarrow) \text{ for each } A \in \mathbf{d}Q : X \cup \{A\} \Vdash \mathbf{d}Q_1$$

Generally speaking, clause (↓) amounts to: if the implying question, Q, has a true direct answer and all the implying d-wffs in X are true, it is impossible that the implied question, Q_1 has no true direct answer.

As for the presented example, condition (↓) is fulfilled non-trivially: there are cats which are neither black nor grey, and the cat could have been let in by someone else.[2]

Who let the cat in: Andrew, Dorothy or Paul?

Andrew let the cat in iff the cat is black.

Dorothy or Paul let the cat in iff the cat is grey.

Is the cat black, or is it grey?

[2] In some cases condition (↓) is fulfilled trivially, because an implied question is safe, that is, generally speaking, must have a true direct answer irrespective of how things are (e.g.:"Is the cat black?"). IEL takes into account both safe and non-safe (i.e. risky) questions.

Forth ↓

$$(\downarrow)\ \text{for each } A \in \mathbf{d}Q : X \cup \{A\} \Vdash \mathbf{d}Q_1$$

Who let the cat in: Andrew, Dorothy or Paul?

Andrew let the cat in iff the cat is black.

Dorothy or Paul let the cat in iff the cat is grey.

Is the cat black, or is it grey?

Who let the cat in: Andrew, Dorothy or Paul?

Andrew let the cat in iff the cat is black.

Dorothy or Paul let the cat in iff the cat is grey.

Is the cat black, or is it grey?

(↑) for each $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \not\models Y$.

Generally speaking, condition (↓) amounts to the following: each direct answer to an implied question, Q_1, *narrows down*, together with the declaratives in X, the class of "possibilities" offered by (the set of direct answers to) the implying question Q. Thus each direct answer to an implied question, Q_1, is (potentially) cognitively useful in resolving the problem expressed by the implying question Q.

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(↑) for each $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \not\models Y$.

Who let the cat in: Andrew, Dorothy or Paul?

Andrew let the cat in iff the cat is black.

Dorothy or Paul let the cat in iff the cat is grey.

Is the cat black, or is it grey?

Who let the cat in: Andrew, Dorothy or Paul?

Andrew let the cat in iff the cat is black.

Dorothy or Paul let the cat in iff the cat is grey.

Is the cat black, or is it grey?

Back ↑

In some cases the “back” (↑) condition is fulfilled due to the fact that *single* direct answers to the implying question are entailed, as in:

Is Andrew lying?
Andrew lies iff he speaks very slowly.
Is Andrew speaking very slowly?

Who discovered polonium?
The discoverer of polonium is also the discoverer of radium.
Who discovered radium?

Regular erotetic implication

If this is a general rule, erotetic implication is called *regular*.

Definition (Regular erotetic implication)

Imr(Q, X, Q_1) iff

1. for each $A \in \mathbf{d}Q : X \cup \{A\} \nvDash \mathbf{d}Q_1$ and
2. for each $B \in \mathbf{d}Q_1$ there exists $C \in \mathbf{d}Q$ such that $X \cup \{B\} \models C$.

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Problem decomposition

- One of the crucial principles which govern effective problem solving is the following:

  (**DP**) (***Decomposition Principle***): *Decompose a principal problem (PP) into simpler sub-problems (SPs) in such a way that solutions to SPs can be assembled into an overall solution to PP.*

- When we consider a problem definite enough to be adequately expressed by a question, its decomposition amounts, generally speaking, to finding an appropriate collection of auxiliary questions.
- A decomposition can be *static*, that is, resulting in finding a set of mutually independent auxiliary questions such that once *all* of them are answered, the initial problem is resolved.
- Yet, a more interesting case is that of *dynamic* decomposition that comes in *stages*: the consecutive auxiliary questions (which constitute the sub-goals of the next stage) depend on how the previous requests for information have been fulfilled.

Erotetic decomposition principle

- The main goal, determined by the principal problem, remains unchanged, but sub-goals are processed in a goal-directed way.
- Moreover, the erotetic decomposition principle:

  (**EDP**) (***Erotetic Decomposition Principle***): *Transform a principal question into auxiliary questions in such a way that: (a) consecutive auxiliary questions are dependent upon previous questions and, possibly, answers to previous auxiliary questions, and (b) once auxiliary questions are resolved, the principal question is resolved as well.*

  is obeyed until the principal problem becomes solved.
- Inferential Erotetic Logic (IEL) offers an account of this kind of problem decomposition.

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