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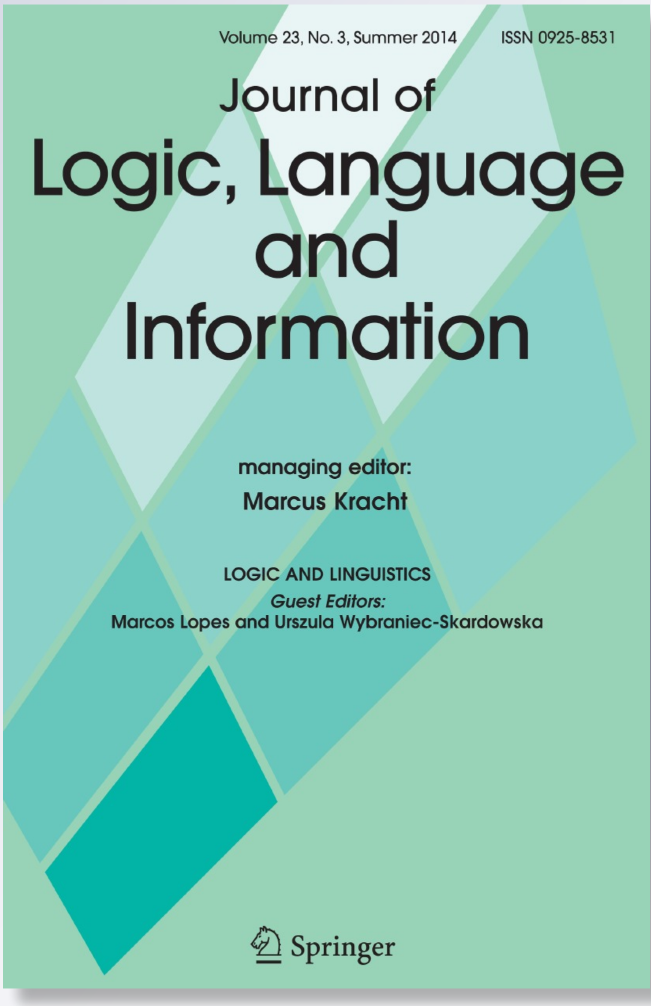

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Support and Sets of Situations

Andrzej Wiśniewski

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Abstract An alternative conceptual setting of the basic system of inquisitive semantics is presented. A situational interpretation of the proposed formalism is discussed.

Keywords Inquisitive semantics · Situational semantics · Truth

1 Introduction

Inquisitive semantics is a research program which has originated from an analysis of questions, but currently is evolving towards a general theory of meaning.[1] In this short paper we propose an alternative conceptual setting of the basic propositional system of inquisitive semantics, labelled InqB. Inquisitive entailment is retained in the setting, but the concept of model used is more general.

Then we sketch a non-standard, "situational" interpretation of InqB.

2 InqB: The Canonical Account

InqB provides a new semantic framework for propositional languages.

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[1] For inquisitive semantics see, e.g., Groenendijk and Roelofsen (2009), Ciardelli and Roelofsen (2011), Groenendijk (2011), Ciardelli et al. (2013), Wisniewski and Leszczynska-Jasion (in print). For recent developments see: http://sites.google.com/site/inquisitivesemantics/.

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Let $\mathcal{L}_{\mathcal{P}}$ be a propositional language built over a non-empty set of propositional variables $\mathcal{P}$, where $\mathcal{P}$ is either finite or countably infinite. The primitive logical constants of the language are: $\bot$, $\vee$, $\wedge$, $\rightarrow$. Well-formed formulas (wffs) of $\mathcal{L}_{\mathcal{P}}$ are defined as usual.

On the canonical account of InqB, $\mathcal{L}_{\mathcal{P}}$ is supposed to be associated with the set of *suitable possible worlds*, $\mathcal{W}_{\mathcal{P}}$, being the *model* of $\mathcal{L}_{\mathcal{P}}$. A possible world, in turn, is conceived either as a subset of $\mathcal{P}$ or as a binary valuation of $\mathcal{P}$. Regardless of which of these solutions is adopted, $\mathcal{W}_{\mathcal{P}}$ is uniquely determined. When possible worlds are conceptualized as sets of propositional variables, $\mathcal{W}_{\mathcal{P}} = \wp(\mathcal{P})$, i.e. the power set of $\mathcal{P}$. If, however, possible worlds are identified with *indices*, that is, binary valuations of $\mathcal{P}$, then $\mathcal{W}_{\mathcal{P}}$ is the set of all indices.

The intended area of applicability of InqB is the information exchange between individuals. It is assumed that the expressions used are interpreted uniformly by the parties involved. This is the main reason for which a language of the considered kind is associated with exactly one model.

2.1 States, Support and Meaning

InqB defines the basic semantic categories in terms of *support*, being a relation between a wff and a set of possible worlds. Sets of possible worlds are called *states*. They are intuitively interpreted as representing information states. Singleton sets/states correspond to information states of maximal consistent information, while $\mathcal{W}_{\mathcal{P}}$ corresponds to the ignorant state, i.e. an information state in which no possible world is excluded. $\emptyset$ represents the absurd state. Formally, a state is a subset of $\mathcal{W}_{\mathcal{P}}$.

Notation The letters A, B, C, D are metalanguage variables for wffs of $\mathcal{L}_{\mathcal{P}}$, and the letters X, Y are metalanguage variables for sets of wffs of the language. $\mathtt{p}$ is used below as a metalanguage variable for propositional variables. The letters σ, τ, γ, with or without subscripts, will refer to states.

Support is defined in InqB by the following clauses (we write "$\sigma \succ A$" for "state σ supports wff A"; it is assumed that $\sigma \subseteq \mathcal{W}_{\mathcal{P}}$):

1. $\sigma \succ \mathtt{p}$ iff $w(\mathtt{p}) = \mathbf{1}$ for any $w \in \sigma$,
2. $\sigma \succ \bot$ iff $\sigma = \emptyset$,
3. $\sigma \succ (A \wedge B)$ iff $\sigma \succ A$ and $\sigma \succ B$,
4. $\sigma \succ (A \vee B)$ iff $\sigma \succ A$ or $\sigma \succ B$,
5. $\sigma \succ (A \rightarrow B)$ iff for each $\tau \subseteq \sigma$: if $\tau \succ A$ then $\tau \succ B$.

Negation is defined by:

$$\neg A =_{df} (A \rightarrow \bot).$$

The definition of support generalizes the standard definition of truth of a wff in a world. To see this, it suffices to put $\sigma = \{w\}$ and then replace "$\{w\} \succ A$" with "A is true in w". We get the usual clauses defining truth of a wff in a world. As a result, classical semantic concepts are definable in InqB. Let us stress, however, that, due to clauses (4) and (5), support by a state does not amount to truth in each world of the state.

The account of meaning offered by inquisitive semantics is two-fold. The semantics assigns to a wff its *informative content* and its *inquisitive content*. Both concepts are defined in terms of support.

In the general setting of inquisitive semantics the informative content of a wff is defined as the union of all states that support the wff. In the case of InqB, however, the informative content of a wff A amounts to the *truth set* of A, that is, the set of all the worlds from $\mathcal{W}_{\mathcal{P}}$ in which A is (classically) true. The inquisitive content of A, in turn, is the family of all the states that support A. A wff A is *inquisitive* iff the informative content of A does not support the wff A. In the case of InqB this amounts to the following claim: the informative content of A does not belong to the inquisitive content of A. The intuition which lies behind the concept of inquisitiveness is: something more than the information the wff provides is needed to settle the issue it raises. Wffs which are not inquisitive are called *assertions*. As for assertions, their informative contents settle the issues they raise.

InqB (and inquisitive semantics in general) enables a differentiation between classical tautologies: some of them are inquisitive, while some others are not. Being a question is a semantic property of a wff: A is a question iff the truth set of A equals $\mathcal{W}_{\mathcal{P}}$. Each question is a classical tautology, but some questions are inquisitive.

2.2 Remarks on Models

As we have mentioned above, on the canonical account the set of propositional variables $\mathcal{P}$ of $\mathcal{L}_{\mathcal{P}}$ uniquely determines the set of possible worlds $\mathcal{W}_{\mathcal{P}}$ associated with the language; this set constitutes the model of $\mathcal{L}_{\mathcal{P}}$. A moment's reflection reveals that when $\mathcal{P}$ is countably infinite, $\mathcal{W}_{\mathcal{P}}$ is uncountably infinite.[2] Finite models exist only for languages over finite sets of propositional variables, and countably infinite models do not exist at all. Yet, InqB simulates information exchange in terms of elimination of worlds from an (initial) information state. As $\mathcal{W}_{\mathcal{P}}$ represents the ignorant information state, it follows that, in the case of languages over countably infinite sets of propositional variables, arriving at states of maximal consistent information (which are singleton sets) requires an elimination of uncountably many possible worlds. On the other hand, as long as we stay within the realm of Classical Recursion Theory, there is no recursive function whose domain is an uncountably infinite set. Thus when we deal with a language over infinite set of propositional variables/atoms, there is no algorithmic procedure which leads from the ignorant information state to a state of maximal consistent information. This is a strong philosophical claim stemming from conceptual decisions only.

3 InqB: An Alternative Account

In this section we propose an alternative conceptual setting of InqB within which inquisitive entailment is retained, but the concept of model is generalized.

2 The reasons are simple. First, there exist uncountably many subsets of a countably infinite set. Second, there exists a 1–1 correspondence between indices defined over countably infinite set $\mathcal{P}$ and countably infinite sequences of logical values, **1** and **0**, and there exist uncountably many such sequences.

Definition 1 (*ι-model*) An ι-model of $\mathcal{L}_{\mathcal{P}}$ is a structure:

$$\langle W, V \rangle \quad (1)$$

where $W \neq \emptyset$ and $V : \mathcal{P} \times W \mapsto \{\mathbf{1}, \mathbf{0}\}$.

The set W will be called the *domain* of the corresponding ι-model. W can be thought of as a set of possible worlds. However, Definition 1 is neutral w.r.t. the controversy what possible worlds are. The only condition imposed on W is non-emptiness. Thus the cardinality of ι-models is not determined by the cardinality of $\mathcal{P}$. Needless to say, there exist ι-models of any cardinality, and many ι-models of a given cardinality. Moreover, it is not excluded that W contains possible worlds which are indistinguishable with respect to the valuation function V.

Remark We use the term "ι-model" for clarity only. Structures of the form (1) are often employed in semantics. For example, they are used as models of (propositional) **S5**.

Let $\mathcal{M} = \langle W, V \rangle$ be an ι-model. By a $\mathcal{M}$-*state* we mean a subset of W. An expression of the form "$\sigma \succ_{\mathcal{M}} A$" reads "$\mathcal{M}$-state σ supports wff A". The support relation $\succ_{\mathcal{M}}$ is defined by:

Definition 2 (*Support in an ι-model*) Let $\sigma \subseteq W$.

1. $\sigma \succ_{\mathcal{M}} \mathtt{p}$ iff $V(\mathtt{p}, w) = \mathbf{1}$ for each $w \in \sigma$,
2. $\sigma \succ_{\mathcal{M}} \bot$ iff $\sigma = \emptyset$,
3. $\sigma \succ_{\mathcal{M}} (A \wedge B)$ iff $\sigma \succ_{\mathcal{M}} A$ and $\sigma \succ_{\mathcal{M}} B$,
4. $\sigma \succ_{\mathcal{M}} (A \vee B)$ iff $\sigma \succ_{\mathcal{M}} A$ or $\sigma \succ_{\mathcal{M}} B$,
5. $\sigma \succ_{\mathcal{M}} (A \rightarrow B)$ iff for each $\tau \subseteq \sigma$: if $\tau \succ_{\mathcal{M}} A$ then $\tau \succ_{\mathcal{M}} B$.

Inquisitive negation is introduced by:

$$\neg A =_{df} (A \rightarrow \bot).$$

Thus we get:

($\mathtt{neg}$) $\sigma \succ_{\mathcal{M}} \neg A$ iff for each $\tau \subseteq \sigma$ such that $\tau \neq \emptyset : \tau \not\succ_{\mathcal{M}} A$.

Remark The concept of support is defined similarly as in the standard setting (see page 2); we have only added a relativization to $\mathcal{M}$.

Note that support by a $\mathcal{M}$-state does not amount to truth in each world of the state: the clauses for disjunction and implication (as well as for negation) are more demanding. The following holds:

Corollary 1 (Persistence) *If* $\sigma \succ_{\mathcal{M}} A$, *then* $\tau \succ_{\mathcal{M}} A$ *for any* $\tau \subseteq \sigma$.

As a consequence we get:

Corollary 2 *If* $\sigma \succ_{\mathcal{M}} A$, *then* $\{w\} \succ_{\mathcal{M}} A$ *for each* $w \in \sigma$.

The converse of Corollary 2 does not hold.

Since we operate with ι-models, the respective semantic concepts of InqB become relativized to models. For clarity, we begin with:

Definition 3 (*Truth in a world of an ι-model*) $\mathcal{M}, w \models A$ iff $\{w\} \succ_{\mathcal{M}} A$.

Definition 4 (*Truth set in an ι-model*) $|A|_{\mathcal{M}} = \{w \in W : \mathcal{M}, w \models A\}$.

Definition 5 (*Inquisitive content in an ι-model*) Let W be the domain of an ι-model $\mathcal{M}$.

$$\|A\|_{\mathcal{M}} = \{\sigma \subseteq W : \sigma \succ_{\mathcal{M}} A\}.$$

One can prove:

Corollary 3 $\|A\|_{\mathcal{M}} = \{\sigma \subseteq |A|_{\mathcal{M}} : \sigma \succ_{\mathcal{M}} A\}$.

Corollary 4 $\emptyset \in \|A\|_{\mathcal{M}}$ *for any wff A and any ι-model $\mathcal{M}$.*

For brevity, we introduce:

Definition 6 $\sigma \succ_{\mathcal{M}} X$ iff $\sigma \succ_{\mathcal{M}} B$ for each $B \in X$.

3.1 Inquisitive Entailment

Inquisitive semantics defines entailment in terms of meaning inclusion: a wff A (inquisitively) entails a wff B iff the informative content of A is included in the informative content of B and the inquisitive content of A is included in the inquisitive content of B. However, in the case of InqB the second clause yields the first. Let us designate by $\|C\|$ the inquisitive content of wff C w.r.t. the canonical model $\mathcal{W}_{\mathcal{P}}$. More precisely:

$$\|C\| = \{\sigma \subseteq \mathcal{W}_{\mathcal{P}} : \sigma \succ C\}.$$

As for InqB, inquisitive entailment, $\models_{\mathsf{InqB}}$, can thus be defined as follows:

Definition 7 (*Inquisitive entailment*) $X \models_{\mathsf{InqB}} A$ iff $\bigcap_{B \in X} \|B\| \subseteq \|A\|$.

Let us now prove that inquisitive entailment can be characterized in terms of ι-models.

Theorem 1 $X \models_{\mathsf{InqB}} A$ *iff for each ι-model $\mathcal{M} = \langle W, V \rangle$ and each $\mathcal{M}$-state σ: if $\sigma \succ_{\mathcal{M}} X$, then $\sigma \succ_{\mathcal{M}} A$.*

Proof It suffices to prove that a state of an ι-model supports (in the model) all the wffs in X but does not support A if, and only if there exists a subset of $\mathcal{W}_{\mathcal{P}}$ which supports X and does not support A—regardless of whether $\mathcal{W}_{\mathcal{P}}$ comprises indices or sets of propositional variables.

(⇒) Suppose that for some ι-model $\mathcal{M} = \langle W, V \rangle$ and some $\mathcal{M}$-state σ it holds that $\sigma \succ_{\mathcal{M}} X$ and $\sigma \not\succ_{\mathcal{M}} A$. For each $w \in W$, let us assign to w the index/valuation, v_w, defined by:

$$v_w(\mathtt{p}) = V(\mathtt{p}, w), \quad \text{for any} \quad \mathtt{p} \in \mathcal{P} \tag{2}$$

Let $\Im$ be the set of indices assigned, in the above manner, to the elements of W. Given the assignment, there exists a surjection $f : W \mapsto \Im$, such that:

$$f(w) = v_w \tag{3}$$

We define:

$$f[\sigma] =_{df} \{f(w) : w \in \sigma\} \tag{4}$$

$f[\sigma]$ is the image of σ under f. Clearly:

(a) $f[\sigma] = \emptyset$ iff $\sigma = \emptyset$,
(b) if $w \in \sigma$, then $v_w \in f[\sigma]$,
(c) if $\tau \subseteq \sigma$, then $f[\tau] \subseteq f[\sigma]$.

Consider the following ι-model:

$$\mathcal{M}^* = \langle \Im, V^* \rangle \tag{5}$$

where $V^*(\mathtt{p}, f(w)) = f(w)(\mathtt{p})$. One can prove by induction that the following:

$$\sigma \succ_{\mathcal{M}} B \quad \text{iff } f[\sigma] \succ_{\mathcal{M}^*} B \tag{6}$$

holds for any $\mathcal{M}$-state σ and each wff B.

Let $B = \mathtt{p}$. We have:

$$\begin{gathered} \sigma \succ_{\mathcal{M}} \mathtt{p} \text{ iff} \\ \forall w \in \sigma : V(\mathtt{p}, w) = \mathbf{1} \text{ iff} \\ \forall w \in \sigma : f(w)(\mathtt{p}) = \mathbf{1} \text{ iff} \\ \forall v \in f[\sigma] : V^*(\mathtt{p}, v) = \mathbf{1} \text{ iff} \\ f[\sigma] \succ_{\mathcal{M}^*} \mathtt{p} \end{gathered}$$

Let $B = \bot$. We get:

$$\sigma \succ_{\mathcal{M}} \bot \text{ iff } \sigma = \emptyset \text{ iff} f[\sigma] = \emptyset \text{ iff} f[\sigma] \succ_{\mathcal{M}^*} \bot$$

Let B be of the form $C \to D$.

Induction hypothesis. For any $\mathcal{M}$-state σ:

$$\begin{gathered} \sigma \succ_{\mathcal{M}} C \quad \text{iff } f[\sigma] \succ_{\mathcal{M}^*} C \\ \sigma \succ_{\mathcal{M}} D \quad \text{iff } f[\sigma] \succ_{\mathcal{M}^*} D \end{gathered} \tag{7}$$

Suppose that $f[\sigma] \not\succ_{\mathcal{M}^*} (C \rightarrow D)$. So there exists $\gamma \subseteq f[\sigma]$ such that $\gamma \succ_{\mathcal{M}^*} C$ and $\gamma \not\succ_{\mathcal{M}^*} D$. Since $\gamma \subseteq f[\sigma]$, for some $\tau \subseteq \sigma$ we have $\gamma = f[\tau]$. Hence, by the induction hypothesis, $\sigma \not\succ_{\mathcal{M}} (C \rightarrow D)$.

Suppose that $\sigma \not\succ_{\mathcal{M}} (C \rightarrow D)$. Thus there exists a $\mathcal{M}$-state $\tau \subseteq \sigma$ such that $\tau \succ_{\mathcal{M}} C$ and $\tau \not\succ_{\mathcal{M}} D$. Clearly, $f[\tau] \subseteq f[\sigma]$. Thus, by the induction hypothesis, $f[\sigma] \not\succ_{\mathcal{M}^*} (C \rightarrow D)$.

The cases of disjunction and conjunction are straightforward.

Since (6) holds, we get:

$$f[\sigma] \succ_{\mathcal{M}^*} X \quad \text{and} \quad f[\sigma] \not\succ_{\mathcal{M}^*} A \tag{8}$$

Assume that $\mathcal{W}_{\mathcal{P}}$ is the set of indices (i.e. valuations of $\mathcal{P}$). Clearly, $f[\sigma] \subseteq \mathcal{W}_{\mathcal{P}}$. By (2), (3), and (4) we get:

$$f[\sigma] \succ_{\mathcal{M}^*} \mathtt{p} \quad \text{iff } f[\sigma] \succ \mathtt{p} \tag{9}$$

Thus the following can be easily proven:

$$f[\sigma] \succ_{\mathcal{M}^*} B \quad \text{iff } f[\sigma] \succ B \tag{10}$$

where B is any wff. Hence, by (8), $\bigcap_{B \in X} \|B\| \not\subseteq \|A\|$ and thus $X \not\models_{\mathsf{InqB}} A$.

Now assume that $\mathcal{W}_{\mathcal{P}} = \wp(\mathcal{P})$. Each index, v, in $f[\sigma]$ uniquely determines a subset, $\mathcal{P}_v$, of $\mathcal{P}$ such that $\mathcal{P}_v = \{\mathtt{p} \in \mathcal{P} : v(\mathtt{p}) = \mathbf{1}\}$. Let $\mathcal{P}_{f[\sigma]}$ stand for the set of subsets of $\mathcal{P}$ determined by $f[\sigma]$. Consider the following ι-model:

$$\mathcal{M}^{**} = \langle \mathcal{P}_{f[\sigma]}, V^{**} \rangle \tag{11}$$

where $V^{**}(\mathtt{p}, \mathcal{P}_v) = \mathbf{1}$ iff $\mathtt{p} \in \mathcal{P}_v$, for each $v \in f[\sigma]$. One can easily prove that the following:

$$f[\sigma] \succ_{\mathcal{M}^*} B \quad \text{iff } \mathcal{P}_{f[\sigma]} \succ_{\mathcal{M}^{**}} B \tag{12}$$

holds for any wff B. Hence, by (8):

$$\mathcal{P}_{f[\sigma]} \succ_{\mathcal{M}^{**}} X \quad \text{and} \quad \mathcal{P}_{f[\sigma]} \not\succ_{\mathcal{M}^{**}} A \tag{13}$$

But $\mathcal{P}_{f[\sigma]} \subseteq \wp(\mathcal{P})$ and the following holds:

$$\mathcal{P}_{f[\sigma]} \succ_{\mathcal{M}^{**}} \mathtt{p} \quad \text{iff } \mathtt{p} \in \mathcal{P}_v \quad \text{for each } v \in f[\sigma] \quad \text{iff } \mathcal{P}_{f[\sigma]} \succ \mathtt{p} \tag{14}$$

One can prove by induction that for each wff B:

$$\mathcal{P}_{f[\sigma]} \succ_{\mathcal{M}^{**}} B \quad \text{iff } \mathcal{P}_{f[\sigma]} \succ B \tag{15}$$

Thus, by (13), $X \not\models_{\mathsf{InqB}} A$.

(⇐) Suppose that $X \not\models_{\mathsf{InqB}} A$. Thus $\bigcap_{B \in X} \|B\| \not\subseteq \|A\|$. Hence there exists a subset σ of $\mathcal{W}_{\mathcal{P}}$ such that $\sigma \succ X$ and $\sigma \not\succ A$.

Assume that σ is a set of indices. We consider the following ι-model:

$$\mathcal{M}^{\backslash} = \langle \sigma, V^{\backslash} \rangle \tag{16}$$

where $V^{\backslash}(\mathrm{p}, v) = \mathbf{1}$ iff $v(\mathrm{p}) = \mathbf{1}$, for each $v \in \sigma$. One can easily prove by induction that for any wff B:

$$\sigma \succ B \quad \text{iff } \sigma \succ_{\mathcal{M}^{\backslash}} B \tag{17}$$

Hence $\sigma \succ_{\mathcal{M}^{\backslash}} X$ and $\sigma \not\succ_{\mathcal{M}^{\backslash}} A$.

Assume that $\sigma \subseteq \wp(\mathcal{P})$. Take the following ι-model:

$$\mathcal{M}^{\backslash\backslash} = \langle \sigma, V^{\backslash\backslash} \rangle \tag{18}$$

where $V^{\backslash\backslash}(\mathrm{p}, v) = \mathbf{1}$ iff $\mathrm{p} \in v$, for each $v \in \sigma$. As above, it can be shown that $\sigma \succ_{\mathcal{M}^{\backslash\backslash}} X$ and $\sigma \not\succ_{\mathcal{M}^{\backslash\backslash}} A$. □

As a consequence of Theorem 1 we get:

Corollary 5 $\models_{\mathsf{InqB}} A$ *iff for each ι-model $\mathcal{M}$ and each $\mathcal{M}$-state σ: $\sigma \succ_{\mathcal{M}} A$.*

Thus the conceptual setting proposed in this section is adequate for InqB: it retains inquisitive entailment and the propositional logic determined by InqB is complete w.r.t. the ι-models semantics.

Remarks As for the standard setting of InqB, a language of the considered kind is associated with exactly one model, the canonical model (see Section 2 for details). When $\mathcal{L}_{\mathcal{P}}$ is a language built over a set of propositional variables $\mathcal{P}$, the model of the language is just $\mathcal{W}_{\mathcal{P}}$. The proof of Theorem 1 shows that $\mathcal{W}_{\mathcal{P}}$ corresponds to an ι-model, $\mathcal{M}_{\mathcal{P}}$, whose domain is $\mathcal{W}_{\mathcal{P}}$, and for which we have $\succ = \succ_{\mathcal{M}_{\mathcal{P}}}$, that is, support in the canonical model $\mathcal{W}_{\mathcal{P}}$ set-theoretically equals support in $\mathcal{M}_{\mathcal{P}}$.[3] Hence the basic concepts of InqB, in particular inquisitiveness and informativeness, usually defined in terms of the canonical model, are definable in terms of ι-models. Note that the domain of $\mathcal{M}_{\mathcal{P}}$ can be thought of as representing the ignorant information state.

4 A Situational Account of InqB

The elements of domains of ι-models have been so far intuitively construed as possible worlds. The only formal restriction imposed on domains of ι-models was non-emptiness. Support is a relation between wffs and *sets* of elements of domains, that is, intuitively, sets of possible worlds. In this section we are going to define

[3] When $\mathcal{W}_{\mathcal{P}}$ comprises indices, the valuation function, V, of $\mathcal{M}_{\mathcal{P}}$ is defined by: $V(\mathrm{p}, v) = \mathbf{1}$ iff $v(\mathrm{p}) = \mathbf{1}$. If $\mathcal{W}_{\mathcal{P}} = \wp(\mathcal{P})$, we put $V(\mathrm{p}, v) = \mathbf{1}$ iff $\mathrm{p} \in v$.

the concept of truth of a wff in a possible world in terms of support. Clearly, this requires a new way of thinking about ι-models and a non-standard account of possible worlds.

4.1 Intuitions

The domain of an ι-model is a non-empty set. It is not excluded that the domain contains elements which are indistinguishable w.r.t. the valuation function, V, of the model: it may happen that for some distinct elements, w and w', of the domain, we have $V(\mathtt{p}, w) = V(\mathtt{p}, w')$ for each propositional variable $\mathtt{p}$ of the language in question. Now think of elements of the domain of an ι-model as *situations*, and of the valuation function V as follows: "$V(\mathtt{p}, w) = \mathbf{1}$" means "$\mathtt{p}$ holds in situation w", while the meaning of "$V(\mathtt{p}, w) = \mathbf{0}$" is: "it is not the case that $\mathtt{p}$ holds in situation w".[4] We do not attempt to define situations in general. As Devlin (1991, p. 70) puts it, "Situations are just that: situations". Yet, we tend to think that situations are, at least partially, determined by subjective factors. An agent categorizes the world according to his/her conceptual apparatus, and conceptualizes some phenomena as situations. But which phenomena are conceived as situations and which are not depends on an agent's decisions, though rarely conscious ones. Our only assumption concerning the realm of situations is: there may exist different situations which, nevertheless, agree on the level of "atomic facts". Thus situations are not supposed to be determined by the "atomic facts" that hold in them (by the logical values of propositional variables). This assumption reflects the underlying idea that situations are predominantly epistemic constructs and, as such, are dependant upon an agent's propositional attitudes and conceptual framework. However, if you are inclined to think of situations in purely ontic terms, the above assumption makes room for an indeterministic perspective.

At a given moment only some situations constitute the "point of reference" of an agent's cognitive or epistemic activities. The set of such situations can be labelled as "the (epistemic) world of situations the agent lives in (at a given moment)" and identified with a *possible world*. To be more precise, we construe the domain, W, of an ι-model $\langle W, V\rangle$ as a *single* possible world, $\widehat{W}$, comprising all the situations from W.[5] In order to keep things simple $\mathcal{M}$-states different from W (i.e. proper subsets of the domain) are not regarded as possible worlds.[6]

[4] Observe that $V(\mathtt{p}, w) = \mathbf{0}$ does not yield "$\neg\mathtt{p}$ holds in w', as V is defined only for atoms/propositional variables.

[5] Possible worlds in our sense are thus not situations, but *sets* of situations. Assuming that we are speaking about well-founded sets, no possible world is a situation. In order to avoid this consequence one can turn to a set theory that allows for non well-founded sets (e.g. Aczel's theory). In this setting $\widehat{W}$ would be the non well-founded set which, besides the elements of W, has also W as a member. It would make no harm to stipulate that $V(\mathtt{p}, W) = \mathbf{1}$ iff $V(\mathtt{p}, w) = \mathbf{1}$ for any $w \in W$ such that $w \neq W$.
A different version of an uncommitted "situational" semantics in which an assignment of a *set* of situations to a wff plays the basic role can be found in Wiśniewski (1997); see Wiśniewski (2013), pp. 33–45 for an English translation.

[6] Of course, each non-empty $\mathcal{M}$-state different from W can serve as the domain of another ι-model and thus determine another world. Our construction is not ontologically loaded.

4.2 Truth

What about truth? Let us introduce a certain auxiliary concept.

Definition 8 (*Truth in an ι-model*) A wff A is *true* in an ι-model $\mathcal{M}$ (in symbols: $\mathcal{M} \models A$) iff $\sigma \succ_{\mathcal{M}} A$ for each $\mathcal{M}$-state σ.

Note that the concept of truth in an ι-model is defined in terms of *support*. Since Corollary 1 holds, one can easily prove:

Corollary 6 *Let W be the domain of an ι-model $\mathcal{M}$.*

$$\mathcal{M} \models A \quad \textit{iff}\, W \succ_{\mathcal{M}} A.$$

Thus a wff is true in an ι-model just in case it is supported by the domain of the model. But recall that the (classical) truth of A in each $w \in W$ does not warrant that W supports A: the concept of support is stronger.

The domain, W, of an ι-model $\mathcal{M} = \langle W, V \rangle$ corresponds to a single possible world, $\widehat{W}$ (construed in the sense described above). Let us now assume that truth in $\widehat{W}$ equals truth in $\mathcal{M} = \langle W, V \rangle$. We get the following characteristics of the resultant concept of truth:[7]

Corollary 7 *Let "$\widehat{W} \models A$" abbreviate "A is true in $\widehat{W}$".*

1. $\widehat{W} \models \mathrm{p}$ *iff* $\|\mathrm{p}\|_{\mathcal{M}} = \wp(W)$,
2. $\widehat{W} \models (A \rightarrow B)$ *iff* $\|A\|_{\mathcal{M}} \subseteq \|B\|_{\mathcal{M}}$,
3. $\widehat{W} \models (A \vee B)$ *iff* $\|A\|_{\mathcal{M}} = \wp(W)$ *or* $\|B\|_{\mathcal{M}} = \wp(W)$,
4. $\widehat{W} \models (A \wedge B)$ *iff* $\|A\|_{\mathcal{M}} = \wp(W)$ *and* $\|B\|_{\mathcal{M}} = \wp(W)$,
5. $\widehat{W} \models \neg A$ *iff* $\|A\|_{\mathcal{M}} = \{\emptyset\}$.

Clause (1) yields that an atomic sentence/propositional variable p is true in $\widehat{W}$ iff p holds in *each* situation being an element of W. (2) states that an implication is true in $\widehat{W}$ just in case each set of situations included in W that supports the antecedent also supports the consequent. (3), in turn, says that a disjunction is true in $\widehat{W}$ iff each set of situations included in W supports a disjunct, while (4) states that a conjunction is true just in case each set of situations included in W supports both conjuncts. According to (5), a negation is true in $\widehat{W}$ iff the empty set is the only set included in W that supports the negated wff.

Needless to say, it can happen that a wff (even an atom/propositional variable!) is not true in $\widehat{W}$ and its negation is also not true in $\widehat{W}$.

So far so good. But the key assumption has been: support is defined as in InqB. Let us now test the consequences of this assumption against some natural-language examples.

Let W be a non-empty set of situations. Take the following sentence:

John is irritated. (19)

[7] Recall that $\|A\|_{\mathcal{M}}$ is the set of all $\mathcal{M}$-states that support A.

For simplicity, assume that "John" is a rigid designator w.r.t. elements of W. The sentence (19) is true in $\widehat{W}$ just in case John is irritated in *each* situation from W, and untrue otherwise.

Now let us analyse:

$$\textit{If John is irritated, then his cat is nervous.} \quad (20)$$

Observe that the truth of (20) in $\widehat{W}$ does not require any of "John is irritated" and "John's cat is nervous" be true in $\widehat{W}$. (20) is true in $\widehat{W}$ just in case each set of situations included in W that comprises situations in which John is irritated is *also* a set of situations in which John's cat is nervous. Thus an implication can be true in $\widehat{W}$ although neither its antecedent nor its consequent is true in $\widehat{W}$.[8] Is this an acceptable consequence? Well, think of counterfactuals and defaults.

Consider, in turn:

$$\textit{John is not irritated.} \quad (21)$$

where "not" is construed as sentential negation. (21) is true in $\widehat{W}$ iff no situation in which John is irritated belongs to W.

Next, let us consider:

$$\textit{John is irritated and his cat is nervous.} \quad (22)$$

(22) is true in $\widehat{W}$ just in case each set of situations included in W supports both conjuncts. Since the conjuncts are atomic sentences, it follows that (22) is true in $\widehat{W}$ iff John is irritated in each situation from W and his cat is nervous in each situation from W.

Let us analyse:

$$\textit{If John stays at home, then if John is irritated, his cat is nervous.} \quad (23)$$

One can easily prove that a formula of the form $A \rightarrow (B \rightarrow C)$ is supported by a state if and only if the corresponding formula $A \wedge B \rightarrow C$ is supported by the state.[9] Hence (23) is true in $\widehat{W}$ just in case each set of situations from W that comprises situations in which John stays at home and is irritated is also a set of situations in which John's cat is nervous.

Now let us consider:

$$\textit{John is irritated or his cat is nervous.} \quad (24)$$

[8] Recall that "A is not true in $\widehat{W}$" means "there is a subset of W that does not support A", and this is not tantamount to "$\neg A$ is true in $\widehat{W}$".

[9] For, suppose that $\sigma \succ_{\mathcal{M}} (A \rightarrow (B \rightarrow C))$, but $\sigma \not\succ_{\mathcal{M}} (A \wedge B \rightarrow C)$. Hence for some $\tau \subseteq \sigma : \tau \succ_{\mathcal{M}} A$, $\tau \succ_{\mathcal{M}} B$ and $\tau \not\succ_{\mathcal{M}} C$. Thus $\tau \succ_{\mathcal{M}} A$ and $\tau \not\succ_{\mathcal{M}} (B \rightarrow C)$, that is, $\sigma \not\succ_{\mathcal{M}} (A \rightarrow (B \rightarrow C))$.
Now suppose that $\sigma \not\succ_{\mathcal{M}} (A \rightarrow (B \rightarrow C))$. It follows that for some $\tau \subseteq \sigma$: $\tau \succ_{\mathcal{M}} A$ and $\tau \not\succ_{\mathcal{M}} (B \rightarrow C)$. Hence for some $\tau' \subseteq \tau$: $\tau' \succ_{\mathcal{M}} B$ and $\tau' \not\succ_{\mathcal{M}} C$. Therefore $\tau' \succ_{\mathcal{M}} (A \wedge B)$ and thus $\sigma \not\succ_{\mathcal{M}} (A \wedge B \rightarrow C)$.

(24) is true in $\widehat{W}$ iff John is irritated in each situation from W or John's cat is nervous in each situation from W. Interestingly enough, the "inquisitive" feature of disjunction disappears in the current setting: what we have got resembles the "disjunction property" known from Intuitionistic Logic, but this time pertaining not only to theses/theorems.

Let us analyse:

$$\textit{If John stays at home or John is irritated, then his cat is nervous.} \tag{25}$$

Now the claim is: each set of situations that comprises situations in which John stays at home is also a set of situations in which his cat is nervous, *and* each set of situations that comprises situations in which John is irritated is also a set of situations in which his cat is nervous. The reason is that a formula of the form $A \vee B \to C$ is supported by a state iff the formula $(A \to C) \wedge (B \to C)$ is supported by the state.

The case of:

$$\textit{If John is irritated when staying at home, then his cat is nervous.} \tag{26}$$

interpreted as an instance of $(p \to q) \to r$, is, despite the non-classical meaning of negation, similar. One can prove that a formula of the form $(A \to B) \to \mathtt{p}$ (where $\mathtt{p}$ is a propositional variable) is supported by a state just in case the corresponding formula $(\neg A \to \mathtt{p}) \wedge (B \to \mathtt{p})$ is supported by the state.[10] Thus the claim of (26) is: each set of situations that does not include a situation in which John stays at home is also a set of situations in which his cat is nervous, and each set of situations that comprises situations in which John is irritated is also a set of situations in which his cat is nervous.

Finally, let us consider:

$$\begin{array}{l}\textit{If John's cat is nervous, then John does not stay at home} \\ \quad \textit{or John is irritated.}\end{array} \tag{27}$$

The claim of (27) is: each set of situations that comprises situations in which John's cat is nervous is also a set of situations that does not include a situation in which John stays at home *or* each set of situations such that John's cat is nervous in any of these situations is also a set of situations in which John is irritated.

It seems that the results of the above considerations comply with intuitions.

[10] Let $\sigma \succ_{\mathcal{M}} ((A \to B) \to \mathtt{p})$, but $\sigma \not\succ_{\mathcal{M}} (\neg A \to \mathtt{p}) \wedge (B \to \mathtt{p})$. Thus: (a) $\sigma \not\succ_{\mathcal{M}} (\neg A \to \mathtt{p})$ or (b) $\sigma \not\succ_{\mathcal{M}} (B \to \mathtt{p})$. If (a) holds, then there exists $\tau \subseteq \sigma$ such that $\tau \succ_{\mathcal{M}} \neg A$ and $\tau \not\succ_{\mathcal{M}} \mathtt{p}$. Hence $\mathcal{M}, w \not\models \mathtt{p}$ for some $w \in \tau$, and, simultaneously, $\mathcal{M}, w \models \neg A$. Therefore $\mathcal{M}, w \not\models ((A \to B) \to \mathtt{p})$ and hence $\sigma \not\succ_{\mathcal{M}} ((A \to B) \to \mathtt{p})$. As for (b), we reason analogously.
Let $\sigma \succ_{\mathcal{M}} (\neg A \to \mathtt{p}) \wedge (B \to \mathtt{p})$, but $\sigma \not\succ_{\mathcal{M}} ((A \to B) \to \mathtt{p})$. Hence for some $\tau \subseteq \sigma$ we have: $\tau \succ_{\mathcal{M}} (A \to B)$ and $\tau \not\succ_{\mathcal{M}} \mathtt{p}$. Thus $\mathcal{M}, w \not\models \mathtt{p}$ for some $w \in \tau$. Therefore, by the initial assumption and Corollary 1 we get: $\mathcal{M}, w \models A$ as well as $\mathcal{M}, w \models \neg B$. On the other hand, if $\tau \succ_{\mathcal{M}} (A \to B)$ and $w \in \tau$, then $\mathcal{M}, w \models (A \to B)$. A contradiction.

4.3 Inquisitive Entailment Again

Inquisitive entailment viewed from the canonical point of view warrants the transmission of support. It can be proven, however, that inquisitive entailment *also* warrants transmission of truth understood in the above-sketched manner.

Theorem 2 $X \models_{\mathsf{InqB}} A$ *iff* A *is true in each* ι*-model in which all the wffs in* X *are true.*

Proof For conciseness, let "$\mathcal{M} \models X$" abbreviate "$\mathcal{M} \models B$ for each $B \in X$".

($\Rightarrow$) By Theorem 1 and Corollary 6.

($\Leftarrow$) Suppose that A is true in each ι-model in which all the wffs in X are true. Let $\mathcal{M} = \langle W, V \rangle$ be an arbitrary but fixed ι-model. Hence $\mathcal{M}$ satisfies the condition:

$$\text{if} \quad \mathcal{M} \models X, \quad \text{then } \mathcal{M} \models A \tag{28}$$

By Corollary 6 we get:

$$\text{if} \quad W \succ_{\mathcal{M}} X, \quad \text{then } W \succ_{\mathcal{M}} A \tag{29}$$

Suppose that for some $\mathcal{M}$-state σ we have:

$$\sigma \succ_{\mathcal{M}} X \quad \text{and} \quad \sigma \not\succ_{\mathcal{M}} A \tag{30}$$

Since $\sigma \not\succ_{\mathcal{M}} A, \sigma \neq \emptyset$. Consider the following ι-model

$$\mathcal{M}^* = \langle \sigma, V^* \rangle \tag{31}$$

where $V^*(\mathrm{p}, w) = V(\mathrm{p}, w)$ for any $w \in \sigma$. By Corollary 6 we get:

$$\mathcal{M}^* \models X \quad \text{and} \quad \mathcal{M}^* \not\models A \tag{32}$$

On the other hand, by assumption each ι-model which makes true all the wffs in X makes true A as well. Thus:

$$\mathcal{M}^* \models A \tag{33}$$

We arrive at a contradiction. Hence the following condition holds:

$$\text{for each} \quad \mathcal{M}\text{-state} \quad \sigma : \quad \text{if} \quad \sigma \succ_{\mathcal{M}} X, \quad \text{then } \sigma \succ_{\mathcal{M}} A \tag{34}$$

Recall that $\mathcal{M}$ is an arbitrary ι-model. Thus the analogue of (34) holds for any model for which the analogue of (28) is true. By assumption, the latter is true for each ι-model. Hence, by Theorem 1, $X \models_{\mathsf{InqB}} A$. □

5 Concluding Remarks

Inquisitive semantics in general, and InqB in particular have been designed as tools of modelling information exchange. This explains some peculiarities of the canonical account of InqB. A language in question has only one model since the "meanings" of the expressions used are supposed to be uniform. An information state of maximal consistent information is assumed to be reachable by linguistic means, and since states of this kind are modelled as singleton sets of worlds, one has to construe possible worlds in a way that excludes the existence of different worlds which agree on the values of propositional variables. Conceiving possible worlds as indices or sets of propositional variables secures this.

As for the ι-models semantics for InqB proposed in this paper, a language in question has many models, and elements of domains need not be distinguishable in terms of valuations. It is an open problem whether, and if so, how the alternative conceptual setting can be profitably used in the modelling of information exchange. Yet, as we have shown, the ι-models approach is more general but still retains InqB-entailment, and, on the other hand, facilitates the use of the formalism outside the realm of its intended applicability.

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