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EROTETIC SEARCH SCENARIOS, PROBLEM-SOLVING, AND DEDUCTION#

ANDRZEJ WIŚNIEWSKI

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1. *Introduction*

To say that questions and answers play important roles in problem-solving is to broadcast a slogan. But to show how they do it isn't that easy. Recently an interesting framework for handling this issue has been proposed by Jaakko Hintikka. This is the so-called Interrogative Model of Inquiry (hereafter IMI)[1].

The concept of *interrogative game* plays a central role in IMI. An interrogative game involves two parties: an Inquirer and an external source of information, called Nature or Oracle. In the simplest case the aim of a game is to prove a predetermined conclusion, which is an answer to the main question. In more complicated cases the aim is to prove at least one among previously specified sentences (which are regarded as possible answers to the main question) or to prove desideratum of the main question (roughly, the desideratum is a proposition which specifies the cognitive state of affairs which the Inquirer wants to be brought about)[2]. In each case it is assumed that the Inquirer has at his/her disposal some initial premises. The Inquirer

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is free to choose between: (a) *deductive moves*, in which conclusions are drawn from what has already been established and (b) *interrogative moves*, in which questions are addressed to a source of information. The choice is a matter of strategy. The only restriction imposed on questions that may occur in interrogative moves is that the presuppositions of these questions have to be established, i.e. must be conclusions of some earlier deductive move(s) or belong to the initial premises. But the inferences which take place in an interrogative game have declarative sentences as premises and conclusions. Questions do not serve as premises or conclusions: they are devices by means of which new relevant information comes into play (of course, with the exception of the main question, which specifies the aim of the game).

But what if we allow questions to be being used as premises for further questions? This step is, in a sense, natural: we often transform questions into further questions whose answers are more accessible by available means. IMI does not have a logical apparatus to handle this, however. But there is a logic, called Inferential Erotetic Logic (IEL for short), which may be helpful here. IEL defines the concept of validity of inferences which involve questions and explicates the concept of search scenario. The aim of this paper is to show some applications of the tools elaborated within IEL in the area of problem solving.

We start by pointing out some changes in the general picture which stem from taking into consideration inferences that have questions as premises and conclusions. Then we turn to the role played by erotetic search scenarios (defined within the framework of IEL) in problem-solving.

2. *The General Setting*

First, we assume that the main problem of an inquiry is expressed by a question. This is a relatively safe assumption. But the second assumption is less safe: we assume that this question is well-defined, that is, satisfies Hamblin's postulate:

> [H] *Knowing what counts as an answer is equivalent to knowing the question.*

This will be explicated as:

> [DA] *One can specify the set of direct answers.*[3]

[3] For different approaches to the problem of answerhood see, e.g., the survey paper Harrah (2002).

Pragmatically, a direct answer is a possible and just-sufficient answer. Thus, intuitively, direct answers are the possibilities offered by a question. Direct answers are declarative sentences. We also assume that any auxiliary question of an inquiry should satisfy the condition [DA]. Of course, the above assumptions restrict the range of applicability of our proposal. But we do not aim at a complete theory of problem-solving.

We use dQ for the set of direct answers to a question Q.

A direct answer is a possible answer and thus may be true or false. A question is said to be *sound* if and only if at least one direct answer to it is true, and *unsound* otherwise. We neither assume nor deny that the main question is sound, and similarly for other questions which may occur in an inquiry.

Problem-solving always takes place in some environment. We model this by saying that there exists a set of *initial premises*; these premises are regarded as reliable and so are their logical consequences.

Yet, we are not interested in procedures which amount to picking up a direct answer to the main question from the initial premises. For this reason we assume that no direct answer to the main question belongs to the set of initial premises. This does not mean, however, that a direct answer to the main question cannot be "extracted" from the initial premises by performing some inferential moves. But we admit the possibility that in order to solve the main problem new information should be acquired.

We assume that during the search for a solution to the problem expressed by a question three kinds of activities are permitted:

- *valid inferential moves*;
- *hypothetical moves*;
- *information-gaining moves*.

2.1. *Inferential Moves*

Inferential moves may be either *standard* or *erotetic*. Standard moves are inferences which have declarative sentences as premises and conclusions. Inferential erotetic moves are erotetic inferences. An *erotetic inference* has a question as the conclusion, whereas the premises consist of a question and possibly some declarative sentence(s).[4] Validity of standard inferential moves is defined in terms of entailment; the transmission of truth is the underlying idea. Validity of erotetic inferences is defined in terms of *erotetic implication*; the underlying ideas are:

[4] In this paper we disregard erotetic inferences which have declarative sentences as premises and questions as conclusions; IEL analyses them as well.

(C_1) (*transmission of soundness/truth into soundness*) if the implying question is sound and all the declarative premises are true, then the implied question must be sound,

(C_2) (*open-minded cognitive usefulness*) each direct answer to the implied question is potentially useful, on the basis of the declarative premises, for finding an answer to the implying question.

Condition (C_2) is then clarified by requiring that each direct answer to the implied question should, together with the declarative premises, *narrow down* the class of possibilities offered by the implying question. This idea is explicated in semantic terms (see below).

Erotetic implication can be defined for a wide class of formalized languages (cf. Wiśniewski 1994, 1995, 1996, 2001). Let us consider the simplest case, however.

2.1.1. *An Illustration: Erotetic Implication and Classical Propositional Calculus*

Let L be the language of Classical Propositional Calculus (hereafter CPC); we assume that the connectives $\neg$, $\rightarrow$, $\wedge$, $\vee$, $\equiv$ occur in L. We use p, q, r, s, t, u, w, $p_1, \ldots$ for propositional variables. The concept of well-formed formula of L is defined as usual. The language *L* is an extension of L. *L* has well-formed formulas (wffs) of two kinds: declarative and erotetic. *Declarative well-formed formulas* (d-wffs for short) of *L* are the well-formed formulas of L, exclusively. Erotetic wffs of *L* are questions of the language. A *question* of *L* is an expression of the form:

(2.1) $\quad ?\,\{A_1, A_2, ..., A_n\}$

where $n > 1$ and $A_1, A_2, ..., A_n$ are nonequiform (i.e. syntactically distinct) d-wffs of *L*. If $?\,\{A_1, A_2, ..., A_n\}$ is a question, then each of the d-wffs $A_1, A_2, ..., A_n$ is called a *direct answer* to this question. Note that each question of *L* has at least two direct answers.

A question of the form (2.1) can be read: "Is it the case that A_1, or is it the case that A_2, ..., or is it the case that A_n?". However, in some special cases a different reading can be recommended. Moreover, for the sake of concision we adopt here some notational conventions. Questions of the form:

(2.2) $\quad ?\,\{A, \neg A\}$

called *simple yes-no questions*, can be read "Is it the case that A?", and are abbreviated as:

(2.3) $?A$

The d-wffs A and $\neg A$ are the *affirmative answer* and the *negative answer*, respectively. Questions falling under the schema:

(2.4) $?\{A \wedge B, A \wedge \neg B, \neg A \wedge B, \neg A \wedge \neg B\}$

can be read "Is it the case that A and is it the case that B?"; they may be called (binary) *conjunctive questions*, and are abbreviated as:

(2.5) $? \pm \mid A, B \mid$

We use the Greek lower-case letters φ, ψ, γ, with subscripts if needed, as metalinguistic variables for d-wffs and questions. The letters A, B, C, D are metalinguistic variables for d-wffs, and the letters X, Y are metalinguistic variables for sets of d-wffs. The symbols Q, Q_1, ... will be used as metalinguistic variables for questions. As above, the set of direct answers to a question Q will be referred to as dQ. In the metatheory we use the standard set-theoretical terminology and notation.

Let *D* be the set of all the d-wffs of *L*. Let $\{1, 0\}$ be the set of truth values (where 1 stands for Truth, and 0 for Falsehood). A CPC-*valuation* is a function from *D* to the set $\{1, 0\}$, defined in the standard manner. Let *v* be a CPC-valuation. We say that a d-wff A is *true* under *v* if $v(A) = 1$, and *false* under *v* if $v(A) = 0$. Since we will be dealing only with CPC- valuations, in what follows by a valuation we mean a CPC-valuation.

A d-wff A is *CPC-valid* iff A is true under every valuation.

Note that a valuation is a function from the set of d-wffs and thus one cannot apply the concept of valuation to questions. We do not assign Truth or Falsehood to questions. But in the case of questions we use the (relativized) concept of soundness. We say that a question Q is *sound* with respect to a valuation *v* (*v*-sound for short) iff at least one direct answer to Q is true under *v*.

Next we introduce the concept of multiple-conclusion entailment (mc-entailment for short; cf., e.g., Shoesmith and Smiley 1978). Mc-entailment is a relation between *sets* of d-wffs of *L*. We say that a set of d-wffs X *multiple-conclusion entails* a set of d-wffs Y iff the following condition holds:

(#) for each valuation *v*: if all the d-wffs in X are true under *v*, then at least one d-wff in Y is true under *v*.

The standard concept of (single-conlusion) entailment can now be defined as mc-entailment of a singleton set (*i.e.* X *entails* A iff X mc-entails $\{A\}$).

Erotetic implication is defined by:

Definition 1: *A question* Q implies *a question* Q_1 on the basis of *a set of d-wffs* X (*in symbols*: $\mathsf{Im}(Q, X, Q_1)$) *iff*

(i) *for each* $A \in \mathsf{d}Q : X \cup \{A\}$ *mc-entails* $\mathsf{d}Q_1$, *and*

(ii) *for each* $B \in \mathsf{d}Q_1$ *there exists a non-empty proper subset* Y *of* $\mathsf{d}Q$ *such that* $X \cup \{B\}$ mc-entails Y.

If $X = \emptyset$, then we say that Q *implies* Q_1 and we write $\mathsf{Im}(Q, Q_1)$.

Since we consider here only questions which have finite sets of direct answers, mc-entailment of $\mathsf{d}Q$ reduces to entailment of a disjunction of all the elements of $\mathsf{d}Q$, and mc-entailment of a finite proper subset of $\mathsf{d}Q_1$ is tantamount to entailment of a disjunction of all the elements of the subset. Thus it is possible to define erotetic implication for *L* without applying the concept of mc-entailment. In the general case, however, one has to use mc-entailment when defining erotetic implication.

2.2. *Further Remarks on Inferential Erotetic Moves*

Once erotetic implication is defined, so is validity of erotetic inferences.

An erotetic inferential move may pertain to the main question of an inquiry. This reflects the usual situation, in which we transform an initial question into a question whose answers are, in some sense, more accessible. An erotetic inferential move may also involve as a premise a question which is the conclusion of a previous erotetic inferential move. Moreover, the declarative premises may involve not only the initial premises, but also items of information introduced in information-gaining moves or hypothetical moves. Let us stress that we do not require that once a question is arrived at, it must be asked and answered. We allow for a situation in which a question which is the conclusion of an erotetic inferential move is used only as a premise of a further erotetic move. Thus auxiliary questions are either *queries* (i.e. questions which are asked and answered) or *non-queries* (i.e. questions which serve only as premises in erotetic inferential moves).

2.3. *Hypothetical Moves and Information-gaining Moves*

A *hypothetical move* is an introduction of a direct answer to a query. An *information-gaining* move amounts to an introduction of a reliable direct answer to a query. Thus when a hypothetical move is performed, a sentence is introduced only for the reason that it is a direct answer to a query. When an information-gaining move is performed, an answer is introduced for stronger reason(s). The relevant reasons are: (a) the just-introduced answer is an initial premise (recall that initial premises are considered as reliable); (b) the just-introduced answer is given by an external source of information which is considered as reliable, or (c) the just-introduced answer is a conclusion of a valid (standard) inference whose premises are considered as reliable.

3. *Erotetic Derivations and Search Scenarios*

Erotetic derivations and erotetic search scenarios have been formally defined elsewhere.[5] But in order to make this paper self-contained we have to say a few words about them. We will start from a semi-formal presentation and concentrate upon the underlying intuitions; some comments and developments presented below are new. The schematic definitions gain exact contents after specifying the syntax and semantics of the language considered. In particular, when we replace "declarative sentences" by "d-wffs" and conceive "entailment" and "erotetic implication" according to the definitions introduced in Section 2.1.1, we get the relevant concepts for the language *L*.

3.1. *Erotetic Derivations*

Inferential moves supplemented with hypothetical moves and/or information-gaining moves can be *arranged into* erotetic derivations.

An erotetic derivation is goal-directed: it starts with a question and aims at a direct answer to it. The remaining items of the derivation are auxiliary questions and/or declarative sentences. An auxiliary question must be (erotetically) implied by some earlier item(s) of the derivation; it is assumed that no auxiliary question is equivalent to (*i.e.* has the same set of direct answers as) the first question. A declarative sentence involved in an erotetic derivation is either an initial premise, or is a (direct) answer to an auxiliary question (in this case it is introduced immediately after the question), or is entailed by some earlier item(s) of the derivation. To be more precise, we have:

[5] Cf. Wiśniewski (2003); see also Wiśniewski (2001).

Definition 2: *A finite sequence* $\mathsf{e} = \varphi_1, ..., \varphi_n$ *is an* erotetic derivation (*e-derivation for short*) of a direct answer A to a question Q on the basis of a set of declarative sentences X *iff any* $\varphi_i (1 \leq i \leq n)$ *is either a question or a declarative sentence,* $\varphi_1 = Q$*,* $\varphi_n = A$*, and the following conditions hold:*

(1) *for each question* φ_k *of* e *such that* $k > 1$:
  (a) $\mathsf{d}\varphi_k \neq \mathsf{d}Q$*, and*
  (b) φ_{k+1} *is either a question or a direct answer to* φ_k;

(2) *for each declarative sentence* φ_j *of* e:
  (a) $\varphi_j \in X$*, or*
  (b) φ_j *is a direct answer to* φ_{j-1}*, where* $\varphi_{j-1} \neq Q$*, or*
  (c) φ_j *is entailed by a certain set of declarative sentences such that each element of this set precedes* φ_j *in* e;

(3) *for each question* φ_k *of* e *such that* $\varphi_k \neq Q$: φ_k *is (erotetically) implied by a certain question* φ_i *which precedes* φ_k *in* e *on the basis of the empty set, or on the basis of a set of declarative sentences such that each element of this set precedes* φ_k *in* e.

An element φ_k (where $1 < k < n$) of an e-derivation $\mathsf{e} = \varphi_1, \ldots, \varphi_n$ is a *query* of e if φ_k is a question and φ_{k+1} is a direct answer to φ_k. Thus queries are defined syntactically. Note that an e-derivation may involve auxiliary questions that are not queries.

There are e-derivations which involve only one question, i.e. the initial one. We call them *erotetically trivial*. Observe that a two-term sequence $< Q, A >$, where $A \in \mathsf{d}Q$, is an e-derivation of A only if $A \in X$. An *erotetically nontrivial* e-derivation must contain at least one query.

3.2. *Erotetic Search Scenarios*

Some families of interconnected e-derivations (of direct answers to a given question) constitute *erotetic search scenarios*.

Definition 3: *A finite family* Φ *of e-derivations is an* erotetic search scenario for a question Q relative to a set of declarative sentences X *iff each element of* Φ *is an e-derivation of a direct answer to* Q *on the basis of* X *and the following conditions hold:*

(1) $\mathsf{d}Q \cap X = \emptyset$;

(2) Φ *contains at least two elements*;

(3) *for each element* $\mathsf{e} = \varphi_1, \varphi_2, \ldots, \varphi_n$ *of* Φ*, for each index* k *such that* $1 \leq k < n$:
  (a) *if* φ_k *is a question and* φ_{k+1} *is a direct answer to* φ_k*, then for each direct answer* B *to* φ_k*, the family* Φ *contains a certain*

e-derivation $\mathsf{e}' = \psi_1, \psi_2, \ldots, \psi_m$ *such that* $\psi_j = \varphi_j$ *for* $j = 1, \ldots, k$*, and* $\psi_{k+1} = B$;

(b) *if* φ_k *is a declarative sentence, or* φ_k *is a question and* φ_{k+1} *is not a direct answer to* φ_k*, then for each e-derivation* $\mathsf{e}' = \psi_1, \psi_2, \ldots, \psi_m$ *in* Φ *such that* $\psi_j = \varphi_j$ *for* $j = 1, \ldots, k$*, we have* $\psi_{k+1} = \varphi_{k+1}$.

We shall use the term "e-scenario" instead of the long expression "erotetic search scenario". The elements of e-scenarios are called *paths*. If a path has a direct answer A as its last element, we say that this path *leads to* A. The elements of an appropriate set X will be called *initial premises*. If Φ is an e-scenario for Q relative to the empty set, we simply say that Φ is an e-scenario *for* Q.

A *query* of an e-scenario is defined as a query of a path of the e-scenario.

Clause (3a) of Definition 1 expresses the idea of *fairness with respect to queries*: if a direct answer C to a query is introduced at a path e, then for *any* direct answer B to the query that is different from C, there exists a path e' which is identical with e to the level of the query, and then has B at the place where e has C. Thus, roughly, for any path and any query on that path there exists a cluster of related paths which have the query and its predecessors in common (that is, "go through" the query and its predecessors), but diverge with respect to the direct answers to the query. Moreover, each direct answer to a query is "used" at some path of the cluster. Clause (3b), in turn, expresses the idea of *regularity*: if $\varphi_k (k < n)$ is a declarative sentence of a path $\mathsf{e} = \varphi_1, \varphi_2, ..., \varphi_n$, or φ_k is a question of e which is not a query, then each path which is identical with e to the level of φ_k has φ_{k+1} as the $k + 1$st element. In other words, declarative sentences and questions that are not queries are "used" within a cluster of related paths in the same manner; only queries are branching points of e-scenarios.

E-scenarios can be displayed in the form of diagrams showing downward trees; the paths of an e-scenario are represented by the branches of a tree, the trunk included. Two simple examples are presented below:

Example 1.

We make use of the following facts about erotetic implication in *L*:

(3.1) $\mathsf{Im}(?\, p, p \equiv q \wedge r, ?\, (q \wedge r))$

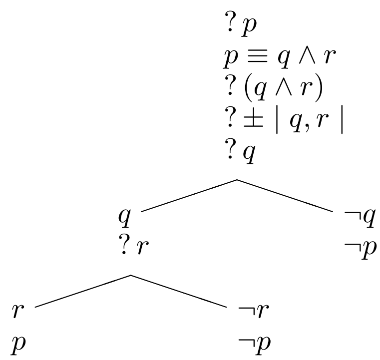
(3.2) $\mathsf{Im}(?\, (q \wedge r), ?\pm \mid q, r \mid)$

(3.3) $\mathsf{Im}(?\pm \mid q, r \mid, ?\, q)$

(3.4) $\mathsf{Im}(?\pm \mid q, r \mid, ?\, r)$

An e-scenario for $?\, p$ relative to $\{p \equiv q \wedge r\}$ is displayed in:

Figure 1.



Note that since we do not have $\mathsf{Im}(?\, (q \wedge r), ?\, q)$, an introduction of $?\pm \mid q, r \mid$ is needed as an intermediate step (in order to retain erotetic implication). Of course, question $?\pm \mid q, r \mid$ is not a query.

Example 2.

The following hold:

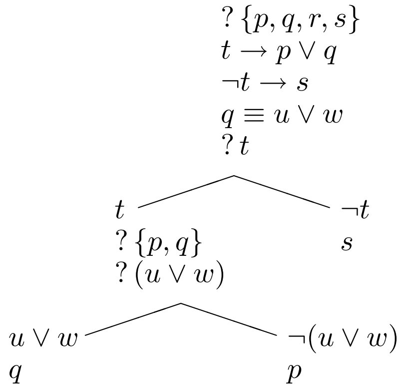
(3.5) $\mathsf{Im}(?\, \{p, q, r, s\}, t \rightarrow p \vee q, \neg t \rightarrow s, ?\, t)$

(3.6) $\mathsf{Im}(?\, \{p, q, r, s\}, t \rightarrow p \vee q, t, ?\, \{p, q\})$

(3.7) $\mathsf{Im}(?\, \{p, q\}, t \rightarrow p \vee q, t, q \equiv u \vee w, ?\, (u \vee w))$

An e-scenario for $?\, \{p, q, r, s\}$ relative to the set $\{t \rightarrow p \vee q, \neg t \rightarrow s, q \equiv u \vee w\}$ is displayed in:

Figure 2.



For further examples see Wiśniewski (2003).

Note finally that e-scenarios are either complete or incomplete. An e-scenario for Q relative to X is *complete* if each direct answer to Q is the endpoint of some path of the scenario, and *incomplete* otherwise. Figure 1 shows an example of a complete e-scenario, whereas the scenario shown in Figure 2 is incomplete (since there is no path which leads to r).

3.3. *Search Plans*

Both e-derivations and e-scenarios are abstract entities: sequences of expressions or families of sequences of them. An erotetically nontrivial (i.e. involving at least one query) e-derivation can be viewed as a result of performing a series of inferential moves together with information-gaining and/or hypothetical moves. But some erotetically nontrivial e-derivations can also be conceived differently, as *series of conditional instructions pertaining to the search for information*. An example may help to clarify matters here. The following is an e-derivation of the answer p to the question $?\,p$ on the basis of $p \equiv q \wedge r$:

(3.8) $?\,p$
(3.9) $p \equiv q \wedge r$
(3.10) $?\,(q \wedge r)$
(3.11) $?\pm \mid q, r \mid$
(3.12) $?\,q$
(3.13) q
(3.14) $?\,r$
(3.15) r
(3.16) p

Questions (3.12) and (3.14) are queries. The affirmative answer p to (3.8) is not entailed by the initial premise (3.9). Yet it is entailed by (3.9) together with the answers (3.13) and (3.15) to the queries. So in order to know that p when we already know that $p \equiv q \wedge r$, it is sufficient to know that (3.13) and (3.15) hold. Let us now put in differently. When you look for an answer to (3.8) on the basis of (3.9), first ask the question (3.12). If you have got the answer (3.13), then ask question (3.14). If you have got the answer (3.15) to it, then, finally, p is the answer to (3.8). We can express this by adding self-explanatory comments ("marks") at the margin ("I-PREM." abbreviates "initial premise"):

(3.8′) $?\,p$ [MAIN]
(3.9′) $p \equiv q \wedge r$ [I.PREM.]
(3.10′) $?\,(q \wedge r)$
(3.11′) $?\pm \mid q, r \mid$
(3.12′) $?\,q$ [ASK]
(3.13′) q [IF]
(3.14′) $?\,r$ [ASK]
(3.15′) r [IF]
(3.16′) p

Auxiliary questions that are not queries are left unmarked; they are necessary premises of erotetic inferences involved (since erotetic implication is not "transitive"), but do not function as requests for information.[6]

But what if $\neg q$ happens to be the true answer to query (3.13)? *There is no instruction that pertains to this case.* However, the above e-derivation is the leftmost path of the e-scenario presented in Figure 1. Let us now supplement the diagram presented in Figure 1 with marks in an analogous manner:

Figure 3.

$?\,p$ [MAIN]
$p \equiv q \wedge r$ [I.PREM.]
$?\,(q \wedge r)$
$?\pm \mid q, r \mid$
$?\,q$ [ASK]

[IF] q — $\neg q$ [IF]
[ASK] $?\,r$ — $\neg p$

[IF] r — $\neg r$ [IF]
p — $\neg p$

[6] Similarly, we do not mark declarative sentences that are neither initial premises nor answers to queries (if there are such sentences; in the case of the present example there are not).

The above diagram provides us with conditional instructions which tell us what auxiliary questions should be asked and when they should be asked. Moreover, it shows where to go if such-and-such a direct answer to a query appears to be acceptable and does so with respect to any direct answer to each query. On the other hand, the answers to queries that occur at a path indicate what information is needed in order to reach the answer that is the endpoint of the path.

One can prove (cf. Wiśniewski 2003) that e-scenarios have the golden path property: *if the main question of an e-scenario is sound and all the initial premises are true*[7]*, then at least one path of the scenario leads to a true direct answer to the main question*; this path involves only sound auxiliary questions and only true declarative sentences (and among them true answers to queries). Thus, roughly, an e-scenario presents not only a search plan, but a "safe" search plan. Moreover, an e-scenario presents a "no-halt" plan: each direct answer to any query opens further possibilities. This is why search scenarios have priority over erotetic derivations[8].

Designing an e-scenario is one thing, executing it is another. Once an e-scenario is ready, the next thing to do is to ask consecutive queries and to make an appropriate use of consecutive answers.

But is it the case that information-gaining moves are inevitable in *any* successful problem-solving procedure based on e-scenarios? The answer to this general question is "no": although new information is needed in most cases, sometimes the *right solution* can be found without performing any information-gaining moves. In these cases *a systematic reflection on possible ways of reaching alternative solutions is sufficient in order to establish*

[7] Of course, truth and soundness have to be relativized to the underlying semantics (thus 'true' means 'true in a model', or 'true under a valuation' etc., and similarly for soundness).

[8] A digression: the e-scenarios displayed above are *information-picking*, that is, fulfill the following condition:

> (#) the last element of a path (i.e. a direct answer to the main question) is entailed by the set made up of all the other declarative sentences that occur at the path, but is not entailed by any proper subset of this set.

Thus, roughly, all the declarative sentences, answers to queries included, are necessary for the outcome. Moreover, the analyzed e-scenarios are in the canonical form (all the initial premises precede the first query) and are concise (the only declarative sentences which occur at their paths are either initial premises, or answers to queries, or answers to the main question). There are e-scenarios which do not have these properties, however. But e-scenarios which have them seem to be especially useful in problem-solving.

the right solution. This systematic reflection is a kind of diagrammatic reasoning which operates on e-scenarios only. We will come back to this issue in Section 4.

3.4. *Systematic Embeddings*

Erotetic search scenarios can be designed by applying some underlying logic of questions (which determines erotetic implication) and logic of declaratives (which determines entailment). Thus it is possible to design a search scenario prior to making an attempt to solve a problem; a reasonable strategy is to construct a scenario whose queries are accessible, i.e. answers to them can be established by available means. Of course, in order to design a domain-specific e-scenario we also need some relevant pieces of knowledge, and a cooperation between a logician and an expert seems inevitable. Assume, however, that we have at our disposal a collection of e-scenarios. Is it possible to design new e-scenarios on the basis of these?

As it is shown in Wiśniewski (2003), e-scenarios can be obtained from e-scenarios by *embedding*. The basic idea is the following. We have an e-scenario Φ for Q relative to X, and a query Q^* of Φ. We also have a complete e-scenario Ψ for Q^* relative to Y. Then we embed Ψ into Φ; the result (provided that some conditions are met) is a new e-scenario for Q relative to $X \cup Y$. Roughly, this new e-scenario includes some instructions as to how (i.e. "by means of what queries") Q^* can be answered, and makes use of possible answers to Q^* in the same manner as it happens in Φ. The operation of embedding is described in exact terms in Wiśniewski (2003). However, since e-scenarios are displayed as trees, an embedding can also be viewed as a kind of diagrammatic operation. It is even possible to formulate diagrammatic rules which enable successful embedding. In what follows we will propose some exemplary rules of this kind. For simplicity, we restrict ourselves to the propositional case.

3.4.1. *An Example from CPC*

Let us now come back to the language *L* characterized in Section 2.1.2. In what follows we will be applying the syntactic and semantic concepts introduced there. The concepts of erotetic derivation and erotetic search scenario are defined according to the patterns presented by Definition 2 and Definition 3, respectively; we replace "declarative sentences" by "d-wffs" and conceive "entailment" and "erotetic implication" accordingly.

It is easy to show that the following are facts about erotetic implication in *L*:

(3.17) $\mathsf{Im}(?\,(B \oplus C), ?\pm \mid B, C \mid)$, where $\oplus$ is any of: $\rightarrow$, $\wedge$, $\vee$, $\equiv$,
(3.18) $\mathsf{Im}(?\,\neg B, ?\,B)$,
(3.19) $\mathsf{Im}(?\pm \mid B, C \mid, ?\,B)$,
(3.20) $\mathsf{Im}(?\pm \mid B, C \mid, ?\,C)$.

Thus the following show schemata of complete e-scenarios for simple yes-no questions that are based on compound d-wffs; we shall call them *standard e-scenarios*:

Figure 4.

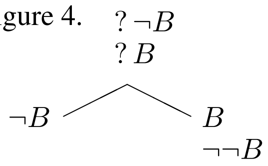


Figure 5.

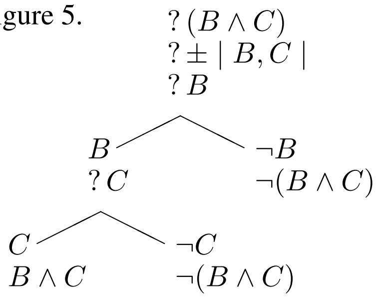


Figure 6.

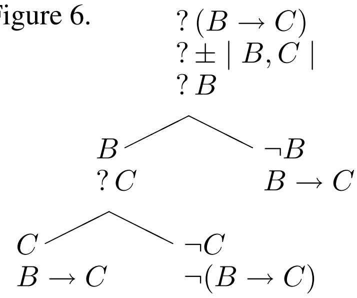


Figure 7.

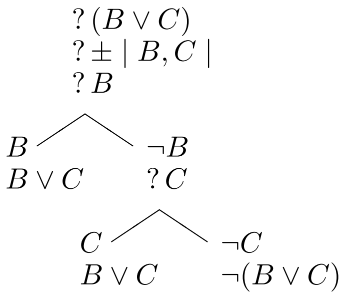
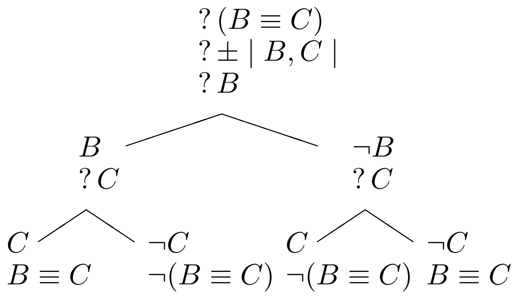


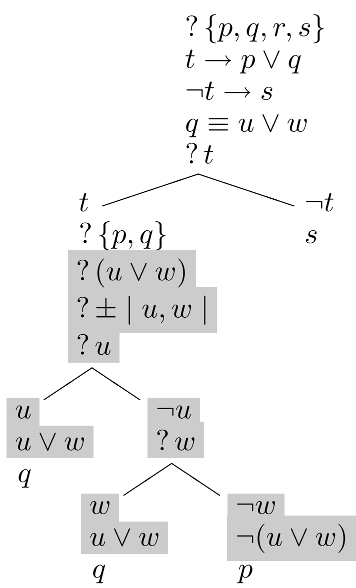
Figure 8.



Note that the above figures show e-scenarios *for* the main question, that is, e-scenarios relative to the empty set.

Now let us come back to the e-scenario displayed in Figure 2 (cf. Section 3.2). This scenario involves the question $?\,(u \vee w)$ as a query. We can transform the scenario by embedding, using Figure 7 as the source of instruction. As the result we get a new e-scenario, displayed in:

Figure 9.



For transparency, we have highlighted the embedded e-scenario.

Observe that what occurs after $u \vee w$ in the initial scenario, now occurs after it in the new scenario, and similarly for $\neg(u \vee w)$. Note that since the embedded scenario does not involve any declarative premises, the new e-scenario is still relative to the same set of d-wffs as the initial e-scenario.

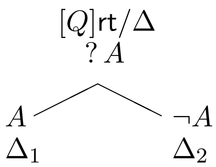
The transition from the initial e-scenario to the new e-scenario may be regarded as an application of a diagrammatic rule. Let us now turn to these rules.

3.4.2. *Some Diagrammatic Rules*

We say that a question Q is *equivalent to* a question Q^* iff $\mathsf{d}Q = \mathsf{d}Q^*$. Thus equivalence of questions is understood here as a set-theoretical relation, and not as an inferential relation.

Recall that a query of an e-scenario is a query of a path of the scenario, and that queries are defined syntactically: an element φ_k (where $1 < k < n$) of an e-derivation $\mathsf{e} = \varphi_1, ..., \varphi_n$ is a *query* of e if φ_k is a question and φ_{k+1} is a direct answer to φ_k. Now assume that φ_k is a query of e. Let us call the sequence $\varphi_1, ..., \varphi_{k-1}$ the *root* of φ_k at e. When e is a path of an e-scenario Φ and φ_k is a query of e, the family Φ splits into two subfamilies: Φ^* and Δ. The family Φ^* comprises all the paths of Φ which begin with the root of φ_k at e, have the question φ_k as a query, and then develop in different directions. The family Δ consists of all the remaining paths of Φ (of course, Δ is empty when φ_k is the first query of every path of the scenario). The following:

(I)

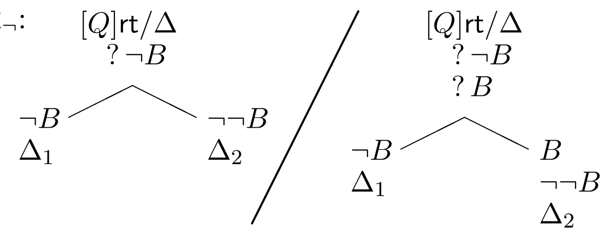


represents an e-scenario, which has Q as the main question (*i.e.* Q is the first element of any path), and is the union of two disjoint families of e-derivations:

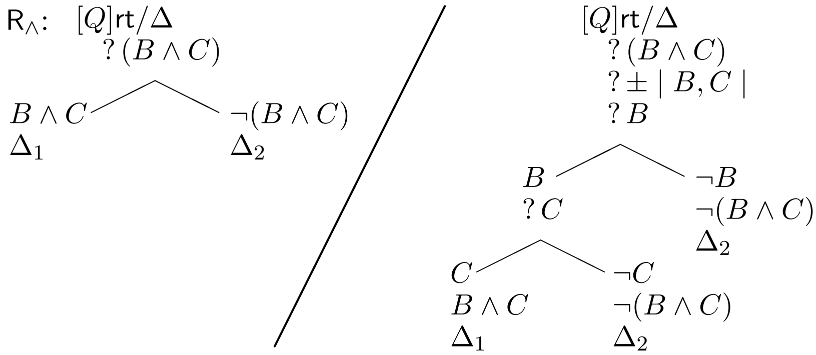
(a) e-derivations which have $?\,A$ as a query and a sequence of wffs rt before the query, and next either have A and a sequence of wffs that belongs to a family Δ_1, or have $\neg A$ and a sequence of wffs which belong to a family Δ_2 (note that rt is the root of the query in each case), and
(b) e-derivations which do not fulfill the condition (a).

Any of Δ, Δ_1, Δ_2 can be empty. By using the above notation we can express *diagrammatic rules of successful embedding*, which enable us to transform e-scenarios into new e-scenarios.

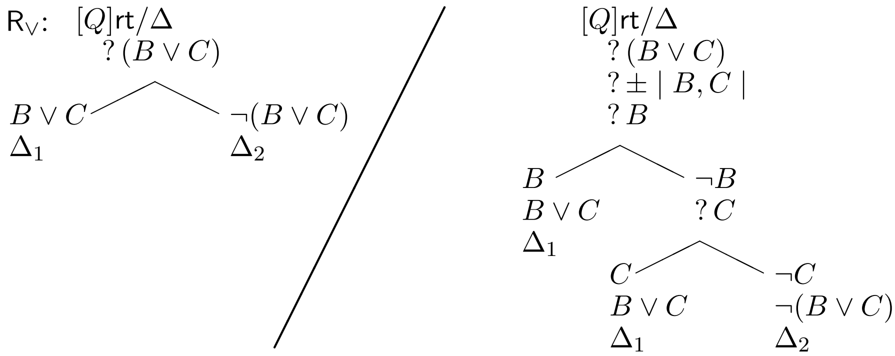
$\mathsf{R}_{\neg}$:



provided that Q *is not equivalent to* $?\,B$.



provided that Q is not equivalent to any of: $?\pm \mid B, C \mid, ?\,B, ?\,C$.



provided that Q is not equivalent to any of: $?\pm \mid B, C \mid, ?\,B, ?\,C$.

A proviso is always needed because of the requirement (1a) of the Definition 2 (of erotetic derivation). Rules $\mathsf{R}_\rightarrow$ and $\mathsf{R}_\equiv$ for e-scenarios which involve queries of the form $?\,(A \rightarrow B)$ and $?\,(A \equiv B)$ can be formulated in a similar manner, by using the schemes displayed in Figures 6 and 8, respectively. We leave this to the reader. We use the symbol Ξ for the set $\{\mathsf{R}_\neg, \mathsf{R}_\wedge, \mathsf{R}_\vee, \mathsf{R}_\rightarrow, \mathsf{R}_\equiv\}$ of rules defined above.

One can easily show that an application of any rule of Ξ produces an e-scenario out of an e-scenario.

Let us stress than when a rule of Ξ is applied, one acts upon a single occurrence of a query. Moreover, the new e-scenario and the old one do not differ with respect to items, which follow consecutive answers to the query that is acted upon: if $\Delta_i (i = 1, 2)$ occurs in the old e-scenario after an answer A, then Δ_i occurs in the resulting e-scenario after A as well.

Observe that the transition from the e-scenario displayed in Figure 2 to the e-scenario displayed in Figure 9 can be viewed as a result of an application of rule $\mathsf{R}_\vee$.

4. *Erotetic Search Scenarios and Deduction*

The rules of Ξ can be applied step by step, until no yes-no query based on a compound d-wff occurs. Sometimes, however, the result of consecutive applications of these rules is somehow surprising at first glance.

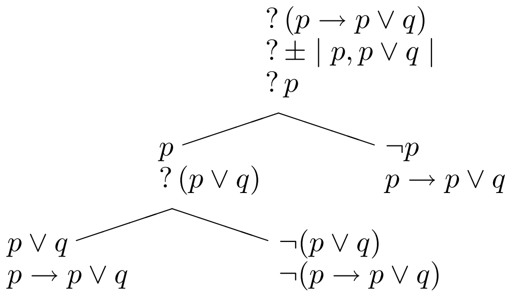
Example 3.

Suppose that one tries to solve the problem expressed by the question:

(4.1) $\quad ?\,(p \rightarrow p \vee q)$

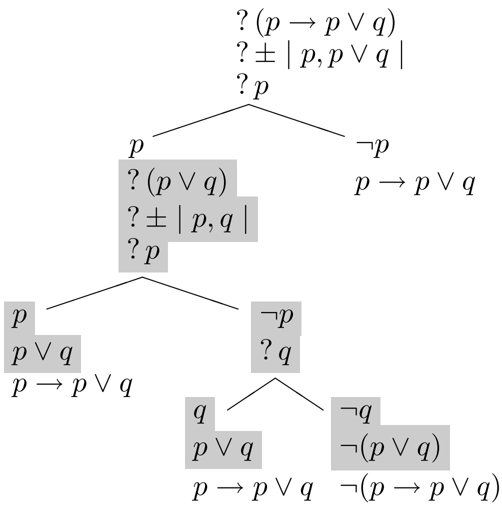
and builds the standard e-scenario for the question, according to the pattern presented in Figure 6. One arrives at the following e-scenario:

Φ_1:



Now we apply rule $\mathsf{R}_\vee$ to Φ_1 with respect to the query $?\,(p \vee q)$, and we get:

Φ_2:



Observe that the path of Φ_2 which leads to the negative answer to the main question involves contradictory d-wffs (that is, p and $\neg p$). Clearly, one cannot expect getting a reliable answer p first, and then getting a reliable answer $\neg p$ (dialetheism apart). On the other hand, the affirmative answer to the main question is a CPC-valid formula.

Example 4.

Suppose that we are looking for an answer to the question:

(4.2) $\quad ?\,((p \to q) \wedge p \to q)$

First, we construct the following standard e-scenario, using Figure 6 as the source of instruction.

Ψ_1:

$?\,((p \to q) \wedge p \to q)$
$?\pm \mid (p \to q) \wedge p, q \mid$
$?\,((p \to q) \wedge p)$

- $(p \to q) \wedge p$
  $?\,q$
  - q
    $(p \to q) \wedge p \to q$
  - $\neg q$
    $\neg((p \to q) \wedge p \to q)$
- $\neg((p \to q) \wedge p)$
  $(p \to q) \wedge p \to q$

Now we apply rule $\mathsf{R}_\wedge$ with respect to the query $?\,((p \to q) \wedge p)$. We obtain:

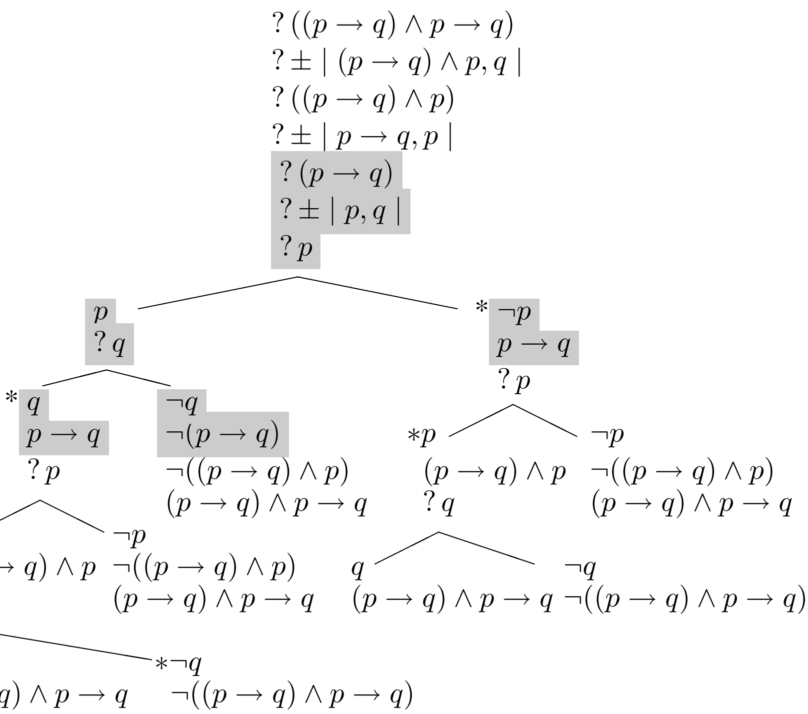
Ψ_2:

$?\,((p \to q) \wedge p \to q)$
$?\pm \mid (p \to q) \wedge p, q \mid$
$?\,((p \to q) \wedge p)$
$?\pm \mid p \to q, p \mid$
$?\,(p \to q)$

- $p \to q$
  $?\,p$
  - p
    $(p \to q) \wedge p$
    $?\,q$
    - q
      $(p \to q) \wedge p \to q$
    - $\neg q$
      $\neg((p \to q) \wedge p \to q)$
  - $\neg p$
    $\neg((p \to q) \wedge p)$
    $(p \to q) \wedge p \to q$
- $\neg(p \to q)$
  $\neg((p \to q) \wedge p)$
  $(p \to q) \wedge p \to q$

The next move is an application of rule $\mathsf{R}_{\rightarrow}$ with respect to the query $?\,(p \rightarrow q)$. We get:

Ψ_3:



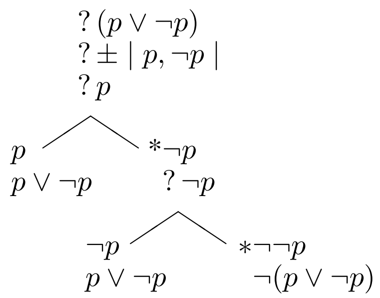
Again, each path of Ψ_3 which leads to the negative answer involves contradictory d-wffs (we have indicated them by *). On the other hand, the affirmative answer is a CPC-valid formula. So the situation is similar as in the case of the previous example.

Can this observation be generalized? The answer is "yes" and we will turn to it in a moment. Now let us only add the following.

First, both Φ_2 and Ψ_3 involve repeated queries, and, as a matter of fact, this is why contradictory formulas show up at paths. However, there is nothing in the definition of e-scenario that prevents us from repeating auxiliary questions.

Second, it happens that we receive an e-scenario with contradictory d-wffs at paths simply by building the appropriate standard scenario. Here is an example:

Figure 10.



Third, and most important: arriving at a complete e-scenario for a yes-no question, which involves contradictory d-wffs at each path leading to the negative answer, is not a disaster. On the contrary, an e-scenario of this kind warrants that the affirmative answer is CPC-valid and thus true as well. Let us now turn to this issue.

The following holds (for a proof, see Wiśniewski 2003):

Theorem 1: *Let* Φ *be an e-scenario for a question* Q *relative to a set of d-wffs* X. *Let* v *be a valuation such that* Q *is* v*-sound and all the d-wffs in* X *are true under* v. *Then the scenario* Φ *contains at least one path* e *such that:*

(a) *each d-wff of* e *is true under* v*; and*
(b) *each question of* e *is* v*-sound; and*
(c) e *leads to a direct answer to* Q *which is true under* v.

If v is a valuation, then by a v-*golden path* of an e-scenario Φ we will mean a path of Φ which satisfies the clauses (a), (b) and (c) of the above theorem with respect to v.

Recall that an *e-scenario for* Q is an e-scenario for Q relative to the empty set. If e is a path of an e-scenario for Q, then by *ans*(e) we designate the set made up of direct answers to all the queries of e which occur at e. (In other words, *ans*(e) consists of all the d-wffs that occur at e immediately after a query of e; according to the definition of e-derivation, these d-wffs are direct answers to queries). One can easily prove the following:

Lemma 1: *If* e *is a path of an e-scenario for* Q *and* e *leads to* A*, then* *ans*(e) *entails* A.

Recall that an e-scenario for Q is complete iff each direct answer to Q is the endpoint of some path of the scenario. Let us now prove:

Lemma 2: *If Φ is a complete e-scenario for $?A$ and v^* is a valuation such that $\mathsf{v}^*(\neg A) = 1$, then there exists a v^*-golden path of Φ which leads to $\neg A$.*

Proof. Since Φ is complete, it has path(s) which lead(s) to A and path(s) which lead(s) to $\neg A$. Since $?A$ is a simple yes-no question, then for any valuation v, the question $?A$ is v-sound. Therefore, by Theorem 1, for each valuation v there exists a v-golden path of Φ. Since $\mathsf{v}^*(\neg A) = 1$, then Φ has at least one v^*-golden path. But since $\mathsf{v}^*(A) = 0$, then each v^*-golden path of Φ leads to $\neg A$. □

Theorem 2: *Let A be a compound d-wff. If there exists a complete e-scenario Φ for $?A$ such that the following condition holds:*

(contr) *for each path e of Φ which leads to $\neg A$ there exists a d-wff B such that $B \in \underline{ans}(\mathsf{e})$ and '$\neg B$' $\in \underline{ans}(\mathsf{e})$*

then A is CPC-valid.

Proof. Assume that A is not CPC-valid. Then for a certain valuation v we have $\mathsf{v}(\neg A) = 1$. Since there exists a complete e-scenario Φ for $?A$, then by Lemma 2 at least one path of Φ which leads to $\neg A$ is v-golden. On the other hand, by condition (contr) no path of Φ which leads to $\neg A$ is v-golden, since each path which leads to $\neg A$ comprises a d-wff and its negation. We arrive at a contradiction. So A is CPC-valid. □

Thus in order to show that A is CPC-valid it suffices to construct a complete e-scenario for $?A$ that has the property (contr). Since validity yields truth, an e-scenario with the property (contr) gives us a solution to the initial problem and a "surplus": it shows that the affirmative answer is not only true, but also valid.

4.1. *Erotetic Proofs. Soundness and Completeness*

As we have shown, complete e-scenarios with the property (contr) can be constructed in a systematic way by applying the rules of Ξ. On the other hand, an e-scenario with the property (contr) justifies the claim that the affirmative answer to the main question is CPC-valid. Let us now express these ideas in more exact terms.

Let A be a compound d-wff. The concept of standard e-scenario for the question $?A$ is defined in the obvious way: if A is of the form $\neg B$, then the standard e-scenario for $?A$ is of the form presented in Figure 4; if A is of the form $B \wedge C$, then the standard e-scenario for $?A$ is of the form presented in Figure 5, and so forth (see Section 4.2). Now we introduce:

Definition 4: *A finite sequence* $\Phi_1, ..., \Phi_n$ *of e-scenarios for* $?\,A$ *is an* erotetic proof *of A iff*

(a) Φ_1 *is the standard e-scenario for* $?\,A$,

(b) Φ_{i+1} *results from* $\Phi_i (1 \leq i < n)$ *by an application of a rule* $\mathsf{r} \in \Xi$,

(c) Φ_n *is a complete e-scenario for* $?\,A$ *such that for each path* e *of* Φ_n *which leads to* $\neg A$ *there exists a d-wff* B *such that* $B \in \underline{ans}(\mathsf{e})$ *and* '$\neg B$' $\in \underline{ans}(\mathsf{e})$.

Certainly, erotetic proofs are rather non-standard entities; here we use the term "proof" for the lack of a better idea. However, by Theorem 2 we immediately get:

Theorem 3: (soundness)*: If there exists an erotetic proof of a d-wff A, then A is CPC-valid.*

We say that a simple yes-no question Q is *based on* a d-wff A iff Q is of the form $?\,A$. By the *degree* of a compound d-wff A we mean the number of occurrences of connectives in A. In order to prove completeness we need:

Lemma 3: *For each compound d-wff A there exists a finite sequence* $\Phi_1, ...,$ Φ_n *of e-scenarios for* $?\,A$ *such that:*

(a) Φ_1 *is the standard e-scenario for* $?\,A$,

(b) Φ_{i+1} *results from* $\Phi_i (1 \leq i < n)$ *by an application of a rule* $\mathsf{r} \in \Xi$, *and*

(c) Φ_n *is a complete e-scenario for* $?\,A$ *such that each query of* Φ_n *is a simple yes-no question based on a propositional variable that occurs in A.*

Proof. Assume that A is of degree 1. In this case the relevant sequence of e-scenarios is a one-term sequence; the element is the standard e-scenario for $?\,A$.

Now assume that A is of degree greater than 1. First, we construct the standard e-scenario Φ_1 for $?\,A$. The e-scenario Φ_1 has at most two queries; moreover, at least one query of Φ_1 is a simple yes-no question based on a compound subformula of A. Suppose that only one query of Φ_1 is a simple yes-no question based on a compound d-wff, say, B. Then we apply the appropriate rule $\mathsf{r} \in \Xi$ to Φ_1; the choice is determined by the main connective of B (if $\wedge$ is the main connective of B, we apply $\mathsf{R}_\wedge$, and so forth). The result is an e-scenario Φ_2 (observe that no "new" question introduced this way has $\{A, \neg A\}$ as the set of direct answers). Now suppose that both queries of Φ_1 are based on compound d-wffs. We establish some order among the queries and, first, by applying the appropriate rule to the first query, we receive an e-scenario Φ_2^*, and second, by applying the appropriate rule to Φ_2^* (which still involves the second query), we get an e-scenario Φ_3. If all the queries of the resultant e-scenarios (i.e. Φ_2 or Φ_3) are simple yes-no questions based on propositional variables, we have already constructed a complete e-scenario for $?\,A$ with the required property. But it may happen that some queries of Φ_2 or of Φ_3 are still based on compound d-wffs. We establish a certain order among such queries and then build up a sequence of e-scenarios by applying the appropriate rules in the established order. Again, it may happen that the last e-scenario of this sequence still has queries based on compound d-wffs. We repeat the procedure until we arrive at a complete e-scenario which has as queries only simple yes-no questions based on propositional variables; it is clear that each of these variables occurs in A. Since each d-wff is of a finite degree, the procedure terminates in a finite number of steps and produces a finite sequence of e-scenarios. □

Theorem 4: (completeness) *If a d-wff A is CPC-valid, then there exists an erotetic proof of A.*

Proof. If A is CPC-valid, then A is a compound d-wff. Assume that there is no erotetic proof of A. Since A is a compound d-wff, then, by Lemma 3, there exists a finite sequence $\mathsf{s} = \Phi_1, ..., \Phi_n$ of e-scenarios for $?\,A$ such that Φ_1 is the standard e-scenario, Φ_{i+1} results from Φ_i by an application of a rule $\mathsf{r} \in \Xi$, and Φ_n involves as queries only simple yes-no questions based on propositional variables that occur in A; moreover, Φ_n is complete. Since s is not an erotetic proof of A, then there exists a path e of Φ_n which leads to $\neg A$ such that for no d-wff B we have both $B \in \underline{ans}(\mathsf{e})$ and '$\neg B$' $\in \underline{ans}(\mathsf{e})$. The queries of e are simple yes-no questions based on propositional variables. Hence $\underline{ans}(\mathsf{e})$ consists of propositional variables and/or negated propositional variables, and does not include a variable and its negation. Therefore there exists a valuation v which makes true all the elements of $\underline{ans}(\mathsf{e})$. But since e leads to $\neg A$, then, by Lemma 1, $\mathsf{v}(\neg A) = 1$ and thus $\mathsf{v}(A) = 0$. Therefore A is not CPC-valid. We arrive at a contradiction. □

Thus by Theorem 3 and Theorem 4 we get:

Theorem 5: *A d-wff A is CPC-valid iff there exists an erotetic proof of A.*

We can also prove:

Theorem 6: *If a d-wff A is CPC-valid, then for each finite sequence $\Phi_1, ...,$ Φ_n of e-scenarios for $?\,A$ such that:*

(a) Φ_1 *is the standard e-scenario for $?\,A$,*

(b) Φ_{i+1} *results from* $\Phi_i (1 \leq i < n)$ *by an application of a rule* $\mathsf{r} \in \Xi$, *and*

(c) Φ_n *is a complete e-scenario for* $?\,A$ *such that each query of* Φ_n *is a simple yes-no question based on a propositional variable that occurs in* A.

the e-scenario Φ_n *has the property* (contr).

Proof. Suppose that there exists a finite sequence $\Phi_1, ..., \Phi_n$ of e-scenarios for $?\,A$ such that conditions (a), (b), and (c) are fulfilled, but Φ_n does not have the property (contr). Thus there exists a path e of Φ_n for which the following hold: (i) e leads to $\neg A$, and (ii) for some valuation v, $\mathsf{v}(B) = 1$ for each $B \in \underline{ans}(\mathsf{e})$. Therefore, by Lemma 1, $\mathsf{v}(\neg A) = 1$ and hence A is not CPC-valid. □

Theorem 6 yields that the order in which rules of Ξ are applied is irrelevant: if only the initial yes-no question is based on a CPC-valid d-wff A, then we will always receive, sooner or later, an erotetic proof of A. Moreover, this will happen even if we do not know in advance that the affirmative answer (*i.e.* A) is CPC-valid. The relevant feature here is CPC-validity itself and not a questioner's knowledge about it.

The following philosophical comment is in order here. The consecutive items of an erotetic proof of a CPC-valid d-wff A are e-scenarios for $?\,A$; the last item is a complete e-scenario for $?\,A$ *which illustrates that in order to get the negation of* A *we would have to receive reliable, but contradictory answers*. This is impossible (dialetheism apart) and therefore we have a good reason for the acceptance of A as CPC-valid.

5. *Final Remarks*

Since there are many proof methods for CPC, the importance of the method presented in Section 4 can be discussed. An erotetic proof proceeds by performing simple steps, but in a complex environment. However, we aimed at showing that there are cases in which a systematic reflection on possible ways of reaching alternative solutions is sufficient in order to establish the right solution, and the considerations of Section 4 justify this claim. According to a common-sense view, each successful problem-solving procedure based on questioning requires information-gaining moves. The expression "by questions" usually means "by questions and answers". As we have shown, however, there are exceptions from this rule.

Let us finally mention two related approaches to provability, which are also based on IEL.

Urbański (cf. Urbański 2001, 2001a, 2002, 2002a) has proposed a new proof method for some propositional calculi. The concept of Synthetic Tableau for a formula is introduced. Roughly, a Synthetic Tableau for a formula is a family of interconnected synthetic derivations of this formula or its negation; a synthetic derivation, in turn, starts with a variable that occurs in the formula or with the negation of such a variable, and ends with the target formula. The remaining items are either proper subformulas of the target formula or negations of its proper subformulas, introduced according to some simple introductory rules. The relevant synthetic derivations are interconnected in such a way that a Synthetic Tableau has a tree-like structure. As far as CPC is concerned, the main result is: a formula A is CPC-valid iff each synthetic tableau for A consists of synthetic derivations of A only (i.e. no synthetic derivation of the tableau ends with $\neg A$). It can be shown that each regular Synthetic Tableau for A can be extended to an e-scenario for $?\,A$ which involves as queries only simple yes-no questions based on propositional variables that occur in A; moreover, a Synthetic Tableau for A can be extracted from an e-scenario for $?\,A$ which has some simple properties (for details, see Urbański 2001a). Thus a regular Synthetic Tableau for a CPC-valid formula A can be extended to an (incomplete) e-scenario for $?\,A$ such that all the queries of this scenario are simple yes-no questions based on propositional variables that occur in A, and all the paths of the scenario lead to A. Looking from the philosophical point of view, an e-scenario of this kind shows that A can be reached after receiving any answers to the relevant queries. On the other hand, the last e-scenario which occurs in an erotetic proof (in our sense) of A shows that in order to get $\neg A$ we have to receive contradictory, but reliable answers. Thus an erotetic proof of A shows that it is impossible to reach $\neg A$.

The second question-theoretic approach to provability is based on the old idea of transforming a main problem into consecutive sub-problems until a solution becomes evident. Proofs now are not sequences of e-scenarios, but sequences of questions ending with a question which has some required properties. An interesting feature of a proof of this kind, called a *Socratic proof*, is that it can be transformed into a Gentzen-style proof; in some cases a transformation into an Analytic Tableau is also possible. For details see Wiśniewski (2004).

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