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Logic, Empiricism, and Rejection*

1. The idea that logic is not immune against revisions performed for empirical reasons is usually attributed to Quine. Recall the famous "field of force" metaphor from *Two dogmas of empiricism*:

> The totality of our so-called knowledge or beliefs, from the most casual matters of geography and history to the profoundest laws of atomic physics or even of pure mathematics and logic, is a man-made fabric which impinges on experience only along the edges. Or, to change the figure, total science is like a field of force whose boundary conditions are experience. A conflict with experience at the periphery occasions readjustments in the interior of the field. Truth values have to be redistributed over some of our statements. Re-evaluation of some statements entails re-evaluation of others, because of their logical interconnections; the logical laws being in turn simply certain further statements of the system, certain further elements of the field. Having re-evaluated one statement we must re-evaluate some others, whether they be statements logically connected with the first or whether they be the statements of logical connections themselves. But the total field is so undetermined by its boundary conditions, experience, that there is much latitude of choice as to what statements to re-evaluate in the light of any single contrary experience. No particular experiences are linked with any particular statements in the interior of the field, except indirectly through considerations of equilibrium affecting the field as a whole. (...) no statement is immune to revision. Revision even of the logical law of the excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle? (Quine 1951, pp. 41–42)

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*The basic idea of this essay had been presented in my paper "O pewnych konsekwencjach postulatu empirycznej falsyfikowalności twierdzeń logiki", *Studia Metodologiczne* **26**, 1991, pp. 129–138.

It is less known that Kazimierz Ajdukiewicz, a prominent representative of the Lvov-Warsaw School, in his 1947 paper *Logika a doświadczenie* (Logic and experience)[1] put forward the idea of empirical falsifiability and verifiability of logical theorems. According to Ajdukiewicz, logic can be confronted with experience in the following way:

> From scientific hypotheses consequences would be drawn by applying not only the primary deductive rules of the language, but also rules derived from the former and from logical theorems treated as auxiliary hypotheses. Should these consequences be in conformity with experience this result could be regarded as a confirmation (verification) of the conjunction composed of the hypothesis and hypothetically assumed logical theorems. If, however, these consequences were not in accordance with experience the ensuing difficulty could be overcome in one of two ways: one could retain logical theorems (thus accepting the completed deduction as correct) and reject the scientific hypothesis; alternatively, one might retain the hypothesis in spite of this contradiction with experience and reject some of the logical theorems which played the role of auxiliary hypotheses; finally, and with the same effect, one might regard the deduction as formally incorrect, i.e. question the correctness of those steps of the deduction which were based on the rule derived from the rejected logical theorem. (Ajdukiewicz 1947)[2]

The similarities are striking, but there are also substantial differences. In particular, the proposals were put forward for different reasons. As for Quine, the holistic account is a way out for an empiricist, since the "two dogmas" of empiricism are, as Quine argues for, untenable. Ajdukiewicz, in turn, modifies his theory of language and meaning in order to open the possibility of confronting logic with experience. In both cases empirically motivated rejection is not a must, but only an option. Needless to say, neither Quine nor Ajdukiewicz was a psychologist in logic; what was supposed to be rejected or falsified were not laws of logic construed as "laws of thought", but logical theorems and rules understood as in modern formal logic in the first part of the twentieth century.

2. Allowing for an indirect, but still empirically motivated, falsifiability of laws of logic is an appealing idea, especially now, when the number of newly

[1]The paper was first published in Polish. Its review occurred in 1948 in *The Journal of Symbolic Logic*; see Kokoszyńska (1948). An abridged English version of the paper was published in 1950 in *Synthese*, VIII, 6/7, pp. 289–299. An English translation of the full paper is included in Ajdukiewicz (1978), pp. 165–181.

[2]English translation excerpted from Ajdukiewicz (1978), pp. 178–179.

constructed/discovered logics is rapidly increasing, gradually overshadowing the old concept of "the logic". Yet, as often happens, what is attractive at first sight raises problems when dealt with afterwards. In the analysed case the problems stem from the degree of entrenchment of laws of logic in a system of knowledge, as well as from the interdependences of logic and metalogic.

3. A simple exercise in metalogic illustrates the point. Let $\mathcal{L}$ be a language for which the set of declarative formulas/sentences, $\mathfrak{D}_{\mathcal{L}}$, is well-defined (i.e. is recursive or at least r.e.). A logic, ℓ, applicable to $\mathcal{L}$, uniquely determines the consequence relation, $\vdash_\ell$, between subsets of $\mathfrak{D}_{\mathcal{L}}$ and elements of $\mathfrak{D}_{\mathcal{L}}$. Let us label the relation $\vdash_\ell$ as *ℓ-consequence*. Assume that $\vdash_\ell$ satisfies the standard conditions:

(Reflexitivity) *If* $A \in X$, *then* $X \vdash_\ell A$.

(Monotonicity) *If* $X \vdash_\ell A$ *and* $X \subseteq Y$, *then* $Y \vdash_\ell A$,

(Transitivity) *If* $X \vdash_\ell A$ *and* $Y, A \vdash_\ell B$, *then* $X, Y \vdash_\ell B$.

for any $X, Y \subseteq \mathfrak{D}_{\mathcal{L}}$ and any $A, B \in \mathfrak{D}_{\mathcal{L}}$.

Now let us introduce:

DEFINITION 1. $\mathbf{Th}_{(\ell,\mathcal{L})} = \{A \in \mathfrak{D}_{\mathcal{L}} : \emptyset \vdash_\ell A\}$.

Elements of $\mathbf{Th}_{(\ell,\mathcal{L})}$ are *instances of laws of logic ℓ expressed in $\mathcal{L}$*; in what follows we will be calling them briefly *laws of ℓ*.

The following holds:

COROLLARY 1. *For any* $A \in \mathfrak{D}_{\mathcal{L}}$ *and any* $B \in \mathbf{Th}_{(\ell,\mathcal{L})} : A \vdash_\ell B$.

Proof. If $\mathbf{Th}_{(\ell,\mathcal{L})} = \emptyset$, then the corollary trivially holds. Assume that $\mathbf{Th}_{(\ell,\mathcal{L})} \neq \emptyset$. Suppose that $A \nvdash_\ell B$ for some $A \in \mathfrak{D}_{\mathcal{L}}$ and some $B \in \mathbf{Th}_{(\ell,\mathcal{L})}$. Since $\emptyset \vdash_\ell B$, by monotonicity we get $A \vdash_\ell B$. A contradiction. □

Corollary 1 states that any law of logic ℓ (more precisely: any instance of a law of ℓ expressed in the language $\mathcal{L}$) is a ℓ-consequence of any declarative sentence/formula of $\mathcal{L}$. The proof clearly shows that this holds due to monotonicity of $\vdash_\ell$.

4. The concept of rejection is not univocal. To reject a law of logic may mean "not to have (an analogue of) it as a theorem". For instance, by saying that Intuitionistic Logic rejects the law of excluded middle one simply means that (an analogue of) the law is not a theorem of Intuitionistic Logic, and possibly explains why it is the case. But "rejects" used in the context of "empirically" is stronger: it means that a statement in question cannot be accepted for (maybe only indirect, but still) empirical reasons. Without analysing the issue in depth let us only observe that the following postulate seems uncontroversial given that the latter concept of rejection is taken into consideration:

(PR) *If B is an ℓ-consequence of A and B is rejected, then A should be rejected.*

5. When we combine the postulate (PR) with Corollary 1, we immediately get:

COROLLARY 2. *If at least one law of logic ℓ expressed in a language $\mathcal{L}$ is rejected, then each declarative sentence/formula of $\mathcal{L}$ should be rejected.*

In other words: once you reject a law of logic, you are obliged to reject any statement.

This is paradoxical and, clearly, constitutes a consequence that an adherent of the sensitivity of logic to experience would have liked to avoid. However, the price to be paid is, in each case, high.

6. First, note that (PR) relies upon the assumption that ℓ-consequence is (or is thought to be) *truth-preserving*, and can be questioned otherwise. For example, if a logic licenses enumerative induction, (PR) fails. So the price would be: the consequence licensed by a logic need not be truth-preserving. Or, to put it differently, a consequence which is not truth-preserving is still a *logical* consequence.

Second, as the proof of Corollary 1 shows, the corollary holds due to the monotonicity of the consequence relation in question. If ℓ determines a non-monotonic consequence relation (or, equivalently, a non-monotonic consequence operation), Corollary 1 need not hold and hence Corollary 2 remains unproven. So the price is: one has to switch to a non-monotonic logic.

Third, although Corollary 1 holds when ℓ is a theoremless, i.e. purely inferential, logic,[3] in such a case it carries no threat since there is nothing that can be rejected with causing damage. At first sight, switching to a purely inferential logic looks to be the most attractive option. But the chosen logic must be *deeply theoremless* in the sense that there is no formalism which, on the one hand, operates with theorems/laws at the object-language level and, on the other, determines the relevant consequence relation. Clearly,

[3]In this case certainly $\mathbf{Th}_{(\ell,\mathcal{L})} = \emptyset$ and the corollary holds trivially.

Classical Logic does not have this property, and similarly for Intuitionistic Logic, as well as for the most widely used modal logics. So the price is still high. Moreover, some logics are *a priori* excluded from the game.

7. To put the conclusion ironically: when the logic you operate with is, generally speaking, either akin to Classical Logic at the metalogical level or is Classical Logic itself, presumably the best option is to grant the privilege of empirical vulnerability to its laws, but never to execute that privilege. If, however, you do wish to execute it, you must be prepared for severe casualties.

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