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Weak Epistemic Logic, Immediate Consequence, and Paraconsistency*

1. Introduction. In Wiśniewski (1995) a system of modal logic, called $\Sigma.0$, was introduced. A semantical analysis of $\Sigma.0$ and some of its subsystems was then given by Świrydowicz (1995); the analysis is carried out in terms of possible worlds semantics and by means of, inter alia, non-normal possible worlds. The system $\Sigma.0$ was interpreted in Wiśniewski (1998) as an epistemic logic, namely, as a logic of occurrent belief.

$\Sigma.0$ is built in a language which results from the language of Classical Propositional Calculus (CPC) by adding modal operators. To be more precise, the vocabulary of the language contains propositional variables p, q, r, …, the connectives $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (implication), $\leftrightarrow$ (material equivalence), two modal operators[1]: $\Box$, $\Diamond$, and brackets. Well-formed formulas (wffs for short) are defined as usual; iterations of modal operators are permitted. We shall designate the language described above by $\mathcal{L}_{\Sigma.0}$ and we use the letters A, B, C, … (with subscripts if needed) as metalinguistic variables for wffs of $\mathcal{L}_{\Sigma.0}$. The Greek lower case letters α, β, γ, … will be used as metalinguistic variables for CPC-formulas. By *Taut* we shall designate the set of wffs of $\mathcal{L}_{\Sigma.0}$ whose elements result from CPC-valid formulas by uniformly replacing the propositional variables by wffs of $\mathcal{L}_{\Sigma.0}$.

Axioms of $\Sigma.0$ fall under the following schemata:

Ax.0. A, where $A \in$ *Taut*,

Ax.1. $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$,

[1] In the original presentation (cf. Wiśniewski 1995), $\Box$ and $\Diamond$ were replaced by S and D, respectively.

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Ax.2. $\Box(A \land B) \to \Box A \land \Box B$,

Ax.3. $\Box A \land \Box B \to \Box(A \land B)$,

Ax.4. $\Box A \to \Diamond A$,

Ax.5. $\Box A \to \neg\Diamond\neg A$,

Ax.6. $\Diamond A \to \neg\Box\neg A$.

The only rule of inference of $\Sigma.0$ is *Modus Ponens* (MP).

$\Sigma.0$ is a very weak modal system. It can be shown that $\Sigma.0$ is a proper subsystem of Łukasiewicz's four-valued modal system **Ł**. Thus $\Sigma.0$ has no theorems of the form $\Box A$ and is not closed under Necessitation.[2] Similarly, $\Sigma.0$ does not contain any theorem of the form $\Diamond A$. Moreover, $\Diamond A \leftrightarrow \neg\Box\neg A$ and $\Box A \leftrightarrow \neg\Diamond\neg A$ are not theorems of $\Sigma.0$ and thus the modalities $\Box$ and $\Diamond$ are not interdefinable in it. In general, $\Sigma.0$ does not reduce modalities.

2. An epistemic interpretation of $\Sigma.0$. There are situations in which weakness is a merit, however. The main shortcoming of most epistemic logics based on modal logics is their strength, since they usually prejudge belief in all logically valid formulas, and come to logical omniscience with respect to deduction: all logical consequences of beliefs are also beliefs.

Needless to say, this is completely unrealistic. Yet, when interpreted as an epistemic logic, $\Sigma.0$ does not produce these effects. The epistemic interpretation we have in mind runs as follows: the box $\Box$ can be construed as referring to an occurrent (or active, if you prefer) belief of a person. An expression $\Box A$ can read "a person x *occurrently believes* that A" or "it is occurrently believed that A". An expression of the form $\Diamond A$, in turn, can read "a person x *occurrently admits* that A" or "it is admissible that A". The system $\Sigma.0$ has no theorems of the form $\Box A$ – but there are no reasons for which an (epistemic) logic should prejudge an occurrent belief in anything (logical theorems included!). Similarly, a logic should not prejudge admissibility of anything – and so does $\Sigma.0$. On the other hand, Ax.1 assigns to an epistemic agent some minimal deductive ability: it says that an occurrent belief that an implication holds transforms into an occurrent belief that its consequent holds when the antecedent of this implication happens to be occurrently believed. Ax.2 says that occurrent belief distributes over conjunction and thus assigns to an epistemic agent some analytical abilities. Ax.3, in turn, assigns to an epistemic agent some synthetic abilities: occurrent belief that the conjuncts hold yields occurrent belief that the whole conjunction holds as well. In other words, axioms Ax.1, Ax.2 and Ax.3 say that occurrent beliefs

[2] By the way, this explains why Ax.2 and Ax.3 are not superfluous.

are closed under *Modus Ponens*, Simplification, and Adjunction. Yet, $\Sigma.0$ says nothing about the closure of occurrent beliefs under other CPC-rules. Moreover, the rule:

$$\frac{A \to B}{\Box A \to \Box B}$$

is *not* valid in $\Sigma.0$ and thus the logical omniscience paradox does not hold with respect to the system.

Since $\Sigma.0$ has no theorems of the form $\Box A$, then $\Sigma.0$ has no theorems of the form $\Box(\alpha \to \beta)$, where $\alpha \to \beta$ is a CPC-valid implication. But the following holds:

(I) if $\vdash_{\mathsf{CPC}} \alpha \to \beta$, then $\vdash_{\Sigma.0} \Box(\alpha \to \beta) \wedge \Box\alpha \to \Box\beta$.

Thus an occurrent belief that an antecedent of a CPC-valid implication holds transforms into an occurrent belief that the consequent of the implication holds given that the implication in question happens to be occurrently believed.

The remaining axioms of $\Sigma.0$ characterize admissibility with respect to occurrent belief. According to Ax.4, an occurrent belief yields admissibility. Ax.5, in turn, says that an occurrent belief that A excludes the admissibility of the negation of A. And finally, Ax.6 says that the admissibility of A excludes an occurrent belief that the negation of A is the case.

3. $\Sigma.0$ **and immediate consequence**. Since $\Sigma.0$ is a very weak epistemic logic, one can try to define the concept of *immediate consequence* in terms of it. Moreover, we may define a non-modal propositional logic of immediate consequence.

We say that a CPC-formula β is a $\Sigma.0$-immediate consequence of a set of CPC-formulas X (in symbols: $X \Vdash_{\Sigma.0} \beta$) iff the following condition holds:

($\star$) for some $\alpha_1, \ldots, \alpha_n \in X$: $\vdash_{\Sigma.0} \Box\alpha_1 \wedge \ldots \wedge \Box\alpha_n \to \Box\beta$.

Of course, a $\Sigma.0$-immediate consequence is always a CPC-consequence, but not conversely. When we have a CPC-formula (valid or not) of the form:

(1) $\alpha_1 \wedge \ldots \wedge \alpha_n \to \beta$

then the consequent β is a $\Sigma.0$-immediate consequence of a set made up of the formulas α_1, ..., α_n *and* the implication (1). For example, since axioms of $\Sigma.0$ say nothing about the behaviour of disjunction in the scope of modal operator $\Box$, there is no reason for which the following:

(2) $\Box(A \vee B) \wedge \Box\neg A \to \Box B$

should be a theorem of $\Sigma.0$. So $\alpha \vee \beta, \neg\alpha \nVdash_{\Sigma.0} \beta$. But the following is a theorem[3] of $\Sigma.0$:

(3) $\Box((A \vee B) \wedge \neg A \to B) \wedge \Box(A \vee B) \wedge \Box\neg A \to \Box B$

and thus we have:

(4) $(\alpha \vee \beta) \wedge \neg\alpha \to \beta, \alpha \vee \beta, \neg\alpha \Vdash_{\Sigma.0} \beta$

Yet, there are cases in which the appropriate implication (i.e. "law of logic") is dispensable. For example, the following hold:

(5)	$\alpha \Vdash_{\Sigma.0} \alpha$	(by Ax.0)
(6)	$\alpha \to \beta, \alpha \Vdash_{\Sigma.0} \beta$	(by Ax.1)
(7)	$\alpha, \beta \Vdash_{\Sigma.0} \alpha \wedge \beta$	(by Ax.3)
(8)	$\alpha \wedge \beta \Vdash_{\Sigma.0} \alpha$	(since $\vdash_{\Sigma.0} \Box(\alpha \wedge \beta) \to \Box\alpha$)
(9)	$\alpha \wedge \beta \Vdash_{\Sigma.0} \beta$	(since $\vdash_{\Sigma.0} \Box(\alpha \wedge \beta) \to \Box\beta$)
(10)	$\alpha \to \beta, \beta \to \gamma, \alpha \Vdash_{\Sigma.0} \gamma$	(since $\vdash_{\Sigma.0} \Box(\alpha \to \beta) \wedge \Box(\beta \to \gamma) \wedge \Box\alpha \to \Box\gamma$)
(11)	$\alpha \to (\beta \to \gamma), \alpha, \beta \Vdash_{\Sigma.0} \gamma$	(since $\vdash_{\Sigma.0} \Box(\alpha \to (\beta \to \gamma)) \wedge \Box\alpha \wedge \Box\beta \to \Box\gamma$)
(12)	$\alpha \to \beta, \alpha \to \gamma, \alpha \Vdash_{\Sigma.0} \beta \wedge \gamma$	(since $\vdash_{\Sigma.0} \Box(\alpha \to \beta) \wedge \Box(\alpha \to \gamma) \wedge \Box\alpha \to \Box(\beta \wedge \gamma)$)
(13)	$\alpha \to (\alpha \to \beta), \alpha \Vdash_{\Sigma.0} \beta$	(since $\vdash_{\Sigma.0} \Box(\alpha \to (\alpha \to \beta)) \wedge \Box\alpha \to \Box\beta$)
(14)	$\alpha \to \beta, \alpha \wedge \gamma \Vdash_{\Sigma.0} \beta \wedge \gamma$	(since $\vdash_{\Sigma.0} \Box(\alpha \to \beta) \wedge \Box(\alpha \wedge \gamma) \to \Box(\beta \wedge \gamma)$)
(15)	$\alpha \to \beta, \gamma \to \delta, \alpha \wedge \gamma \Vdash_{\Sigma.0} \beta \wedge \delta$	(since $\vdash_{\Sigma.0} \Box(\alpha \to \beta) \wedge \Box(\gamma \to \delta) \wedge \Box(\gamma \wedge \delta) \to \Box(\beta \wedge \delta)$)

Weak as it is, $\Vdash_{\Sigma.0}$ still fulfils the following standard conditions (we assume that X, Y stand for sets of CPC-formulas):

(**Reflexivity**) *If* $\alpha \in X$, *then* $X \Vdash_{\Sigma.0} \alpha$.

(**Monotonicity**) *If* $X \Vdash_{\Sigma.0} \alpha$ *and* $X \subseteq Y$, *then* $Y \Vdash_{\Sigma.0} \alpha$.

[3]For conciseness, we will speak of metalinguistic schemata as theorems or non-theorems.

(**Transitivity**) *If* $X \Vdash_{\Sigma.0} \alpha$ *and* $Y, \alpha \Vdash_{\Sigma.0} \beta$, *then* $X, Y \Vdash_{\Sigma.0} \beta$.

Reflexivity holds since $\Box\alpha \rightarrow \Box\alpha$ is a theorem of $\Sigma.0$, whereas Monotonicity holds due to condition ($\star$). As long as Transitivity is concerned, there are two possibilities: (i) $Y \Vdash_{\Sigma.0} \beta$ and thus $X, Y \Vdash_{\Sigma.0} \beta$ by Monotonicity; (ii) $Y \nVdash_{\Sigma.0} \beta$. In the latter case it suffices to observe that if $\vdash_{\Sigma.0} \Box\delta_1 \wedge \ldots \wedge \Box\delta_n \rightarrow \Box\alpha$ and $\vdash_{\Sigma.0} \Box\alpha \wedge \Box\gamma_1 \wedge \ldots \wedge \Box\gamma_k \rightarrow \Box\beta$, then, by Ax.0 and MP, $\vdash_{\Sigma.0} \Box\delta_1 \wedge \ldots \wedge \Box\delta_n \wedge \Box\gamma_1 \wedge \ldots \wedge \Box\gamma_k \rightarrow \Box\beta$.

Thus a consequence operation $\mathsf{Cni}_{\Sigma.0}$ defined in the following manner:

($\star\star$) $\alpha \in \mathsf{Cni}_{\Sigma.0}(X)$ iff $X \Vdash_{\Sigma.0} \alpha$

satisfies the conditions of Tarski:

(Cd.1) $X \subseteq \mathsf{Cni}_{\Sigma.0}(X)$

(Cd.2) $\mathsf{Cni}_{\Sigma.0}(\mathsf{Cni}_{\Sigma.0}(X)) \subseteq \mathsf{Cni}_{\Sigma.0}(X)$

(Cd.3) If $X \subseteq Y$, then $\mathsf{Cni}_{\Sigma.0}(X) \subseteq \mathsf{Cni}_{\Sigma.0}(Y)$

Of course, $\mathsf{Cni}_{\Sigma.0}$ is also finitary. Note that the empty set has no $\Sigma.0$-immediate consequences!

The *propositional logic of immediate consequence based on* $\Sigma.0$ (in symbols: $\mathsf{IC}_{\Sigma.0}$) can now be defined as the structure $\langle \mathcal{L}_{\mathsf{CPC}}, \Vdash_{\Sigma.0} \rangle$, where $\mathcal{L}_{\mathsf{CPC}}$ is the set of CPC-formulas and $\Vdash_{\Sigma.0}$ is the relation of $\Sigma.0$-immediate consequence defined above.[4]

4. The problem of paraconsistency. Surprisingly enough, the logic $\mathsf{IC}_{\Sigma.0}$, despite its weakness, is not paraconsistent. We have:

COROLLARY 1. $\vdash_{\Sigma.0} \Box A \wedge \Box\neg A \rightarrow \Box B$.

The proof goes as follows (with the exception of the first, we omit the applied instances of Ax.0; for conciseness, we write only one line when MP is applied twice):

[4]Alternatively, $\mathsf{IC}_{\Sigma.0}$ might have been defined as the pair $\langle \mathcal{L}_{\mathsf{CPC}}, \mathsf{Cni}_{\Sigma.0} \rangle$. Another possibility is to define $\mathsf{IC}_{\Sigma.0}$ as the set of all CPC-formulas of the form $\alpha \rightarrow \beta$ for which the following condition holds:

(#) $\vdash_{\Sigma.0} \Box\alpha \rightarrow \Box\beta$

Since we have:

(##) $\vdash_{\Sigma.0} \Box\alpha_1 \wedge \ldots \wedge \Box\alpha_n \rightarrow \Box\beta$ iff $\vdash_{\Sigma.0} \Box(\alpha_1 \wedge \ldots \wedge \alpha_n) \rightarrow \Box\beta$

then the following are equivalent:

(i) $\alpha_1 \wedge \ldots \alpha_n \rightarrow \beta$ is a theorem of $\mathsf{IC}_{\Sigma.0}$,

(ii) $\vdash_{\Sigma.0} \Box\alpha_1 \wedge \ldots \Box\alpha_n \rightarrow \Box\beta$,

(iii) $\{\alpha_1, \ldots, \alpha_n\} \Vdash_{\Sigma.0} \beta$.

(i)	$\Box A \to (\neg\Box A \to \Box B)$	(Ax.0)
(ii)	$\neg\Box A \to (\Box A \to \Box B)$	(Ax.0; (i))
(iii)	$\Box A \to \Diamond A$	(Ax.4)
(iv)	$\Diamond A \to \neg\Box\neg A$	(Ax.6)
(v)	$\Box A \to \neg\Box\neg A$	(Ax.0; (iii), (iv))
(vi)	$\Box\neg A \to \neg\Box A$	(Ax.0; (v))
(vii)	$\Box\neg A \to (\Box A \to \Box B)$	(Ax.0; (ii), (vi))
(viii)	$\Box A \to (\Box\neg A \to \Box B)$	(Ax.0; (vii))
(ix)	$\Box A \wedge \Box\neg A \to \Box B$	(Ax.0; (viii))

Note that the above theorem can also be proved by means of Ax.5 instead of Ax.6, *viz*.

(iii′)	$\Box\neg A \to \Diamond\neg A$	(Ax.4)
(iii″)	$\neg\Diamond\neg A \to \neg\Box\neg A$	(Ax.0; (iii′))
(iv′)	$\Box A \to \neg\Diamond\neg A$	(Ax.5)
(v)	$\Box A \to \neg\Box\neg A$	(Ax.0; (iii″), (iv′))

Anyway, we have:

COROLLARY 2. $\alpha, \neg\alpha \Vdash_{\Sigma.0} \beta$

Hence any CPC-formula is a $\Sigma.0$-immediate consequence of a set made up of a CPC-formula and its negation and thus $\mathsf{IC}_{\Sigma.0}$ is not paraconsistent.

The proof of Corollary 1 shows that the axioms Ax.6 or Ax.5 together with the axiom Ax.4 are responsible for the lack of paraconsistency of $\mathsf{IC}_{\Sigma.0}$. Observe that Ax.4 and Ax.6 (as well as Ax.4 and Ax.5) yield the following theorem of $\Sigma.0$:

(16) $\Box A \to \neg\Box\neg A$

Moreover, (16) is $\Sigma.0$-equivalent to:

(17) $\neg\Box(A \wedge \neg A)$

Formula (16) says that occurrent belief that A excludes occurrent belief that the negation of A is the case. Formula (17), in turn, says that no contradiction is occurrently believed. Note that a modal system which results from $\Sigma.0$ by abandoning axioms Ax.4, Ax.5 and Ax.6, and adding formula (16) (or formula (17)) as a new axiom would still have the formula:

(18) $\Box A \wedge \Box\neg A \to \Box B$

as a theorem. Hence if the concept of immediate consequence and the corresponding logic were defined (in the ways presented above) in terms of the new system, the lack of paraconsistency would still emerge. On the other hand, the epistemic readings of formulas (16) and (17) exhibit the epistemic roots of non-paraconsistency of a propositional logic of immediate consequence.

The following formula:

(19) $\neg\Box\neg A \rightarrow \Box A$

is not a theorem of $\Sigma.0$ (cf. Wiśniewski 1998). Yet, if $\Sigma.0$ (or its modified version mentioned above) were extended by adding (19) as a new axiom, the following would be provable in the new system:

(20) $\Box(A \rightarrow B) \wedge \Box\neg B \rightarrow \Box\neg A$

(21) $\Box A \rightarrow \Box\neg\neg A$

(22) $\Box\neg\neg A \rightarrow \Box A$

(23) $\Box(A \rightarrow B) \wedge \Box(A \rightarrow \neg B) \rightarrow \Box\neg A$

Thus the corresponding propositional logic of immediate consequence would be substantially richer, but of course also not paraconsistent. Let us add that both formula (16) and formula (19) are used in the proofs of (20) – (23).

Note finally that (19) is equivalent to:

(24) $\Box A \vee \Box\neg A$

Formula (24), however, is unrealistic when interpreted in terms of occurent belief.

5. Paraconsistent immediate consequence. The system $\Sigma.0$, however, can also be weakened in different ways. One of them was proposed by Świrydowicz (1995). The system analysed by him (we call it here $\Sigma.0^\star$) is expressed in a language $\mathcal{L}_{\Sigma.0^\star}$ which differs from $\mathcal{L}_{\Sigma.0}$ in the absence of the modal operator $\Diamond$ and the lack of iterations of the modal operator $\Box$. Axioms of $\Sigma.0^\star$ fall under the schemata Ax.0, Ax.1, Ax.2 and Ax.3; MP is the only rule of inference. It can be shown (see Świrydowicz 1995) that the formula:

(25) $\Box(A \wedge \neg A) \rightarrow \Box B$

is not a theorem of $\Sigma.0^\star$. Therefore formula (18) is not a theorem of $\Sigma.0^\star$.

If the concept of immediate consequence is defined in terms of $\Sigma.0^\star$ in the following way:

(•) $X \Vdash_{\Sigma.0^\star} \beta$ iff $\vdash_{\Sigma.0^\star} \Box\alpha_1 \wedge \ldots \wedge \Box\alpha_n \rightarrow \Box\beta$ for some $\alpha_1, \ldots, \alpha_n$ in X.

the analogues of (5) – (15) as well as of (**I**) hold for $\Vdash_{\Sigma.0^\star}$, but $\alpha, \neg\alpha \Vdash_{\Sigma.0^\star} \beta$ does not hold. Moreover, $\Vdash_{\Sigma.0^\star}$ is still reflexive, monotonic and transitive.

The corresponding propositional logic of immediate consequence based on $\Sigma.0^\star$ ($\mathsf{IC}_{\Sigma.0^\star}$ for short) can be defined as the structure $\langle \mathcal{L}_{\mathsf{CPC}}, \Vdash_{\Sigma.0^\star}\rangle$. Of course, $\mathsf{IC}_{\Sigma.0^\star}$ is paraconsistent, but also extremely weak.

In order to overcome weakness but retain paraconsistency we can extend $\Sigma.0^\star$ by adding new axioms. For example, we can add formula (19) to the axioms of $\Sigma.0^\star$. Then the following are provable in the new system:

(26) $\Box(A \to \neg A) \to \Box\neg A$

(26) $\Box(\neg A \to A) \to \Box A$

(27) $\Box(A \to B) \wedge \Box(\neg A \to B) \to \Box B$

In general, there are many propositional logics of immediate consequence which are both paraconsistent and properly include $\mathsf{IC}_{\Sigma.0^\star}$.

Since the formula:

(17) $\neg\Box(A \wedge \neg A)$

expresses the law of non-contradiction in modal terms, one might expect that (17) or any of its equivalents (for example (16)) are the weakest axioms which, when added to $\Sigma.0^\star$, result in the non-paraconsistency of the corresponding propositional logic of immediate consequence. Unfortunately, this is not the case. For example, let us extend $\Sigma.0^\star$ by adding the following axiom:

(28) $\Box\neg A \to \Box(A \to B)$

The proof of (16) now runs as follows (again, we omit the applied instances of Ax.0):

(i)	$\Box(A \to B) \to (\Box A \to \Box B)$	(Ax.1)
(ii)	$\Box\neg A \to (\Box A \to \Box B)$	(Ax.0; (i), (28))
(iii)	$\Box A \wedge \Box\neg A \to \Box B$	(Ax.0; (ii))

When we extend $\Sigma.0^\star$ by adding the following formulas as new axioms:

(29) $\Box(A \vee B) \to \Box(\neg A \to B)$

(30) $\Box A \to \Box(A \vee B)$

the situation is similar, *viz.*:

(iv)	$\Box A \to \Box(\neg A \to B)$	(Ax.0; (29), (30))
(v)	$\Box(\neg A \to B) \to (\Box\neg A \to \Box B)$	(Ax.1)
(vi)	$\Box A \wedge \Box\neg A \to \Box B$	(Ax.0; (iv), (v))

Thus the roots of non-paraconsistency of (even weak) consequence relations reach deeper and wider than one usually suspects.

The only consolation is that formulas (28) and (30), when interpreted in terms of occurent belief, are not intuitive.

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