Andrzej Wiśniewski

Logic and Sets of Situations*

1. "Situations are just that: situations."[1] This statement, made by Keith Devlin, the author of one of the most interesting books devoted to situational semantics, will be a motto of our inquiries. Although the concept of situation plays the basic role in this essay, it will not be defined here in detail. We will be only assuming that there occur actual situations as well as non-actual ones. If we defined situations in general, this would determine what actual situations are. However, logic is not in a position to claim anything about the structure of the world. Without deciding what situations are, we simply assume that there exist non-empty *sets of situations*. On the basis of set theory, which constitutes the background of our considerations, this implies that there also exists a non-empty set of situations being the union of all the sets of situations. We call it the *universe of situations*. We consider languages in which there occur atomic sentences, that is, sentences which have no proper parts being sentences themselves, and compound sentences which result from atomic sentences by means of connectives. We will be assuming that each atomic sentence refers to a *set* of situations. If the relevant set is non-empty, then, looking from the intuitive point of view, the set comprises all these situations in which (the claim made by) the atomic sentence holds; in the case of any atomic sentence A, the expression "A holds in situation s" will be regarded as comprehensible enough. For instance, the sentence:

(1) *Andrzej Wiśniewski works.*

refers to the set of all the situations in which Andrzej Wiśniewski works, whereas the sentence:

[1]K. Devlin, *Logic and Information*, Cambridge University Press, Cambridge 1991. The quotation comes from page 70.

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(2) *Andrzej Wiśniewski levitates.*

refers to the set of all the situations in which Andrzej Wiśniewski levitates. The set of situations to which the sentence (1) refers includes some actual situations, while the set of situations that corresponds to the sentence (2) does not include any actual situation.[2] The set of situations referred to by the following sentence:

(3) *A table is brave.*

is, presumably, empty.

Thus, on our account, what is assigned to atomic sentences are sets of situations, and not single situations. Moreover, the relevant sets of situations are neither supposed to be non-empty nor have to be singleton sets. Hence, whatever situations are, we do not construe the concept of situation as "what is represented by (the content of) a sentence".

We assume that the set of situations assigned to an atomic sentence is a subset of the universe of situations **U**. Logic is not in a position to claim anything about the cardinality of **U**.

2. Consider the following compound sentence:

(4) *Andrzej Wiśniewski works and Andrzej Wiśniewski stays in Zielona Góra*

Let us ask: what set of situations corresponds to the sentence (4)? At first sight, one can think that the set should have been defined as the set of all situations such that for each situation s in the set it is the case that Andrzej Wiśniewski works and Andrzej Wiśniewski stays in Zielona Góra. However, in order to clarify the condition we would have to define what is the reference of the connective "and" in a (single) situation s. This, in turn, leads to inquiries concerning the structure of situations and gives rise to all the well-known difficulties (occurring also in the case of other connectives). Yet, we would like to avoid them. Moreover, this can be accomplished easily, by defining the analysed set directly. The set of situations that corresponds to the sentence (4) is the *intersection* of: the set of situations that is assigned to the sentence (1), and the set of situations that corresponds to the sentence "Andrzej Wiśniewski stays in Zielona Góra".

Similarly, in the case of the following compound sentence:

(5) *Andrzej Wiśniewski works or Andrzej Wiśniewski has a good time.*

the set of situations assigned to (5) can be identified with the *union* of: the

[2]The reader is kindly requested to trust the author.

set of situations that corresponds to the sentence (1), and the set of situations assigned to the sentence "Andrzej Wiśniewski has a good time".[3]

Now consider the following sentence:

(6) *It is not the case that Andrzej Wiśniewski has a good time.*

If we were to clarify the meaning of "it holds in situation s that it is not the case that Andrzej Wiśniewski has a good time", we would entangle in the well-know difficulties concerning the reference of the negation connective. But the following statement seems relatively unproblematic: the set of situations assigned to the sentence (6) is the (set-theoretic) *difference* of the universe of situations and the set assigned to the sentence "Andrzej Wiśniewski has a good time".[4]

Consider, in turn, the following sentence:

(7) *If Andrzej Wiśniewski works, then Andrzej Wiśniewski has a good time.*

A moment's reflection will suffice to notice that the set of situations in which the sentence (7) *does not* hold equals the intersection of: the set of situations assigned to the sentence (1), and the set of situations assigned to the sentence (6). Hence the set of situations that corresponds to the sentence (7) equals the union of: the set of situations assigned to the sentence "It is not the case that Andrzej Wiśniewski works", and the set of situations that corresponds to the sentence "Andrzej Wiśniewski has a good time."

In the case of:

(8) *Andrzej Wiśniewski works if, and only if Andrzej Wiśniewski has a good time.*

the relevant set of situations equals the intersection of: the set of situations assigned to the sentence (7) and the set of situations that corresponds to the following sentence:

(9) *If Andrzej Wiśniewski has a good time, then Andrzej Wiśniewski works.*

3. The compound sentences analysed above are built directly from atomic sentences. It is obvious, however, that the approach can be generalized so that it will pertain to any compound sentence of a language of the considered kind. Let $\cup$, $\cap$, $\setminus$ denote set-theoretic union, intersection, and difference of sets, respectively. Assume that the classical connectives of negation $\neg$, con-

[3]Cf. Barwise (1981), p. 30.

[4]Cf. Cooper & Kamp (1991), pp. 316–320.

junction $\wedge$, disjunction $\vee$, implication $\rightarrow$, and equivalence $\leftrightarrow$ are the only connectives that occur in a language of the considered kind; the set of sentences of the language comprises atomic sentences and compound sentences built from atomic sentences by means of the above connectives. Assume that to each atomic sentence A there corresponds exactly one set of situations $S(A)$ being a subset of the universe of situations $\mathbf{U}$. If $S(A)$ is a non-empty set, then, looking from the intuitive point of view, it comprises all the situations in which the sentence A holds, and only them. Now we can define a function V that assigns to each sentence of a language of the considered kind the corresponding set of situations. We do it in the following way.

1. If A is atomic, then $V(A) = S(A)$.
2. If A is of the form $\neg B$, then $V(A) = \mathbf{U} \setminus V(B)$.
3. If A is of the form $B \wedge C$, then $V(A) = V(B) \cap V(C)$.
4. If A is of the form $B \vee C$, then $V(A) = V(B) \cup V(C)$.
5. If A is of the form $B \rightarrow C$, then $V(A) = V(\neg B) \cup V(C)$.
6. If A is of the form $B \leftrightarrow C$, then $V(A) = (V(\neg B) \cup V(C)) \cap (V(\neg C) \cup V(B))$.

The value of V for a given sentence A can be called the set of the situations in which the sentence A holds. Note that for some sentences the set may be empty! By using the above notion we can now define the concept "a compound sentence A holds in situation s" as follows:

($\star$) *a compound sentence A holds in situation s iff $s \in V(A)$.*

If, for a given compound sentence A, the set $V(A)$ is empty, we can say that there is no situation in which it holds (and, simultaneously, that the negation of A holds in each situation from the universe of situations $\mathbf{U}$). Due to our assumptions, it is not excluded that for some atomic sentences A_1, A_2, …, A_i, … $(i \geq 1)$, the sets $S(A_1)$, $S(A_2)$, …, $S(A_i)$, … are empty. If this is so, we can say that the negations of these sentences hold in each situation belonging to the universe of situations $\mathbf{U}$.

4. The considerations presented above are rather sketchy: we neither clarified what the sets of situations that are assigned to atomic sentences are nor stated anything definite about the universe of situations. However, even what has been said above enables us to show that there are compound sentences with the following property: the set of situations in which a sentence holds is always empty – regardless of what the universe of situations is. In particular, the following sentence has this property:

(10) *Andrzej Wiśniewski stays in Zielona Góra and it is not the case*

that Andrzej Wiśniewski stays in Zielona Góra

Let A^* stand for the sentence "Andrzej Wiśniewski stays in Zielona Góra". Regardless of what **U** is, we have:

$$V(A^* \wedge \neg A^*) = V(A^*) \cap V(\neg A^*) = V(A^*) \cap (\mathbf{U} \setminus V(A^*)).$$

But $V(A^*)$ is a subset of **U**. Therefore:

$$V(A^* \wedge \neg A^*) = \emptyset$$

Hence the set of situations in which sentence (10) holds is empty. The status of the following sentence is analogous:

(11) *Dopey is handsome and it is not the case that Dopey is handsome.*

One can also show that there exist compound sentences with the property: they hold in each situation from the universe of situations – regardless of what the universe of situations is. Here is an example:

(12) *Andrzej Wiśniewski stays in Zielona Góra or it is not the case that Andrzej Wiśniewski stays in Zielona Góra.*

We have:

$$V(A^* \vee \neg A^*) = V(A^*) \cup V(\neg A^*) = V(A^*) \cup (\mathbf{U} \setminus V(A^*)).$$

Yet, $V(A^*)$ is a subset of **U**. Thus:

$$V(A^* \vee \neg A^*) = \mathbf{U}$$

Clearly, the following sentence:

(13) *Dopey is handsome or it is not the case that Dopey is handsome.*

also has the analysed property, irrespective of the way in which Dopey exists.

Sentences (12) and (13) are instantiations of a thesis of Classical Propositional Calculus. The following questions arise: (a) does any sentence being an instantiation of a thesis of Classical Propositional Calculus hold in each situation from the universe of situations, irrespective of the nature of the universe of situations?, (b) is each sentence having the above property an instantiation of a thesis of Classical Propositional Calculus? In order to answer these questions – answer in the affirmative! – we will build some semantics for Classical Propositional Calculus based on the intuitions presented above.

5. For clarity, let us first characterize the language of the version of Classical Propositional Calculus (hereafter: CPC) used below. The vocabulary of the language includes: countably infinitely many propositional variables p_1, p_2, ..., the connectives: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, and brackets. The set of well-formed formulas $\mathsf{Form}_{\mathsf{CPC}}$ of the language is the smallest set that includes all the propositional variables and fulfils the following conditions: (a) if α belongs to $\mathsf{Form}_{\mathsf{CPC}}$, then an expression of the form $\neg\alpha$ belongs to $\mathsf{Form}_{\mathsf{CPC}}$ as well; (b) if α, β belong to $\mathsf{Form}_{\mathsf{CPC}}$, then expressions of the form: $(\alpha \wedge \beta)$, $(\alpha \vee \beta)$, $(\alpha \rightarrow \beta)$, $(\alpha \leftrightarrow \beta)$ also belong to $\mathsf{Form}_{\mathsf{CPC}}$. The Greek lower case letters α, β, ... will be used as metalinguistic variables for well-formed formulas of the language of CPC; we adopt the usual conventions concerning omitting brackets. In the metalanguage we assume the Zermelo-Fraenkel set theory.[5]

As it is well-known, propositional variables can be interpreted either substitutionally or referentially. According to the substitutional interpretation, propositional variables are not variables in the exact sense of the word, but are schematic letters that represent sentences of a language. The referential interpretations diverge with respect to what is conceived as the universe run over by the variables. The most popular solutions are: (a) propositional variables run over a two-element universe whose elements are Truth and Falsity (Frege), (b) propositional variables run over a set of logical values, in which Truth and Falsity are the distinguished elements (Łukasiewicz, Post), (c) propositional variables run over a universe of situations (Wittgenstein, Suszko), and (d) propositional variables run over sets of possible worlds (Kripke, Fine).[6]

The intuitions that underlie the semantics for CPC proposed below can be described as follows. Propositional variables are schematic letters that represent these sentences of a natural language which do not involve the connectives of negation, conjunction, disjunction, implication, and equivalence.[7] However, natural languages diverge. Moreover, even if we fix the natural language under consideration, there are still many possible assignments of the analysed sentences to propositional variables and hence compound sentences can be represented in the language of CPC in different manners. On the other hand, as we have already pointed out, each atomic sentence corresponds to a set of situations that, assuming its non-emptiness, comprises all the situations in which the atomic sentence holds. Moreover, each natural language includes counterparts of the CPC-connectives; the sets of situations assigned to compound sentences can be defined in the manner described above. But, due to the indeterminacy/untranslatability phenomenon, one can hardly expect the unions of sets of situations assigned to atomic sen-

[5]When situational semantics is concerned, non-standard set theories are sometimes used, in particular, Aczel's theory of hypersets (non-well founded sets); see Aczel (1988), cf. also Paśniczek (1994). The reasons for that are described in Barwise (1986). For our simple considerations ZF is sufficient, however.

[6]See Omyła (1991), p. 118.

[7]Note that, according to the above statement, the relevant sentences can involve, e.g., quantifiers.

tences of different languages be identical. What is more, one cannot *a priori* define the universe of situations. All this leads us to the conclusion that, in the case of the language of CPC, we have to assume that there are many situational models. These models differ with respect to universes of situations as well as their subsets assigned to consecutive propositional variables. The concept of situational model we have in mind can be defined as follows ($\subseteq$ stands for the sign of inclusion, and $\wp(U)$ is the power set of U):

DEFINITION 1. *A* situational model *of the language of* CPC *is an ordered pair* $\langle U, f \rangle$, *where* U *is a non-empty set and* f *is a function defined over the set* $\mathsf{Form}_{\mathsf{CPC}}$ *whose values belong to* $\wp(U)$ *such that:*

1. *for each propositional variable* p_i, $f(p_i) \subseteq U$,
2. *for any* $\alpha, \beta \in \mathsf{Form}_{\mathsf{CPC}}$:
   - $f(\neg\alpha) = U \setminus f(\alpha)$,
   - $f(\alpha \wedge \beta) = f(\alpha) \cap f(\beta)$,
   - $f(\alpha \vee \beta) = f(\alpha) \cup f(\beta)$,
   - $f(\alpha \to \beta) = f(\neg\alpha) \cup f(\beta)$,
   - $f(\alpha \leftrightarrow \beta) = (f(\neg\alpha) \cup f(\beta)) \cap (f(\neg\beta) \cup f(\alpha))$.

If $\langle U, f \rangle$ is a situational model of the language of CPC, the set U is called the *universe* of the model.

As for Definition 1, the only assumption concerning the universe of a situational model is its non-emptiness. A given universe will be thought of, intuitively, as a set of situations. Yet, since we have not defined what situations are, we cannot impose any condition, besides non-emptiness, on the universe of a situational model. As we will see, however, no further condition is necessary for the accomplishment of our tasks. The value of the function f for a given well-formed formula α can be conceived, intuitively, as the set of all the situations in which formula α "holds". The set can be empty.

Let us now introduce the concept of situational tautology.

DEFINITION 2. *A well-formed formula* α *of the language of* CPC *is a* situational tautology *iff for each situational model* $\langle U, f \rangle$ *of the language we have:* $f(\alpha) = U$.

The intuition that underlies Definition 2 is the following: formula α is a situational tautology just in case α "holds" in each situation: regardless of what set of situations is taken into consideration and what subsets of the set are assigned to propositional variables, the set of situations that corresponds to α equals the whole universe chosen.

Well-formed formulas of the language of CPC represent sentences of a natural language. Thus each sentence represented by a situational tautology has the following feature: the claim made by the sentence holds in each situation from any universe of situations, irrespective of what situations are and what is *the* universe of situations.[8]

6. One can prove that Classical Propositional Calculus is sound and complete with respect to the semantics presented above, that is, each thesis of an axiomatic system of CPC is a situational tautology and each situational tautology is a thesis of the system.

We will consider the following axiomatic system of CPC. The axioms are all the well-formed formulas of the language falling under the schemata:

$$\alpha \to (\beta \to \alpha) \tag{14}$$

$$(\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma)) \tag{15}$$

$$(\neg\beta \to \neg\alpha) \to (\alpha \to \beta) \tag{16}$$

The rules of inference are: *Modus Ponens* (MP) and Definitional Replacement (DR) with regard to the following definitions:

$$\alpha \vee \beta =_{df} \neg\alpha \to \beta \tag{17}$$

$$\alpha \wedge \beta =_{df} \neg(\alpha \to \neg\beta) \tag{18}$$

$$\alpha \leftrightarrow \beta =_{df} \neg((\alpha \to \beta) \to \neg(\beta \to \alpha)) \tag{19}$$

The concept of proof is understood in the standard way; a thesis of CPC is a well-formed formula of the language of CPC which has at least one proof in the axiomatic system of CPC.

In what follows, by "situational models" we will mean situational models of the language of CPC.

One can easily prove:

LEMMA 1. *For each situational model $\langle U, f \rangle$:*

1. $f(\alpha \to (\beta \to \alpha)) = U$,
2. $f((\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma))) = U$,
3. $f((\neg\beta \to \neg\alpha) \to (\alpha \to \beta)) = U$.

Thus each axiom is a situational tautology.

LEMMA 2. *If $f(\alpha \to \beta) = U$ for each situational model $\langle U, v \rangle$, and $f(\alpha) = U$ for each situational model $\langle U, f \rangle$, then $f(\beta) = U$ for each situational model $\langle U, f \rangle$.*

[8] Let us recall that we have imposed only two conditions on the universe of situations $\mathbf{U}$: (1) non-emptiness, and (2) for each atomic sentence A, the set $S(A)$ is included in $\mathbf{U}$.

Proof. Suppose that the assumptions hold and that $\langle U^*, f^*\rangle$ is a situational model such that $f^*(\beta) \neq U^*$. By assumption, $f^*(\alpha) = U^*$ and $f^*(\alpha \rightarrow \beta) = U^*$. Hence $(U^* \setminus f^*(\alpha)) \cup f^*(\beta) = U^*$. But since $f^*(\alpha) = U^*$, it follows that $f^*(\beta) = U^*$. We arrive at a contradiction. □

According to Lemma 2, each well-formed formula that results from situational tautologies by an application of *Modus Ponens* (MP) is a situational tautology as well.

One can also easily prove:

LEMMA 3. *For each situational model* $\langle U, f\rangle$*:*

1. $f(\alpha \vee \beta) = f(\neg\alpha \rightarrow \beta)$,
2. $f(\alpha \wedge \beta) = f(\neg(\alpha \rightarrow \neg\beta))$,
3. $f(\alpha \leftrightarrow \beta) = f(\neg((\alpha \rightarrow \beta) \rightarrow \neg(\beta \rightarrow \alpha)))$.

Lemma 3 yields that each well-formed formula that results from a situational tautology by an application of Definitional Replacement (DR) is a situational tautology.

Thus the following is true:

THEOREM 1. *Each thesis of* CPC *is a situational tautology.*

Let us now prove:

THEOREM 2. *Each situational tautology is a thesis of* CPC*.*

Proof. As it is well-known, a CPC-formula α is a thesis of CPC iff for each valuation $\mathbf{w}$ (understood as a denumerable sequence of the logical values $\mathbf{1}$, $\mathbf{0}$), the value of α under $\mathbf{w}$ equals $\mathbf{1}$. The concept of value of a CPC-formula under a valuation $\mathbf{w}$ is construed here in the usual way[9]; we designate the value of formula α under $\mathbf{w}$ by $\alpha[\mathbf{w}]$. Thus it suffices to prove that the following condition holds for any CPC-formula α:

(○) *If* $\alpha[\mathbf{w}] = \mathbf{0}$ *for some valuation* $\mathbf{w}$*, then there exists a situational model* $\langle U, f\rangle$ *such that* $f(\alpha) \neq U$.

Suppose that $\alpha[\mathbf{w}] = \mathbf{0}$ for some valuation $\mathbf{w}$. For each CPC-formula there exists a CPC-equivalent formula in conjunctive normal form. Let β be a conjunctive normal form of α. Thus the following:

(a) $\alpha \rightarrow \beta$,

(b) $\beta \rightarrow \alpha$

are theses of CPC. The formula β, as being in CNF, is of the form $\gamma_1 \wedge \ldots \wedge \gamma_k$, where $\gamma_1, \ldots, \gamma_k$ are elementary disjunctions and $k \geq 1$. Since $\beta \rightarrow \alpha$ is a

[9] See, e.g., Batóg (1994), Chapter 1.

thesis of CPC, by the Completeness Theorem for CPC we get $(\beta \to \alpha)[\mathbf{w}] = \mathbf{1}$. So if $\alpha[\mathbf{w}] = \mathbf{0}$, then $\beta[\mathbf{w}] = \mathbf{0}$ and hence $(\gamma_1 \wedge \ldots \wedge \gamma_k)[\mathbf{w}] = \mathbf{0}$. Thus the value under $\mathbf{w}$ of at least one of the elementary disjunctions $\gamma_1, \ldots, \gamma_k$ equals $\mathbf{0}$. Let γ_j $(1 \leq j \leq k)$ be an elementary disjunction such that $\gamma_j[\mathbf{w}] = \mathbf{0}$. Since γ_j is an elementary disjunction, γ_j is of the form $\delta_1 \vee \ldots \vee \delta_m$,. where $m \geq 1$ and δ_h, for $1 \leq h \leq m$, is either a propositional variable or a negated propositional variable. But if $\alpha \to \beta$ is a thesis of CPC, then the following formula:

(c) $\alpha \to \delta_1 \vee \ldots \vee \delta_m$

is a thesis of CPC as well. At the same time we have:

$$(\delta_1 \vee \ldots \vee \delta_m)[\mathbf{w}] = \mathbf{0}$$

that is, $\delta_h[\mathbf{w}] = \mathbf{0}$ for $1 \leq h \leq m$.

Let us consider a situational model $\langle\{\mathbf{1}, \mathbf{0}\}, f\rangle$, where f satisfies the following condition:

(•) *for each propositional variable* p_i, $f(p_i) = \{p_i[\mathbf{w}]\}$.

It is clear that $f(\delta_h) = \{\mathbf{0}\}$ for any h such that $1 \leq h \leq m$. But since the formula (c) is a thesis of CPC, by Theorem 1 we have $f(\alpha \to \delta_1 \vee \ldots \vee \delta_m) = \{\mathbf{1}, \mathbf{0}\}$. It follows that $(\{\mathbf{1}, \mathbf{0}\} \setminus f(\alpha)) \cup \{\mathbf{0}\} = \{\mathbf{1}, \mathbf{0}\}$. Therefore either $f(\alpha) = \{\mathbf{0}\}$ or $f(\alpha) = \emptyset$. Thus $f(\alpha) \neq \{\mathbf{1}, \mathbf{0}\}$. This completes the proof. □

As a consequence of theorems 1 and 2 we get the following Completeness Theorem for CPC with respect to the semantics introduced above.

THEOREM 3. *A* CPC*-formula* α *is a thesis of* CPC *iff* α *is a situational tautology.*

It follows that the scope of the concept of situational tautology equals the scope of the standard concept of tautology. These concepts, however, differ in content.

7. The concept of truth has not been used in the considerations presented above. This was intended, although it does not reflect any special aversion of the author towards the concept. The ideas underlying our semantics for CPC are, in principle, not new[10], though the way we have used them is presumably a novelty. The intuition that the claims made by sentences being

[10] In Kripke's semantics propositional variables run over sets of possible worlds, and valuations assign sets of possible worlds to propositional variables. However, it is a semantics for modal calculi. Yet, the idea can be transferred into semantics for CPC in the following way. Call a *model* an ordered pair $\langle W, g\rangle$, where W is a non-empty set of "possible worlds" and g is a valuation of propositional variables, that is, a function that assigns subsets of W to propositional variables. Let $w \in W$. The concept "α holds in world w" (in symbols: $w \models \alpha$)

instances of theses of CPC hold in any situation is one of the basic intuitions concerning logic. Our aim was to express this intuition by means of relatively few auxiliary concepts each of which is intuitively grounded. Let us stress that we achieved our goal without conceptualizing the concept of situation in detail.[11] If the concept of situation and/or of the universe of situations are specified in some way or another, one can argue that some non-classical logics are, generally speaking, adequate with respect to the corresponding semantics based on the intuitions concerning concepts of situation used[12], or that they perform the role of ontology of the universe of situations.[13] In the case of Classical Propositional Calculus, however, we are permitted to say that, whatever situations are, the claims made by sentences that are instances of theses of the calculus hold in any situation. The reader is free to decide whether this constitutes an argument against the calculus or against the proposed semantics.

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can be introduced as follows: (a) $w \vDash p_i$ iff $w \in g(p_i)$; (b) $w \vDash \neg\alpha$ iff $w \nvDash \alpha$; (c) $w \vDash \alpha \wedge \beta$ iff $w \vDash \alpha$ and $w \vDash \beta$; (d) $w \vDash \alpha \vee \beta$ iff $w \vDash \alpha$ or $w \vDash \beta$; (e) $w \vDash \alpha \rightarrow \beta$ iff $w \nvDash \alpha$ or $w \vDash \beta$; (f) $w \vDash \alpha \leftrightarrow \beta$ iff $w \vDash \alpha$ just in case $w \vDash \beta$. The set of all possible worlds from W in which α holds can now be defined by $\{w \in W : w \vDash \alpha\}$.

Intuitions underlying the concept of situation, however, are different from those which underlie the concept of possible world (cf. Barwise 1989a). Possible worlds are sometimes defined in terms of situations (see Zalta 1991).

An observant reader certainly notices similarities between the semantics presented in this essay and the topological semantics for Heyting's Intuitionistic Propositional Calculus proposed by Tarski (1938, 1983). The similarity becomes easily visible when Tarski's semantics is formulated in a manner slightly departing from the original formulation (cf. Krajewski 1987). Let (X, Int) be a topological space (X is here a non-empty set, and Int is a closure operator). Let us consider an ordered pair $\langle (X, Int), g\rangle$, where g is a function whose domain is the set of well-formed formulas. The function g is supposed to satisfy the following conditions: (a) $g(p_i)$ is an open set of the space (X, Int), for each propositional variable p_i; (b) $g(\neg\alpha) = Int(X \setminus g(\alpha))$; (c) $g(\alpha \wedge \beta) = g(\alpha) \cap g(\beta)$; (d) $g(\alpha \vee \beta) = g(\alpha) \cup g(\beta)$; (e) $g(\alpha \rightarrow \beta) = Int((X \setminus g(\alpha)) \cup g(\beta))$. Tarski proved that formula α is a thesis of Heyting's Intuitionistic Propositional Calculus iff $g(\alpha) = X$ for each structure $\langle (X, Int), g\rangle$ satisfying the conditions specified above. As for CPC, we can say (following some remarks of Tarski; see Tarski 1983, p. 448) that α is a thesis of CPC iff $g(\alpha) = X$ for any structure $\langle (X, Int), g\rangle$ that satisfies the specified conditions and is such that $Int(Y) = Y$ for each $Y \subseteq X$.

[11]Although the concept of situation plays a crucial role in situational semantics, no commonly accepted definition of the concept has been elaborated. By the way, this is a manifestation of a wider phenomenon. Jon Barwise (in Barwise 1989) lists nineteen questions, the arrays of possible answers to which constitute backgrounds of different research paradigms in the field. This does not mean, however, that there were no attempts to define the concept of situation. The author of this essay personally thinks that the following papers and books are worthy of special attention: Barwise (1986), Devlin (1991), Seligman (1991), Wolniewicz (1985), Zalta (1991). *Added in 2013.* This essay was written in 1997. For later developments in situational semantics see, e.g., Devlin (2006), or Mares, Seligman and Restall (2011).

[12]See, e.g. Fenstad et al. (1987), Chapter 4, or Plotkin (1990).

[13]See Omyła (1986), Omyła (1991).

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