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SOME FOUNDATIONAL CONCEPTS OF EROTETIC SEMANTICS*

The aim of this paper is to define some semantical concepts which may be useful in the analysis of questions and questioning. Some of them have been already introduced elsewhere (see the papers Wiśniewski 1989, 1991, 1994a, 1994b, and Buszkowski 1989); we shall give here more extended motivation for the proposed definitions. Some concepts we introduce are in principle borrowed from Belnap's erotetic semantics (cf. Belnap and Steel 1976, Chapter 3); yet, contrary to Belnap, in our analysis we make use of the concept of multiple-conclusion entailment. In general, this concept is one of the main tools applied throughout this paper.

We shall be considering here a class of formalized languages whose meaningful expressions are either declaratives or questions; a language of the analyzed kind consists of the assertoric part being a standard first order language with identity and the erotetic part with questions as the key expressions. Yet, first order languages can be supplemented with questions in many ways and it is not the case that the concepts we are going to introduce are applicable regardless of the way of constructing a question-and-answer system. So let us start with some general information about the existing logical theories of questions and answers.

1. Questions and answers

1.1. Questions: reductionism vs. non-reductionism

As we read in one of the very few monographs on the logic of questions, "Different authors developing logical theories of questions accept different answers to the question 'What is a question?'" (Kubiński 1971, p. 97). This statement, written down more than twenty five years ago, still gives us a realistic description of the situation within erotetic logic.

To speak generally, the approaches to questions proposed by different logicians and formal linguists can be divided into *reductionist* and *non-reductionist* ones. Inside the reductionist approach, in turn, the *radical* and *moderate* standpoints can be distinguished.

1.1.1. *Radical reductionism.* According to the radical view, questions are not linguistic entities. The reduction of questions to sets of sentences or propositions is most often adopted here. Sometimes any set of sentences is allowed to be a question, but usually questions are identified with sets of answers of some distinguished category. Stahl (cf., e.g., Stahl 1962) identifies questions with sets of their sufficient answers; these answers are declarative formulas of a strictly defined kind. Hamblin (cf. Hamblin 1973) identifies a question with the set of its possible answers, whereas Karttunen (cf. Karttunen 1978) identifies questions with sets of their true answers; in both cases the relevant answers are propositions in the sense of some intensional logic. Questions are also identified with functions defined on possible worlds; the set of values of a function of this kind consists of truth-values, or of sets of individuals, or of sets of sets of individuals (cf., e.g., Tichy 1978, and Materna 1981). Also in this case some intensional logic serves as the basis of analysis. An analysis of questions and interrogatives in terms of (some versions of) Montague intensional logic is to be found e.g. in Groenendijk and Stokhof (1984). Some linguists developed the so-called categorial approach to questions: according to this view, questions are to be considered as functions from categorial answers to propositions (cf., e.g., Hausser 1983); a categorial answer may be a full sentence, but may also be a part of it, e.g. a noun phrase, an adverb, etc. There are philosophers of language who tend to identify questions with speech acts rather than with expressions. Let us stress, however, that in all of the above cases a distinction is made between an interrogative (or an interrogative sentence) and a question: whereas interrogatives are linguistic entities, questions are claimed not to be.

Let us finally add that some linguists (cf. Keenan and Hull 1973, and Hiż 1978) proposed theories in which the semantically meaningful units are not questions, but question-answer pairs. Sometimes questions are also analyzed as ordered pairs consisting of interrogative terms and statements expressing the relevant presuppositions.

1.1.2. *Moderate reductionism.* The moderate reductionist view considers questions as linguistic entities which, however, can be reduced to expressions of some other categories. To be more precise, it is claimed here that every question can be adequately characterized as an expression which is synonymous (or synonymous to some reasonable degree) to a certain expression of a different syntactical category. Or, to put it differently, each question can be adequately paraphrased as an expression belonging to some other syntactic category and then formalized within some logic which, although not primarily designed as the logic of questions, can thus be regarded as providing us with the foundations of erotetic logic.

Some theorists propose the reduction of questions to declarative formulas of strictly defined kind(s). Sometimes questions are identified with declarative formulas having free variables, that is, with sentential functions (cf., e.g., Cohen 1929). But questions are also identified with sentences, that is, declarative formulas with no free variables. According to the early proposal of David Harrah (cf. Harrah 1961, 1963) whether-questions are to be understood and then formalized as declarative sentences having the form of exclusive disjunctions, whereas which-questions should be identified with existential generalizations.

Questions are also identified with imperatives of a special kind. The imperative-epistemic approach is the most popular here with Lennart Åqvist and Jaakko Hintikka as its most eminent representatives.

According to Åqvist (cf., e.g., Åqvist 1971, 1972, 1975), a question can be paraphrased as an imperative-epistemic expression of the form "Let it (turn out to) be the case that φ", where φ is a formula which describes the epistemic state of affairs which should be achieved. Pragmatically, a question is thus understood as an imperative which demands of the respondent to widen the questioner's knowledge. Questions are formalized within the framework of some imperative-epistemic logic; on the level of formal analysis we deal with interrogatives. Each interrogative consists of an interrogative operator and its arguments. Interrogatives are defined as abbreviations of certain formulas of the language of the considered imperative-epistemic logic.

The imperative-epistemic approach to questions is also adopted by Jaakko Hintikka in his theory of questions and answers (cf., e.g., Hintikka 1974, 1976, 1978, 1983). Hintikka interprets questions as requests for information or knowledge: according to his view, each question can be paraphrased as an expression which consists of the operator "Bring it about that" followed by the so-called *desideratum* of the question. The desideratum describes the epistemic state of affairs the questioner wants the respondent to bring about. Although the main ideas of Åqvist and Hintikka are similar, they are elaborated on in different ways.

As far as the moderate reductionist view is concerned, the imperative-epistemic approach is the most widely developed one. Yet, there are also other proposals. In particular, there is an old idea (which goes back at least to Bolzano) that the paraphrase of a question should contain an optative operator. (It is worth noticing that Hintikka sometimes calls the operator "Bring it about that" an "imperative or optative operator.") Apostel (cf. Apostel 1969) claims that questions can be reduced to expressions which contain epistemic, deontic and alethic operators as well as the assertion operator; yet, Apostel also claims that the deontic operator "ought to" used by him should be replaced by some optative or imperative operator when a more adequate analysis of such operators will be available. In a short note Åqvist (cf. Åqvist 1983) sketched an outline of the imperative-assertoric analysis of questions; according to this proposal, questions should be paraphrased (and then formalized) as imperatives which contain the assertoric operator "You tell me truly that" instead of the epistemic operator "I know that." Let us stress, however, that none of the above proposals has been elaborated on.

1.1.3. *Non-reductionism.* According to the non-reductionist approach, questions are specific expressions of a strictly defined form; they are not reducible to expressions of other syntactic categories.

The most widespread proposal here is to regard a question as an expression which consists of an interrogative operator and a sentential function. This view is accepted, among others, by Ajdukiewicz (cf., e.g., Ajdukiewicz 1974), Hiż (cf. Hiż 1962), Kubiński (cf., e.g., Kubiński 1960, 1971, 1980), Koj (cf. Koj 1972), Leszko (cf., e.g., Leszko 1983), and Harrah in his later papers (cf., e.g., Harrah 1975, 1984). Some prominent authors (e.g. Carnap, Reichenbach, Cresswell) who only incidentally paid attention to questions also shared this view. Yet, the above idea is most widely elaborated on in the books and papers of Tadeusz Kubiński.

Kubiński's analysis is mainly a syntactical one: questions of a formalized language are defined as expressions which consist of interrogative operators and sentential functions. Interrogative operators, in turn, consist of both constants and variables. The only free variables in the sentential functions which occur in questions are the variables of the corresponding interrogative operators; these variables are "bound" by the interrogative operators. The variables which occur in questions may belong to various syntactical categories. Roughly, the categories of variables indicate the (ontological) categories of objects which are asked about. For example, a question whose interrogative operator contains only individual variables asks about individuals. If the relevant variables run over sentential connectives, then the corresponding questions are about either the existence of some state(s) of affairs or some connection(s) between states of affairs. Questions with predicate variables, in turn, ask about properties or relations. When a question contains only sentential variables, it is a question about logical values (truth and falsehood). Kubiński considers also "mixed" questions, that is, questions whose interrogative operators contain variables belonging to two or more different categories.

Although the "interrogative operator-sentential function" view is shared by most of the adherents of the non-reductionist approach to questions, there are also other proposals. Among them special attention should be paid to Nuel D. Belnap's theory of questions and answers (cf., e.g., Belnap 1963, and Belnap and Steel 1976).

Belnap distinguishes between: (a) natural language questions, (b) interrogatives and (c) questions understood as abstract (set-theoretical) entities. Interrogatives are expressions of some formalized languages. They are not only formal counterparts of natural language questions, but they also express questions understood as abstract entities.

A simple interrogative consists of the question mark ?, the lexical subject and the lexical request. The question mark is interpreted as a sign of the function which assigns to the lexical subject and the lexical request of a simple interrogative the corresponding (abstract) question; such a question consists, in turn, of the abstract subject and the abstract request. A compound interrogative can be obtained from simple interrogatives by performing some (logical of Boolean) operations on them or their lexical subjects.

The basic idea of Belnap's approach is that an interrogative "presents" a set of alternatives together with some suggestions or indications as to what kind of choice or selection among them should be made; the situation is analogous in the case of the corresponding questions. The function of the lexical subject of an interrogative is to offer the relevant (nominal) alternatives, whereas the role of the lexical request is to characterize the required kind of selection. The lexical request consists of three parts: the lexical selection-size specification, the lexical completeness-claim specification, and the lexical distinctness-claim specification. Roughly, the lexical selection-size specification informs how many (that is, how many exactly, how many at least and/or how many at most) of the alternatives offered by the lexical subject of the interrogative are called for. The lexical completeness-claim specification, in turn, informs about the amount of true nominal alternatives called for; there are interrogatives which call for all the true alternatives presented by them, but there are also interrogatives which do not demand so much. Finally, the lexical distinctness-claim specification tells whether the alternatives called for should be semantically different.

1.2. Answers

Most logical theories of questions pay at least as much attention to answers to questions as to questions themselves. It is usually assumed that a question can have many answers; the phrase "answer to a question" *is not* used synonymously with "the true answer to a question." In other words, the analyzed answers are usually *possible* answers: their logical values are not prejudged. Yet, it is not the case that all possible answers are equally interesting to erotetic logicians. The standard way of proceeding is to define some *basic* category of possible answers. They are called *direct answers* (Åqvist, Belnap, Harrah, Kubiński), *proper answers* (Ajdukiewicz), *sufficient answers* (Stahl), *conclusive answers* (Hintikka and his associates), *indicated replies* (Harrah in his later papers), etc. Those "principal" possible answers (let us use this general terms here) are supposed to satisfy some general conditions, usually expressed in pragmatic (in the traditional sense of the word) terms. For example, direct answers in Kubiński's sense are "these sentences which everybody who understands the question ought to be able to recognize as the simplest, most natural, admissible answers to this question" (Kubiński 1980, p. 12). Direct answers in Belnap's sense are the answers which "are directly and precisely responsive to the question, giving neither more nor less information than what is called for" (Belnap 1969a, p. 124). Direct answers in Harrah's sense are replies which are complete and just-sufficient answers (cf. Harrah 1963, p. 26 et al.). In the light of Hintikka's theory a reply is called conclusive just in case it completely satisfies the epistemic request of the questioner, that is, brings about the epistemic state of affairs the questioner wanted to be brought about.

Let us stress that although the above conditions are formulated in pragmatic terms, some of the logical theories of questions define the principal possible answers to questions or interrogatives of *formalized languages* in terms of syntax and/or semantics. The questions (interrogatives) of formalized languages, however, are usually formalizations of natural language questions. Consequently, the principal possible answers are usually defined in such a way that the natural language sentences which correspond to them are those answers to the analyzed natural language questions which have the above-mentioned pragmatic properties. Or, to be more realistic, to define them in such a way is the aim of the enterprise. Yet, there are natural language questions which admit many readings and thus many formalizations. There are also natural language questions which seem to have no well-defined sets of answers; why-questions are often recalled in this context. Moreover, most theories admit questions or interrogatives which have no counterparts in natural languages, but nevertheless have well-defined sets of the "principal" possible answers.

It is not the case that only the principal possible answers are of interest to erotetic logicians. Most theories provide us also with definitions of other kinds of possible answers: partial answers, complete answers, just-complete answers, incomplete answers, corrective answers, etc. These answers are usually defined in terms of the "principal" answers; yet, it also happens that they (or some of them) are defined independently. Their definitions differ from theory to theory; no well-established terminology has been elaborated yet.

Let us finally add that sometimes replies which are not statements (e.g. noun phrases, nods, grunts) are regarded as answers; in most cases, however, answers are assumed to be statements and replies of other kinds are regarded as abbreviations of the corresponding statements.

2. General assumptions

As the above sketchy presentation shows, first-order languages (as well as other formalized languages) can be supplemented with questions in different ways. Yet, regardless of the way how the extension goes on, it requires, first, the introduction of new symbols to the vocabulary of some basic extensional formalized language. Questions (or interrogatives) are made up of the new symbols and the old ones. Then the assignment of the principal possible answers to questions takes place; this is sometimes done in purely syntactical terms, but also on the level of semantics or/and pragmatics. The principal possible answers are usually expressions of the basic extensional language; in most cases they are declarative well-formed formulas of it. The result is a question-and-answer system.

In what follows we will be adopting the following general assumptions. First, we assume that some first-order language with identity $\mathcal{L}_0$ is given. Second, we assume that the vocabulary of $\mathcal{L}_0$ is supplemented with some new symbols in order to constitute the vocabulary of some new (formalized) language $\mathcal{L}$. By *terms* and *declarative well-formed formulae* (d-wffs for short) of $\mathcal{L}$ we shall mean those of $\mathcal{L}_0$. We assume that the language $\mathcal{L}$ contains some new category of meaningful expressions (built up, int.al, by means of the new symbols) which are called *questions*; yet, we do not forejudge the way of constructing questions of $\mathcal{L}$. We then assume that to each question of $\mathscr{L}$ there is assigned an at least two-element set of principal possible answers, which are sentences (d-wffs with no free variables) of $\mathscr{L}$. For brevity we shall call them *direct answers* to the question.

Although the above conditions are rather general, it cannot be said that they are uncontroversial; it even cannot be said that they are fulfilled by each existing question-and-answer system. The motivation for some of them is philosophical. We require each question to have at least two principal possible (i.e. direct) answers because we think that a necessary condition of being a question is to present at least two "alternatives" or conceptual possibilities among which some selection can be made. The "Hobson choice" questions are thus excluded, but rhetorical questions are allowed – the selection need not be rational. Some logical theories of questions (for example, Belnap's theory or Kubiński's theory) allow questions which have only one direct answer, but it seems that this step is motivated rather by the pursuit of generality than other reasons. Concerning the condition according to which each direct answer is a sentence: in the light of semantics we are going to propose the free variables are interpreted in the "generalizing" manner (not as "dummy" names): a sentential function of the form Ax_i is true if and only if, to speak generally, it is satisfied by all the possible values of x_i. Under this interpretation a sentential function expresses a condition which may be satisfied by some object and not satisfied by others; on the other hand, we want the principal possible answers to be answers in the very serious sense of the word. But the most controversial is of course the assumption according to which each question *has* a set of principal possible answers. It cannot be said that each question analyzed in any logical theory of questions fulfills this condition; it also cannot be said that any natural-language question fulfills it. Yet, some of them do, and most question-and-answer systems distinguish questions having this property. So the above assumption may be viewed as restricting the range of applicability of the concepts we are going to propose. In other words, our further considerations pertain to first-order languages with identity enriched with questions in such a way that each question has at least two-element set of principal possible answers; these answers are sentences of the basic extensional language. Yet, let us stress, the details of the extension procedure are irrelevant from the point of view of applicability of the concepts we are going to propose.

We will be using the letters A, B, C, ..., possibly with subscripts, as metalinguistic variables for d-wffs, and the symbols X, X_1, ..., Y, Y_1, ..., Z, Z_1, ... as metalinguistic variables for sets of d-wffs. The symbols Q, Q_1, ... will be used as metalinguistic variables for questions. The set of (all) direct answers to a question Q will be referred to as dQ. On the metalanguage level we assume the von Neumann–Bernays–Gödel version of the set theory (we choose this version because we want to have the possibility of speaking about both sets and classes). We shall use the standard set-theoretical terminology and notation. The expression "iff" is an abbreviation of "if and only if."

3. Basic concepts

3.1. Interpretations, satisfaction and truth

According to what has been said above the language $\mathscr{L}$ results from the first-order language with identity $\mathscr{L}_0$. To speak generally, our first step is to supplement the language $\mathscr{L}_0$ with a standard model-theoretical semantics. To be more precise, we shall first define some concepts which do not pertain to questions of $\mathscr{L}$.

Let us temporarily assume that the vocabulary of $\mathscr{L}$ contains apart from some predicate symbol(s) also some function symbol(s) and individual constant(s). By non-logical constants of $\mathscr{L}$ we mean below individual constants, function symbols and predicate symbols of this language.

DEFINITION 1. An *interpretation* of the language $\mathscr{L}$ is an ordered pair $<\mathbf{M}, \mathbf{f}>$, where $\mathbf{M}$ is a non-empty set and $\mathbf{f}$ is a function defined on the set of non-logical constants of $\mathscr{L}$ which fulfills the following conditions:

(i) for each individual constant a_i, $\mathbf{f}(a_i) \in \mathbf{M}$,

(ii) for each n-argument function symbol F_i^n, $\mathbf{f}(F_i^n)$ is a n-argument function defined on the set $\mathbf{M}$ and whose values belong to the set $\mathbf{M}$,

(iii) for each n-place predicate symbol P_i^n, $\mathbf{f}(P_i^n)$ is a n-ary relation in $\mathbf{M}$.

If $<\mathbf{M}, \mathbf{f}>$ is an interpretation, the set $\mathbf{M}$ is called the *domain* of this interpretation, whereas the function $\mathbf{f}$ is called the *interpretation function*. Note that the new symbols which enable to form questions of $\mathscr{L}$ are not arguments of an interpretation function!

When speaking about interpretations we will be using the symbols $\mathfrak{I}$, $\mathfrak{I}'$, ...

The definition of the concept of interpretation should be adjusted in an obvious way if the vocabulary of $\mathscr{L}$ contains no function symbols or no individual constants.

Let $\mathfrak{I} = \langle \mathbf{M}, \mathbf{f} \rangle$ be an arbitrary but fixed interpretation of $\mathcal{L}$. A $\mathfrak{I}$-*valuation* is a denumerable sequence of elements of the domain of the interpretation $\mathfrak{I}$ (by "denumerable" we mean here and below "countably infinite"). The concept of *value* of a term of $\mathcal{L}$ in the interpretation $\mathfrak{I}$ with respect to a given $\mathfrak{I}$-valuation $\mathbf{s}$ is defined in the standard way. Similarly, the concept of *satisfaction* of a d-wff in the interpretation $\mathfrak{I}$ by a $\mathfrak{I}$-valuation $\mathbf{s}$ is defined in the usual manner.

A d-wff A of $\mathcal{L}$ is *true in an interpretation* $\mathfrak{I}$ of $\mathcal{L}$ if and only if A is satisfied in $\mathfrak{I}$ by each $\mathfrak{I}$-valuation. If a d-wff A is true (is not true) in $\mathfrak{I}$, we write $\mathfrak{I} \models A$ (we write $\mathfrak{I}$ *non* $\models A$). By a *model* of a set of d-wffs X we mean any interpretation of the language in which all the d-wffs in X are true. If an interpretation $\mathfrak{I}$ is a model (is not a model) of a set of d-wffs X, we write $\mathfrak{I} \models X$ (we write $\mathfrak{I}$ *non* $\models X$).

Let us stress that the concepts of satisfaction and truth do not pertain to questions of $\mathcal{L}$.

3.2. Normal interpretations and consistency

Let us now assume that the class of interpretations of $\mathcal{L}$ includes a non-empty subclass (not necessarily a proper subclass) of *normal interpretations*. The reasons for distinguishing normal interpretations from the remaining ones can be various. For instance, one may intend to construe some non-logical constant(s) in a way that complies with some intuitions; in this case normal interpretations can be defined as those which make true some definition(s) or meaning postulate(s) worded in $\mathcal{L}$. Normal interpretations can also be defined as models of some first-order theory worded in (the declarative part of) $\mathcal{L}$. They can also be defined as those models of such a theory which fulfill some additional conditions; the so-called standard models of Peano's arithmetics give us a simple example here. Another possibility lies in defining normal interpretations as those which make true some sentences which are regarded as "laws of science", that is, first-order counterparts of some scientific laws (expressed in $\mathcal{L}$ which serves as the language of logical analysis). Normal interpretations can also be identified with the "intended models" in the sense of philosophy of science. But normal interpretations can also be distinguished for purely "erotetic" reasons. For example, in Belnap's theory of questions special attention is paid to those interpretations of the basic "assertoric" languages in which, to speak generally, the objects assigned to terms that belong to the nominal category determined by a given category condition are among the objects that satisfy this category condition[1]. When questions about objects that satisfy some sentential function or functions are considered, it seems natural to call normal interpretations only those interpretations in which each element of the domain has a name: by doing so we can avoid the situation that there are objects which satisfy the appropriate sentential function(s), but nevertheless the analyzed questions have no true direct answers.

It seems impossible to define the general concept of "normalness" of interpretation: this concept varies from language to language. As far as the language $\mathcal{L}$ is concerned we only assume that the class of normal interpretations of it exists and is non-empty. Moreover, the remaining semantic concepts pertaining to the language $\mathcal{L}$ will be defined here by means of the concept of normal interpretation of $\mathcal{L}$. Thus their definitions remain schematic until the concept of normal interpretation of $\mathcal{L}$ will be defined in detail. But $\mathcal{L}$ is assumed to be an arbitrary but fixed formalized language which fulfills some general conditions and the situation is analogous in the case of its semantics. To speak generally, we thus leave room for different possibilities. Let us stress that we even do not assume (but also do not deny) that the class of normal interpretations of $\mathcal{L}$ is a *proper* subclass of the class of all interpretations of $\mathcal{L}$: it may happen that all interpretations of some language are regarded as normal.

A d-wff A of $\mathcal{L}$ is called a *tautology* of $\mathcal{L}$ if and only if A is true in each normal interpretation of $\mathcal{L}$. A d-wff A of $\mathcal{L}$ is said to be a *contradictory d-wff* (or *contradiction*) of $\mathcal{L}$ if and only if there is no normal interpretation $\mathfrak{I}$ of $\mathcal{L}$ such that some $\mathfrak{I}$-valuation satisfies A in $\mathfrak{I}$. A d-wff A of $\mathcal{L}$ is a *synthetic d-wff* of $\mathcal{L}$ just in case A is neither a tautology of $\mathcal{L}$ nor a contradictory d-wff of $\mathcal{L}$.

By a *normal model* of a set of d-wffs X of $\mathcal{L}$ we mean any normal interpretation of $\mathcal{L}$ being a model of X. A set of d-wffs X of $\mathcal{L}$ is said to be *consistent* just in case there is a normal model of X; otherwise X is said to be *inconsistent*. We shall use the symbol ***Inc*** for the family of all inconsistent sets of d-wffs of $\mathcal{L}$.

4. Entailment and multiple-conclusion entailment

4.1. Entailment

Let us assume again that the class of normal interpretations of $\mathcal{L}$ is defined in some way or another. The semantic concept of *entailment in a language* can now be defined as follows:

DEFINITION 2. A set of d-wffs X of $\mathcal{L}$ *entails in* $\mathcal{L}$ a d-wff A of $\mathcal{L}$ iff A is true in each normal interpretation of $\mathcal{L}$ in which all the d-wffs in X are true.

Let us stress that the above concept of entailment is relativized to the class of normal interpretations of the considered language. Thus entailment in a given language need not be tantamount to logical entailment; of course these concepts coincide if each interpretation is regarded as normal. The concept of logical entailment we have in mind here is defined as follows: X *logically entails* A iff A is true in each interpretation of the language in which all the d-wffs in X are true.[2] In general, the stronger the conditions we impose on normal interpretations the wider becomes the range of entailment in the language. But sometimes this is the effect we intended to achieve: if we intend to reflect in a formal language some non-logical implicatures which are already present in the natural language which is the subject of formalization, the simplest way is to define entailment according to the pattern presented by Definition 2 and to define normal interpretations in the appropriate manner: as those which make true some meaning postulates, or some theory, or some "laws", etc.[3]

We shall use the symbol $\models$ for entailment in a language.

We say that two d-wffs A, B of $\mathcal{L}$ are *equivalent* if and only if B is entailed in $\mathcal{L}$ by A and A is entailed in $\mathcal{L}$ by B.

Let us now introduce the concept of compactness of entailment. A relation $\models$ of entailment in a language is said to be *compact* if whenever $X \models A$ there exists a *finite* subset Y of X such that $Y \models A$. We neither assume nor deny here, however, that entailment in $\mathcal{L}$ is compact; there are languages of the considered kind in which entailment is compact, but there are also languages in which it is not. The compactness of entailment depends on the conditions imposed on the class of normal interpretations.

4.2. Multiple-conclusion entailment

When we are dealing with questions whose sets of direct answers are well-defined, we may think about the direct answers as offering some possibilities or "alternatives" among which some selection should be made. Thus some notion of, to speak generally, "entailing a set of possibilities" is needed. There is a logic, however, within which such a notion has been elaborated on: it is multiple-conclusion logic (cf. Shoesmith and Smiley 1978; see also Scott 1974, and Zygmunt 1984).

Multiple-conclusion logic generalizes the concept of entailment: now a *set* of conclusions is allowed. Any such set is regarded as, intuitively speaking, setting out the field within which the truth must lie if the premises are all true. Let us then introduce the concept of *multiple-conclusion entailment*.

DEFINITION 3. A set of d-wffs X of $\mathcal{L}$ *multiple-conclusion entails in* $\mathcal{L}$ a set of d-wffs Y of $\mathcal{L}$ if and only if the following condition holds:

(*) whenever all the d-wffs in X are true in some normal interpretation of $\mathcal{L}$, then there exists at least one d-wff in Y which is true in this interpretation of $\mathcal{L}$.

To speak generally: X multiple-conclusion entails Y just in case Y must contain at least one true d-wff if all the d-wffs in X are true; "must" in the sense that it is impossible that all the d-wffs in Y are not true in any normal interpretation of the language which makes true all the d-wffs in X.

We will be using the term "mc-entailment" instead of the long expression "multiple-conclusion" entailment.

We shall use the symbol $\Vdash$ for mc-entailment in a language. If Y is not mc-entailed by X, we write X *non* $\Vdash Y$. If a set of d-wffs Y is mc-entailed by a singleton set $\{A\}$, we simply say that Y is mc-entailed by the d-wff A.

The concept of multiple-conclusion entailment is a generalization of the standard (i.e. "single-conclusion") concept of entailment. As an immediate consequence of Definition 2 and Definition 3 we get:

COROLLARY 1. $X \models A$ *iff* $X \Vdash \{A\}$.

Corollary 1 says that a set of d-wffs X entails a d-wff A just in case the set X mc-entails the singleton set whose element is the d-wff A. Thus entailment is definable in terms of mc-entailment. On the other hand, in the general case mc-entailment is not definable in terms of entailment. It happens that a set of d-wffs X mc-entails some set of d-wffs Y, but does not entail (in the standard sense of the word) any d-wff in Y. The sets of the form $\{A \vee \neg A\}$ and $\{A, \neg A\}$, where A is an atomic sentence, give us a simple example here.

The basic properties of mc-entailment are characterized by the following corollaries ($\Vdash$ stands below for mc-entailment in $\mathcal{L}$; let us recall that $\mathcal{L}$ is an arbitrary but fixed language that fulfills the conditions listed above):

COROLLARY 2. *If* $X \cap Y \neq \emptyset$, *then* $X \Vdash Y$.
COROLLARY 3. *If* $X \subseteq X_1$, $Y \subseteq Y_1$ *and* $X \Vdash Y$, *then* $X_1 \Vdash Y_1$.

COROLLARY 4. *If* $X \cup Z_1 \models\!\!\!\!\mid\; Y \cup Z_2$ *for any* Z_1, Z_2 *such that* $Z_1 \cap Z_2 = \emptyset$ *and* $Z_1 \cup Z_2 = Z$, *then* $X \Vdash Y$.

Corollaries 2 and 3 are immediate consequences of Definition 3. For the proof of Corollary 4 assume that X *non* $\Vdash Y$. So there exists a normal interpretation $\mathfrak{I}$ of $\mathcal{L}$ such that $\mathfrak{I} \models X$ and for each $B \in Y$, $\mathfrak{I}$ *non* $\models B$. Let Z_1 be the set of all the d-wffs of $\mathcal{L}$ which are true in $\mathfrak{I}$ and let Z_2 be the set of all the d-wffs of $\mathcal{L}$ which are not true in $\mathfrak{I}$. We now have $X \cup Z_1$ *non* $\Vdash Y \cup Z_2$ as required.

Thus mc-entailment in any of the considered languages is a multiple-conclusion consequence in the sense of the monograph Shoesmith and Smiley (1978) (see also below).

A relation $\Vdash$ of mc-entailment is said to be *compact* if whenever $X \Vdash Y$ there exist *finite* subsets X_1 of X and Y_1 of Y such that $X_1 \Vdash Y_1$. We have:

COROLLARY 5. *Mc-entailment in* $\mathcal{L}$ *is compact iff entailment in* $\mathcal{L}$ *is compact.*

P r o o f:
(⇒) By Corollary 1.
(⇐) Let us first observe that if entailment in $\mathcal{L}$ is compact, then the class of normal interpretations of $\mathcal{L}$ fulfills the following condition:

(:) for each set of d-wffs X of $\mathcal{L}$: if each finite subset of X has a normal model, then the set X has a normal model.

For, if there is a set of d-wffs, say, X_1, such that each finite subset of X_1 has a normal model, but X_1 has no normal model, then X_1 is an infinite set; moreover, X_1 entails some contradictory sentence, say, A, which is not entailed by any finite subset of X_1. It follows that entailment in $\mathcal{L}$ is not compact; so if entailment in $\mathcal{L}$ is compact, the condition (:) holds.

Since the compactness of entailment yields the condition (:), it suffices to prove that if $X \Vdash Y$ and the condition (:) holds, then there are finite sets X_1, Y_1 such that $X_1 \subseteq X$, $Y_1 \subseteq Y$ and $X_1 \Vdash Y_1$. Moreover, it suffices to consider the cases in which X or Y are infinite sets.

Assume that Y is an infinite set. Let $uc(Y)$ be the set of universal closures[4] of the d-wffs of Y. Let us designate by $\neg uc(Y)$ the set of negations of the sentences of the set $uc(Y)$. Suppose that $X \Vdash Y$. Hence $X \cup \neg uc(Y) \in$ ***Inc***. By condition (:) we get that there exists a finite and non-empty subset Z of the set $X \cup \neg uc(Y)$ such that $Z \in$ ***Inc***. There are three possibilities: (a) $Z \subseteq X$, (b) $Z \subseteq \neg uc(Y)$, (c) $Z \subseteq X \cup \neg uc(Y)$, where $Z \not\subseteq X$ and $Z \not\subseteq \neg uc(Y)$. If the possibility (a) holds, then for each finite subset Y_1 of the set Y we have $Z \Vdash Y_1$; at the same time Z is a finite subset of the set X. If the possibility (b) takes place, then some finite subset of the set Y is mc-entailed by the empty set and thus also by each finite subset of the set X. It is obvious that if the possibility (c) holds, then some finite subset of the set Y is mc-entailed by some finite subset of the set X.

Assume that X is an infinite set. If $Y = \emptyset$, then $X \in \mathit{\mathbf{Inc}}$. By condition (:) we get that there exists a finite subset X_1 of the set X such that $X_1 \in \mathit{\mathbf{Inc}}$. Thus $X_1 \Vdash \emptyset$; on the other hand, $\emptyset$ is a finite subset of Y. If $Y \neq \emptyset$, we proceed analogously as in the case in which Y is an infinite set. □

Thus mc-entailment in a language is compact just in case entailment in this language is compact. Let us stress that Corollary 5 speaks of any language of the considered kind. Since we neither assume nor deny here that entailment in $\mathcal{L}$ is compact, the same holds true in the case of mc-entailment in $\mathcal{L}$: we leave room for different possibilities.

By a sentence we mean here a d-wff with no free variables; otherwise a d-wff is said to be a sentential function. One can easily prove:

COROLLARY 6. *If* $A_1, \ldots, A_n$ *are sentences, then* $\{A_1 \vee \ldots \vee A_n\} \Vdash \{A_1, \ldots, A_n\}$.

COROLLARY 7. *If* $A_1, \ldots, A_n$ *are sentences, then:* $X \Vdash \{A_1, \ldots, A_n\}$ *iff* $X \models A_1 \vee \ldots \vee A_n$.

According to Corollary 6, a finite and non-empty set of sentences of a given language is mc-entailed in this language by a disjunction of all its elements (to be more precise, by a singleton set which contains this disjunction). Corollary 7 says that a finite and non-empty set of sentences is mc-entailed by a set of d-wffs X just in case the set X entails some disjunction of all the elements of this set. Let us stress that the corollaries 6 and 7 describe some general properties of mc-entailment: they are true with respect to any language of the considered kind. Let us also stress that the assumption that $A_1, \ldots, A_n$ are sentences is essential: in the case of sentential functions the situation is different. For, let us consider a sentential function $P(x_i)$ (where P is a one-place predicate symbol) of some language and let us assume that there is a normal interpretation $\mathfrak{I}$ of the language such $P(x_i)$ is satisfied in $\mathfrak{I}$ only by some $\mathfrak{I}$-valuations, but not by all of them. The set $\{P(x_i) \vee \neg P(x_i)\}$ does not mc-entail in the analyzed language the set $\{P(x_i), \neg P(x_i)\}$, but does entail the sentence $P(x_i) \vee \neg P(x_i)$. To give a more concrete example: let us imagine that P stands for "is a prime" and the domain of the interpretation consists of all the natural numbers.

The universal closure[5] of a d-wff A is referred to as $\bar{A}$. The following is a consequence of the corollaries 3, 5 and 7:

COROLLARY 8. *If entailment in* $\mathcal{L}$ *is compact, X is a set of d-wffs of* $\mathcal{L}$ *and Y is a non-empty set of d-wffs of* $\mathcal{L}$, *then* $X \Vdash Y$ *iff either there is* $A \in Y$ *such that* $X \models \bar{A}$ *or there are* $A_1, \ldots, A_n \in Y$ *such that* $X \models \bar{A}_1 \vee \ldots \vee \bar{A}_n$.

Thus if entailment is compact, mc-entailment of a non-empty set of d-wffs Y reduces either to entailment of the universal closure of a single d-wff of Y or to entailment of some disjunction of universal closures of d-wffs of Y. It does not mean, however, that the concept of mc-entailment is superfluous: there are languages of the considered kind in which entailment (and thus mc-entailment as well) is not compact.

Historical note. The idea of multiple-conclusion consequence goes back to Gentzen (1934); one of the possible ways of looking at a valid sequent of the form $A_1, \ldots, A_m \vdash B_1, \ldots, B_n$ is to construe it, to speak generally, as stating multiple-conclusion entailment of the set made up of the formulas referred to by $B_1, \ldots, B_n$ from the set made up of the formulas referred to by $A_1, \ldots, A_m$. Under this interpretation the turnstile $\vdash$ is a relation symbol and a calculus of sequents is a (single-conclusion) metacalculus for a multiple-conclusion object-calculus. Yet, there is also another possibility: a sequent $A_1, \ldots, A_m \vdash B_1, \ldots, B_n$ is a notation for a formula $A_1 \wedge \ldots \wedge A_m \rightarrow B_1 \vee \ldots \vee B_n$ and Gentzen's calculi of sequents are variants of the corresponding conventional calculi. Shoesmith and Smiley claim that Gentzen interpreted his calculi of sequents in this latter way. If this is so, it is Carnap who for the first time introduced the concept of multiple-conclusion entailment (cf. Carnap 1943; Carnap uses the term "involution"). The concept of multiple-conclusion consequence was incorporated into the general theory of logical calculi by Dana Scott (1974). Multiple-conclusion consequence and related concepts (multiple-conclusion calculus, multiple-conclusion rules, etc.) are analyzed in detail in the monograph Shoesmith and Smiley (1978); for developments see also the book Zygmunt (1984). Let us add that the approach presented by Shoesmith and Smiley is much more general than ours: it is not restricted to first-order languages supplemented with model-theoretical semantics. Assume that L is a formalized language and let us designate by *Form*(L) the set of formulas of L; the only condition imposed on *Form*(L) is the non-emptiness clause. A relation $\vdash$ between sets of formulas of L is a called a *multiple-conclusion conse-*

quence if and only if $\vdash$ fulfills the following conditions for any $X, Y, Z \subseteq Form(L)$:

(C_1)(*Overlap*): *If* $X \cap Y \neq \emptyset$, *then* $X \vdash Y$.
(C_2)(*Dilution*): *If* $X \subseteq X_1$, $Y \subseteq Y_1$ *and* $X \vdash Y$, *then* $X_1 \vdash Y_1$.
(C_3)(*Cut for sets*): *If* $X \cup Z_1 \vdash Y \cup Z_2$ *for any* Z_1, Z_2 *such that* $Z_1 \cap Z_2 = \emptyset$ *and* $Z_1 \cup Z_2 = Z$, *then* $X \vdash Y$

This is the syntactical concept of multiple conclusion-consequence. But there is also a semantical one. Assume that L is supplemented with some semantics rich enough to define some (relativized) concept of truth for formulas. A *partition of* L is an ordered pair $<T, U>$ such that $T \cap U = \emptyset$ and $T \cup U = Form(L)$; we may think of the elements of the set T of a partition $<T, U>$ as of consisting of truths in the sense of the underlying semantics and of the set U as of consisting of untrue formulas. Let Δ be a class of partitions of L. A relation $\vdash$ between sets of d-wffs of L is the *multiple-conclusion consequence relation characterized by* Δ iff for each partition $<T, U> \in \Delta$ and for each $<X, Y> \in \vdash$, $X \cap U \neq \emptyset$ or $Y \cap T \neq \emptyset$: this is the semantical concept of multiple-conclusion consequence. It may be proved that each relation being a multiple-conclusion consequence in the syntactical sense of the word is a multiple-conclusion consequence in the semantical sense, that is, a multiple-conclusion consequence characterized by some class of partitions of the language. It can also be proved that for each class of partitions of the language, the multiple-conclusion consequence characterized by this class is a multiple-conclusion consequence in the syntactical sense of the word, that is, fulfills the conditions (C_1), (C_2) and (C_3). For proofs, see Shoesmith and Smiley (1978), p. 30.

What we have called above multiple-conclusion entailment in $\mathcal{L}$ would presumably be called by Shoesmith and Smiley multiple-conclusion consequence characterized by the class of normal interpretations of $\mathcal{L}$, or, to be more precise, by the class of partitions of the set of d-wffs of $\mathcal{L}$ determined by the class of normal interpretations of $\mathcal{L}$.

By means of the concept of mc-entailment we can define certain useful erotetic concepts in a simple, general and natural way; as a matter of fact, this concept will be the most useful tool of our further analysis. To our best knowledge, the idea of applying the concept of mc-entailment in erotetic logic appeared for the first time in the papers Buszkowski (1987) and Wiśniewski (1987)[6] and in a marginal form in the dissertation Wiśniewski (1986); see also the papers Buszkowski (1989) and Wiśniewski (1989) for more extended expositions. For the application of this concept in erotetic logic see also the books Wiśniewski (1990a) and (1995) and the papers Wiśniewski (1990b), (1991), (1994a) and (1994b).

5. Erotetic concepts

5.1. Soundness, safety and riskiness

The semantic concepts introduced so far do not pertain to questions. Let us now define the semantical erotetic concepts the introduction of which is the main goal of this paper.

The first important step is a negative one: we do not assign here truth and falsehood to questions. The reason is that it is doubtful whether all questions can express thoughts and describe states of affairs. Yet, we shall introduce here a more neutral semantic concept of *soundness* of a question in a given interpretation of a language.

DEFINITION 4. A question Q of $\mathcal{L}$ is *sound in an interpretation* $\Im$ of the language $\mathcal{L}$ iff at least one direct answer to Q is true in $\Im$.

The basic idea of this definition was suggested by Sylvain Bromberger (cf. Bromberger 1992, p. 146). Soundness understood in the above sense is called by Belnap (nominal) truth (cf. Belnap and Steel 1976, p. 119); yet, we prefer to use here the more neutral term.

There are questions which are sound in each normal interpretation of the language, questions which are sound only in some such interpretation(s) and questions which are not sound in any normal interpretation of the language. Following Belnap (cf. Belnap and Steel 1976, p. 130; we omit the relativization to a set of quasiformulae) we shall introduce here the concepts of *safety* and *riskiness* of a question.

DEFINITION 5. A question Q of $\mathcal{L}$ is said to be *safe* iff Q is sound in each normal interpretation of $\mathcal{L}$; otherwise Q is said to be *risky*.

Thus a safe question is a question which has at least one true direct answer in each normal interpretation of the language. Let us observe that safe questions might also have been defined as questions whose sets of direct answers are mc-entailed by the empty set; we can easily prove:

COROLLARY 9. *A question Q of $\mathcal{L}$ is safe iff the set of direct answers to Q is mc-entailed in $\mathcal{L}$ by the empty set.*

Simple yes–no questions, that is, questions whose sets of direct answers consist of a sentence and its negation, are paradigmatic examples of safe questions. Let us stress, however, that they are not the only safe questions. For instance, any questions having at least two direct answers which contradict each other and any questions which have tautologies among their direct answers are safe questions. Yet, it can be proved that each safe question is reducible to a set of questions made up of simple yes–no questions; it can also be proved that if Q is a safe question which has a finite number of direct answers or entailment in the language is compact, then Q is reducible to a finite set of simple yes–no question. For that and related results see Wiśniewski (1994b).

5.2. Just-complete answers and partial answers

If direct answers are defined in syntactic terms, it may happen that a sentence which is equivalent to a direct answer need not be a direct answer; on the other hand, such a sentence may perform the same pragmatic functions as a direct answer. Let us then introduce the semantic concept of just-complete answer; the definition given below is a slight modification of that proposed by Belnap (cf. Belnap and Steel 1976, p. 126).

DEFINITION 6. A sentence A of $\mathcal{L}$ is a *just-complete answer* to a question Q of $\mathcal{L}$ iff there is a direct answer B to Q such that B entails in $\mathcal{L}$ the sentence A and A entails in $\mathcal{L}$ the answer B.

In other words, a just-complete answer is a sentence which is equivalent to some direct answer.

It seems natural to call a partial answer to a question any sentence which is not equivalent to any direct answer to the question, but which is true if and only if a true direct answer belongs to some specified *proper* subset of the set of all the direct answers to the question. In other words, a partial answer is a sentence which is neither direct nor just-complete answer, but whose truth guarantees that a true direct answer can be found in some "restricted area" and whose truth is guaranteed by this fact. By means of the concept of mc-entailment we can express this intuition as follows:

DEFINITION 7. A sentence A of $\mathcal{L}$ is a *partial answer* to a question Q of $\mathcal{L}$ iff A is not a just-complete answer to Q and there exists a non-empty proper subset Y of the set of direct answers to Q such that: (i) Y is mc-entailed in $\mathcal{L}$ by A and (ii) A is entailed in $\mathcal{L}$ by each element of Y.

Let us stress that our concept of partial answerhood differs from those analyzed by Harrah, Belnap, and Kubiński.

If a question has more than two direct answers, then each disjunction of at least two but not all direct answers which is not equivalent to any single direct answer is a partial answer to the question. If entailment is compact, then each partial answer is either a disjunction of at least two but not all direct answers or a sentence which is equivalent to such a disjunction. The above definition yields, however, that questions with exactly two direct answers have no partial answers (let us recall that each question was assumed to have at least two direct answers). But, looking from the pragmatic point of view, a question which has exactly two direct answers requires a selection of one of them and does not leave room for a partial selection. Moreover, such a question can have answers of other kinds: incomplete, corrective, etc. We will not define here, however, these concepts of answers. Let us finally add that if questions with exactly one direct answer were allowed, no such question would have a partial answer in our sense.

5.3. Presuppositions of questions

The concept of a *presupposition* of a question is defined in various logical theories of questions in different ways. We will define here this concept following the general idea proposed by Belnap (cf. Belnap 1969b; see also Belnap and Steel 1976, p. 119).

DEFINITION 8. A d-wff A of $\mathcal{L}$ is a *presupposition* of a question Q of $\mathcal{L}$ iff A is entailed in $\mathcal{L}$ by each direct answer to Q.

The main advantage of the above definition is that is expresses a clear logical intuition: a presupposition of a question is a d-wff whose truth is necessary for the soundness (having a true direct answer) of the question.

The set of presuppositions of a question Q will be referred to as Pres$Q$.

It can happen that the truth of some presupposition is not only necessary, but also sufficient condition of soundness of the question. Let us then introduce the concept of *prospective presupposition* of a question:

DEFINITION 9. A presupposition A of a question Q of $\mathcal{L}$ is a *prospective presupposition* of Q iff A mc-entails in $\mathcal{L}$ the set of direct answers to Q.

To speak generally, a prospective presupposition is thus a presupposition which, if true, guarantees the existence of a true direct answer to a question.[7] The set of prospective presuppositions of a question Q will be denoted by PPres$Q$.

The set of presuppositions of a question is always nonempty (at least the tautologies of the language belong to it), but it need not be the case with the set of prospective presuppositions. There are languages of the considered kind which contain questions that have no prospective presuppositions; an example will be given below. But each question which has only finitely many direct answers has a non-empty set of prospective presuppositions: by Corollary 6 a disjunction of all the direct answers is a prospective presupposition of it. Let us also add that if a question has prospective presuppositions, all of them are equivalent.

Prospective presuppositions should be distinguished from *maximal presuppositions*.

DEFINITION 10. A presupposition A of a question Q of $\mathcal{L}$ is a *maximal presupposition* of Q iff A entails in $\mathcal{L}$ each presupposition of Q.

A maximal presupposition is thus a presupposition which entails any presupposition. The set of maximal presuppositions of a question Q will be referred to as mPres$Q$.

There is no general reason why each question of any language of the considered kind should have a maximal presupposition. But each question whose set of direct answers is finite does have maximal presuppositions: any disjunction of all the direct answers to it perform this function. Let us also observe that if a question has prospective presuppositions, all of them are maximal. This is due to:

COROLLARY 10. PPres$Q \subseteq$ mPres$Q$.

P r o o f: Assume that $A \in$ PPres$Q$ and that $A \notin$ mPres$Q$. So there is a presupposition B of Q such that A *non* $\models B$; it follows that there is a normal interpretation $\Im$ of the considered language such that $\Im \models A$ and $\Im$ *non* $\models B$. But B is a presupposition of Q; so for each $C \in$ dQ we have $\Im$ *non* $\models C$. Thus A *non* $\Vdash$ dQ and hence $A \notin$ PPres$Q$. We arrive at a contradiction. □

The converse of Corollary 10 need not be true: there are languages of the considered kind which contain questions that have maximal presuppositions which are not prospective presuppositions. Some example may be helpful here. Let us assume that $\mathcal{L}^{\#}$ is a language of the considered kind whose terms and d-wffs are those of some first-order language with identity and infinitely many individual constants; assume also that among questions of $\mathcal{L}^{\#}$ there is a question whose set of direct answers consists of all the sentences of the form $P(t)$, where P is a (concrete) one-place predicate symbol of $\mathcal{L}^{\#}$ and t is an arbitrary closed term of $\mathcal{L}^{\#}$. Let us designate this question by Q^{*}. Assume that the class of normal interpretations of $\mathcal{L}^{\#}$ consists of all the interpretations of the language. Clearly the sentence:

(1) $\exists x\, P(x)$

is a presupposition of Q^{*}, but is not a prospective presupposition of it: there are (normal) interpretations of the language in which the sentence (1) is true, but no sentence of the form $P(t)$ (i.e. no direct answer to Q^{*}) is true. At the same time (1) is a maximal presupposition of Q^{*}. For, let us assume that there is a presupposition, say, A, of Q^{*} which is not entailed in $\mathcal{L}^{\#}$ by the sentence (1). So the set:

(2) $\{\exists x\, P(x),\ \neg\bar{A}\}$

where $\bar{A}$ is the universal closure of A, has a model. But it can be proved that if X is a set of d-wffs of $\mathcal{L}^{\#}$ such that there are infinitely many individual constants of $\mathcal{L}^{\#}$ which do not occur in the d-wffs of X, then X has a model if and only if there is an interpretation $\Im = \langle \mathbf{M}, \mathbf{f} \rangle$ of $\mathcal{L}^{\#}$ which is a model of X and fulfills the following condition:

(i) for each $y \in \mathbf{M}$ there exists a closed term t of $\mathcal{L}^{\#}$ such that for each $\Im$-valuation $\mathbf{s}$, y is the value of t in $\Im$ with respect to $\mathbf{s}$.

(To speak generally, the condition (i) amounts to saying that each element of the domain of $\Im$ has a name in $\mathcal{L}^{\#}$). The proof goes along the lines of the Henkin-style proof of Gödel's theorem of the existence of a model: the only difference is that we use the individual constants which do not occur in the d-wffs of X as the "witnesses". Clearly there are infinitely many individual constants of $\mathcal{L}^{\#}$ that do not occur in the d-wffs of the set (2); so there is an interpretation, say, $\Im'$, of $\mathcal{L}^{\#}$ which fulfills the condition (i) and which is a model of the set (2). It follows that for some fixed sentence of the form $P(t)$, say, $P(t^{*})$, $\Im'$ is also a model of the set:

(3) $\{\exists x\, P(x),\ \neg\bar{A},\ P(t^{*})\}$

But this is impossible, since $P(t^*)$ is a direct answer to Q^* and thus entails the sentence $\bar{A}$ (let us recall that A was assumed to be a presupposition of the question Q^*). We arrive at a contradiction: so the sentence (1) is a maximal presupposition of Q^*. On the other hand, (1) is not a prospective presupposition of the analyzed question.

The example analyzed above is instructive for some other reason as well: it presents a language which contains questions that have no prospective presuppositions. If the question Q^* of $\mathcal{L}^{\#}$ had a prospective presupposition, this presupposition would be entailed by the maximal presupposition (1). So (1) would be a prospective presupposition of Q^*; since it is not, the question Q^* has no prospective presuppositions. Let us stress, however, that we do not claim here that Q^* and similar questions cannot have prospective presuppositions in any language. The concepts of presuppositions introduced above are defined by means of the concepts of entailment and mc-entailment *in a language* and there are languages of the considered kind in which these questions do have prospective presuppositions.

Let us finally introduce the concepts of factual presupposition and maximal factual presupposition, which may be especially useful in the philosophy of science.

DEFINITION 11. A presupposition A of a question Q of $\mathcal{L}$ is a *factual presupposition* of Q iff A is a synthetic d-wff of $\mathcal{L}$.

DEFINITION 12. A factual presupposition A of a question Q of $\mathcal{L}$ is a *maximal factual presupposition* of Q iff A entails in $\mathcal{L}$ each factual presupposition of Q.

It is not the case that each question has factual presuppositions. Moreover, there is no general reason why each question which does have factual presuppositions should have maximal factual presuppositions.

5.4. Relative soundness. Normal questions and regular questions

Let us now introduce the concept of *relative soundness* of a question, which seems to be of basic importance to erotetic logic. The underlying intuition is that a question Q is sound relative to a set of d-wffs X just in case the question Q *must* have a true direct answer *if* all the d-wffs in X are true. This intuition can be expressed in terms of mc-entailment via

DEFINITION 13. A question Q of $\mathcal{L}$ is *sound relative to* a set of d-wffs X of $\mathcal{L}$ iff the set X mc-entails in $\mathcal{L}$ the set of direct answers to Q.

In other words, Q is sound relative to X just in case there is no normal interpretation of the language in which all the d-wffs in X are true, but no direct answer to Q is true.

If Q is sound relative to a singleton set $\{A\}$, we say that Q is sound relative to the d-wff A.

Let us stress that the concept of relative soundness introduced above must be carefully distinguished from the concept of soundness of a question in an interpretation of the language introduced in Section 5.1: these are different concepts.

A safe question is sound relative to any set of d-wffs. This is the trivial case; in order to distinguish the non-trivial cases let us introduce the following concept:

DEFINITION 14. A question Q of $\mathcal{L}$ is *made sound* by a set of d-wffs X of $\mathcal{L}$ iff X mc-entails in $\mathcal{L}$ the set of direct answers to Q although the set of direct answers to Q is not mc-entailed in $\mathcal{L}$ by the empty set.

Thus Q is made sound by X just in case Q is not sound relative to the empty set, but Q is sound relative to X. It is clear that only risky questions can be made sound by sets of d-wffs.

Each question has a non-empty set of presuppositions; the truth of these presuppositions is a necessary condition for having a true direct answer. Yet, when we look in the opposite direction, the truth of all the presuppositions need not be a sufficient condition of having a true direct answer: there are languages of the considered kind in which some questions do not have this property. The language $\mathcal{L}^{\#}$ mentioned in Section 3.3 is a case in point here: since the sentence (1) is a maximal presupposition of the question Q^* but not a prospective presupposition of it, it happens that all the presuppositions of Q^* are true in some normal interpretation of the language, but no direct answer to Q^* is true in it. In order to distinguish questions whose presuppositions guarantee the existence of a true direct answer from the remaining ones let us introduce the concept of *normal question*:[8]

DEFINITION 15. A question Q of $\mathcal{L}$ is said to be *normal* iff the set of direct answers to Q is mc-entailed in $\mathcal{L}$ by the set of presuppositions of Q.

A normal question is thus a question which is sound relative to the set of its presuppositions. Note that since being a normal question depends on mc-entailment in a language (and thus basically on the conditions imposed on the class of normal interpretations), the same question occurring in one language may be normal in it without being normal in some other language. Again, the question Q^* gives us a simple example here: as far as the language $\mathcal{L}^{\#}$ is concerned, Q^* is not normal in it. It can be shown, however, that the question Q^* would be normal if it occured in a language in which the class of normal interpretations would be defined in such a way that the condition (i) of Section 5.3 would be satisfied by each normal interpretation of it. But any question which has only finitely many direct answers is normal in any language of the considered kind; moreover, each safe question is normal. The remaining questions, however, may or may not be normal: it depends on the semantics of the language.

Let us now introduce a more specific concept of *regular question*. To speak generally, a question Q will be called regular if there is a single presupposition of Q whose truth implies the existence of a true direct answer to Q. To be more precise, we adopt

DEFINITION 16. A question Q of $\mathcal{L}$ is said to be *regular* iff there is a presupposition A of Q such that A mc-entails in $\mathcal{L}$ the set of direct answers to Q.

In other words, a question is regular if it is sound relative to some of its presuppositions. Note that each question whose set of direct answers is finite is regular. We also have:

COROLLARY 11. *A question Q is regular iff Q has prospective presuppositions.*

COROLLARY 12. *Each regular question is normal.*

COROLLARY 13. *If entailment in $\mathcal{L}$ is compact, then each normal question of $\mathcal{L}$ is regular.*

Corollary 11 shows that the concept of regular question can be defined in terms of prospective presuppositions. Corollaries 12 and 13 imply that "being a normal question" and "being a regular question" coincide if entailment in a language and thus also mc-entailment in it are compact. Yet, there are languages of the considered kind in which entailment is not compact.

5.5. Self-rhetoricity and informativeness. Proper questions

Let us now define the concept of a self-rhetorical question.

DEFINITION 17. A question Q of $\mathcal{L}$ is said to be *self-rhetorical* iff there is a direct answer A to Q such that A is entailed in $\mathcal{L}$ by the set of presuppositions of Q.

By proposing the above definition we are not going to explicate the general notion of rhetoricity of a question: this notion is clearly a pragmatic one (in the traditional sense of the word). Our aims are limited: we attempt to explicate the concept of "rhetoricity for logical reasons." The following are examples of self-rhetorical questions: questions having tautologies as direct answers, questions all of whose direct answers are contradictory sentences, questions which have only one direct answer being a synthetic sentence. Note that if questions with exactly one direct answer were allowed, these questions would be self-rhetorical questions.

Each self-rhetorical question is normal in the sense of Definition 15. In order to distinguish normal questions which are not self-rhetorical from the remaining ones we introduce:

DEFINITION 18. A question Q is *proper* iff Q is normal but not self-rhetorical.

Let us finally introduce a certain concept of *informativeness* of a question.

Looking from the intuitive point of view, a question is informative relative to a given set of d-wffs just in case each direct answer to the question conveys some information which cannot be legitimately drawn from the analyzed set of d-wffs. In order to explicate this notion let us first introduce the concept of *content* of a set of d-wffs. Let X be a set of d-wffs of $\mathcal{L}$. The *content* of the set X (in symbols: $\mathbf{Ct}(X)$) is defined as follows:

DEFINITION 19.

$$\mathbf{Ct}(X) = \{A\colon A \text{ is entailed in } \mathcal{L} \text{ by } X \text{ and } A \text{ is not a tautology of } \mathcal{L}\}$$

In other words, the content of X consists of all the d-wffs which are non-trivially entailed by X (i.e. are not tautologies).[9]

The relativized concept of informativeness of a question can now be defined as follows:

DEFINITION 20. A question Q is *informative relative to* a set of d-wffs X iff for each $A \in \mathrm{d}Q$, $\mathbf{Ct}(X)$ is a proper subset of $\mathbf{Ct}(X \cup \{A\})$.

Thus a question Q is informative relative to a set of d-wffs X just in case the content of X is a *proper* subset of the content of any set which results from X by adding a direct answer to Q: any such set entails more non-trivial consequences than the initial set X. It is easily seen, however, that Q is informative relative to X if and only if no direct answer to Q is entailed by X.

By proposing the above definition we identify the possibility of extracting information from a set of d-wffs with the entailment of the sentence conveying this information. No doubt, there is plenty of idealization in such an approach; consequently, Definition 20 presents an idealized concept of informativeness of a question. The same holds true in the case of our definition of self-rhetoricity. We can easily prove:

COROLLARY 14. *A question Q is not self-rhetorical iff Q is informative relative to the set of presuppositions of Q.*

Let us finally note:

COROLLARY 15. *A question Q is proper iff Q is normal and Q is informative relative to the set of presuppositions of Q.*

Thus a proper question is a question which is both normal and informative relative to the set of its presuppositions.

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NOTES

* This paper was completed in February, 1995. Some material from it was then used in the book *The Posing of Questions: Logical Foundations of Erotetic Inferences*, which was completed later.

1 Cf. Belnap and Steel (1976), p. 10 et al.

2 Let us observe that according to this definition a d-wff of the form $P(x_i)$ logically entails its universal generalization, i.e. $\forall x_i\, P(x_i)$. This is intuitively acceptable only if the free variable x_i in $P(x_i)$ is not construed as a name of some unspecified object, but is interpreted in the "generalizing" manner, as referring to any element of the domain. This interpretation of free variables underlies the above definitions of logical entailment

and entailment in a language. Let us add that the system of classical predicate calculus with identity which is complete with respect to logical entailment defined in the above manner can be built, int.al., as follows: (1) *Axioms*: substitution-instances of valid formulas of classical propositional calculus as well as the standard axioms for identity; (2) *Rules*: modus ponens, substitution of terms, elimination of ∀, introduction of ∀, elimination of ∃, introduction of ∃.

3 Cf. e.g. Wójcicki (1982).

4 If A is a sentence, then the universal closure of A is equal to A itself. If A is a sentential function and $x_{i1}, \ldots, x_{in}$ (where $x_{i1} < \ldots < x_{in}$) are the all free variables of A, then the universal closure of A is of the form $\forall x_{i1} \ldots \forall x_{in} A$. The universal closure of a d-wff A will be designated by $\bar{A}$.

5 Cf. note 4.

6 The paper Wiśniewski (1987) is an abstract of the paper presented at the VIII International Congress of Logic, Methodology and Philosophy of Science, Moscow 1987. For some mysterious reasons the organizers retyped the manuscript before printing without making any proof reading. As a result this paper presumably wins the world record for misprints.

7 In Belnap's terminology a prospective presupposition is a d-wff which *expresses the presupposition* of the question. Belnap, however, defines this concept without applying the concept of mc-entailment.

8 Buszkowski, who in fact introduced this concept (cf. Buszkowski 1989), uses here the term *correct question*.

9 The definition proposed above is similar to that given by Popper (cf. Popper 1959, p. 120), who, however, uses the concept of derivability.

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