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Effectiveness of Question-Answer Systems*

1. Questions and answers. The question "What is a question?" still expresses an open issue. Although the beginnings of the modern logic of questions date back to late fifties and early sixties of the 20th century, no commonly accepted, paradigmatic theory has been elaborated so far.

1.1. ***Levels of analysis.*** The lack of consensus is not surprising, however, since the term "question" can be understood:

- syntactically: as referring to a sentence of a particular kind, that is, to an *interrogative sentence*,
- semantically: as referring to the *semantic content* of an interrogative sentence (and possibly some other expressions),
- pragmatically: as referring to a *speech act* that is typically performed in uttering questions syntactically and/or semantically construed.

These concepts differ not only in content, but also in scope. There are questions (in the semantic or pragmatic sense of the word) which are not expressed by interrogative sentences. Similarly, interrogative sentences do not always express questions semantically or pragmatically conceived.

Some theories of questions ignore the pragmatic level, while others overestimate it. In most theories, however, all the levels are taken into consideration, although with emphasis put on one or other of them. But even if the primary level of analysis is fixed, alternative accounts are still advocated by different theorists.[1]

[1]The survey paper Harrah (2002) provides a comprehensive exposition of logical theories of questions elaborated up until the late 1990s. Supplementary information about more linguistically oriented approaches can be found, e.g., in Groenendijk & Stokhof (1997), Lahiri

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1.2. ***Answers.*** Most theorists pay at least as much attention to answers to questions as to questions themselves. The analysed answers are usually *possible answers*. The relevant "erotetic" concept of possibility has never been made precise. Yet, it is clear that "possible" does not yield "true": some possible answers are true and some are not. Thus, in most theories, the phrase "an answer to a question" does not amount to "the true answer to a question". Moreover, "possible" presupposes neither "being known" nor "being believed in", although, of course, some possible answers happen to be carriers of items of knowledge or belief.[2] Finally, "possible" implicates "not the only one". It is usually assumed that a question may have many possible answers.

In most theories, some epistemically basic category of possible answers is distinguished. These answers are called, depending on the theory, *direct*, or *conclusive*, or *proper*, or *sufficient*, or *exhaustive*, or *complete*, or *congruent*, etc. Those *principal possible answers* (let us use this general term here), ppa's for short, are supposed to satisfy some general conditions, usually expressed, inter alia, in pragmatic terms. For example:

- Harrah (1963): direct answers are replies which are complete and just-sufficient answers;
- Harrah (2002, p. 1): a direct answer "gives exactly what the question calls for. (...) The label 'direct' (...) connotes both logical sufficiency and immediacy";
- Belnap (1969, p. 124): direct answers are answers which "are directly and precisely responsive to the question, giving neither more nor less information than what is called for";
- Kubiński (1980, p. 12): direct answers are "these sentences which everybody who understands the question ought to be able to recognize as the simplest, most natural, admissible answers to the question";
- Hintikka (1978): a reply is called a conclusive answer if it completely satisfies the epistemic request of the questioner;
- Ginzburg (1995, p. 461): ppa's form the "class of responses that a querier would consider optimal."

Depending on the theory, ppa's are either syntactic or semantic entities. Moreover, it is not commonly accepted that all ppa's are declarative sentences or are expressed by such sentences. Sometimes "short" answers being

(2002), and Krifka (2011). Ginzburg (2011) provides a survey of recent developments in the research on questions, both in logic and in linguistics. A general overview of approaches to questions and their semantics can also be found in Wiśniewski (201x).

[2] One has to distinguish between knowing that p is a possible answer to a given question and knowing or believing that p is true. These are independent issues.

subsentential expressions (e.g. names, noun phrases, or other phrases of a special form) or semantic counterparts of such expressions are taken into account as ppa's; more often, however, short answers are regarded as coded ppa's.

Besides ppa's, other categories of (possible) answers are characterized as well. Sometimes they are defined in terms of, inter alia, ppa's. Sometimes, however, a more general concept of answer is introduced first, and ppa's are just very special cases. The "non-ppa" answers are labelled with adjectives like "partial", "incomplete", "indirect", "corrective", "eliminative", etc. The meanings of these terms vary from theory to theory: so far no commonly accepted, unified account has been elaborated.

2. Question-answer systems. Whatever questions and answers are conceptualized in detail, theories of questions (logical or even linguistic) provide *formalisms* for representing questions and answers. Such a formalism includes *e-formulas*[3], which are either translations of natural-language interrogative sentences or correspond to/express questions semantically construed. Let us stress that e-formulas need not be specific; one can prefer a reductionistic approach to questions and use as e-formulas some imperative-epistemic expressions, or simply certain epistemic formulas, or declarative formulas, etc. When a non-reductionistic approach to questions is adopted, e-formulas differ syntactically from other formulas.

Similarly, ppa's are represented by *nominal ppa's*, being expressions of the formalism.

The outcome of an analysis, in turn, can be viewed as a *question-answer system*.

DEFINITION 1. *A* question-answer system *is an ordered triple* $\langle \Upsilon, \Theta, \mathcal{R} \rangle$, *where:*

1. Υ *is the set of well-formed expressions of a language,*
2. Θ *is a non-empty set of e-formulas of the language,*
3. Θ *is a proper subset of* Υ,
4. Υ *includes a non-empty set of declaratives of the language,*
5. $\mathcal{R} \subseteq \Theta \times \Upsilon$, *where* $\mathcal{R} \neq \emptyset$, *is the answerhood relation, i.e. the set of ordered pairs* $\langle Q, \psi \rangle$ *such that* ψ *is a nominal principal possible answer to* Q.

Let us stress that Υ and Θ are construed here as sets of linguistic entities, that is, expressions of a language. Moreover, the answerhood relation $\mathcal{R}$ is

[3] The letter "e" alludes to "erotetic".

understood (only) *set-theoretically*, as a set of ordered pairs. But recall that a question-answer system is supposed to display the outcome of an analysis, not the ways in which the analysis is performed.

The set $\mathsf{d}Q$ defined by:

$$\mathsf{d}Q =_{df} \{\psi \in \Upsilon : \langle Q, \psi \rangle \in \mathcal{R}\}$$

is the set of all the nominal principal possible answers to Q. Note that for a given e-formula Q, the set $\mathsf{d}Q$ is always unique.

It is not prejudged that sets of nominal ppa's comprise only declarative sentences/formulas. It is permitted (but not assumed!) that $\mathsf{d}Q$ is empty for some e-formula(s) of the system.

3. Effective question-answer systems. Question-answer systems can be evaluated in many respects. Undoubtedly, the following conditions characterize highly desirable properties of a question-answer system:

1. if an expression is an e-formula, this can be effectively established/computed,
2. if an expression is a nominal principal possible answer to an e-formula, this can be effectively established/computed,
3. the set of declaratives is decidable.

In terms of recursion theory: the set of e-formulas should be recursively enumerable, the answerhood relation should be recursively enumerable, and the set of declaratives of the system should be recursive.

In formulating the above requirements we, in principle, follow Harrah (1969).

The above insights can be expressed in our conceptual setting by introducing the concept of *effective question-answer system*.

DEFINITION 2. *A question-answer system* $\langle \Upsilon, \Theta, \mathcal{R} \rangle$ *is* effective *iff*

1. *the set of e-formulas* Θ *of the system is recursively enumerable,*
2. *the answerhood relation* $\mathcal{R}$ *of the system is recursively enumerable, and*
3. *the set of declaratives of the system is recursive.*

3.1. ***Remark***. We have used above, and will be using below, concepts taken from (classical) recursion theory. This presupposes some coding of the expressions considered, that is, expressions of the language of a question-answer system (sentences, questions, and other expressions of the language) by natural numbers. But for our purposes it suffices to assume that these expressions can be coded by natural numbers, i.e. there exists a coding method according to which each expression is coded by an unique natural number.

Given the coding, we may say that a set of expressions is recursive iff the corresponding set of numerical codes of the expressions of the set is recursive, and similarly for recursive enumerability.

As usual, we abbreviate "recursively enumerable" as "r.e.".

4. Harrah's Theorem. The effectiveness conditions (1) – (3) and their counterparts included in Definition 2 reflect some fundamental insights. However, in some cases their joint satisfaction can put us into some trouble. David Harrah writes:

> Let us assume that the language L has a finite alphabet, and that the expressions of L are finite strings of letters of the alphabet, so that the expressions of L can be alphabetically ordered. Let us suppose that the set of questions of L is recursively enumerable. (...) Suppose next that each question either has denumerably many direct answers or can be assigned denumerably many in a harmless way (...). Suppose further that, given a question q, the direct answers to q are recursively enumerable (...). Finally, for simplicity, we may suppose that the direct answers are sentences, or are expressed by sentences, and that *sentence of* L is recursive. Assuming all this, we can use Cantor's diagonal method to construct a set of sentences which is not the set of direct answers to any question in the initial enumeration of questions. In fact, we can construct indefinitely many such new sets (...). (Harrah 1969, p. 160)

Harrah also describes a pattern of constructing the "new" sets and observes that its slight modification produces sets whose elements share some recursive properties. Yet, he does not claim that the "new" sets *are* recursive.

In what follows we are going to show one of the ways in which Harrah's Theorem can be generalized and strengthened.

5. Double frames. Let us now introduce the concept of double frame.[4]

DEFINITION 3. *A* double frame *is an ordered triple* $\langle \Phi, \Gamma, R\rangle$ *such that* Φ *and* Γ *are non-empty sets, and* $R \subseteq \Gamma \times \Phi$ *is a relation whose domain is* Γ.

Looking from a purely formal point of view, a question-answer system understood in the sense of Definition 1 is a double frame. Similarly, the ordered triple:

$$\langle \mathcal{D}, \Theta, \mathcal{R}|\mathcal{D}\rangle$$

where $\mathcal{D}$ is the set of declaratives of a question-answer system $\langle \Upsilon, \Theta, \mathcal{R}\rangle$ constitutes a double frame. Thus we can apply general results pertaining

[4]Cf. Wiśniewski & Pogonowski (2010).

to double frames in the analysis of question-answer systems. However, we need some some auxiliary concepts first.

Let $R^{\rightarrow}x = \{z \in \Phi : x\mathcal{R}z\}$. Elements of $R^{\rightarrow}x$ are called *R-images* of x.

DEFINITION 4. *A double frame $\langle \Phi, \Gamma, R \rangle$ is:*

1. numerical *if Φ and Γ are sets of natural numbers,*
2. effective *if it is numerical and Φ is recursive, Γ is r.e., and R is an r.e. relation,*
3. deeply infinite *if Φ and Γ are countably infinite sets, and each set $R^{\rightarrow}x$ is infinite, for all $x \in \Gamma$.*

We will make use of the following theorem:[5]

The Recursive Jump Theorem (Wiśniewski & Pogonowski 2010). *For any deeply infinite effective double frame $\langle \Phi, \Gamma, R \rangle$ there exists an infinite family Ξ of infinite recursive subsets of Φ such that each element of Ξ is different from any $R^{\rightarrow}x$, for all $x \in \Gamma$.*

6. ω-questions. We need one more auxiliary concept.

DEFINITION 5. *By an* ω-question *we mean an e-formula which fulfils the following conditions: (1) each nominal ppa to it is a declarative, and (2) the set of nominal ppa's to the e-formula is denumerable.*

By "denumerable" we mean, here and below, "countably infinite". The letter ω is used in order to indicate that the set of ppa's is equinumerous with the set of natural numbers. Let us stress that the ppa's to ω-questions are not numerals, but declaratives.

7. Harrah's Theorem strengthened. Let us first prove the following:

THEOREM 1. *Let $\langle \Upsilon, \Theta, \mathcal{R} \rangle$ be an effective question-answer system such that the set of declaratives of the system is denumerable, and Θ consists of ω-questions. There exists a denumerable family of infinite recursive sets of declaratives of the system such that no element of the family is the set of nominal ppa's to an e-formula of the system.*

Proof. Suppose that Θ is denumerable. Let $\mathcal{D}$ be the set of declaratives of the system. By assumption, $\mathcal{D}$ is denumerable. The system is effective and this presupposes some coding of well-formed expressions of the language. Let f be the coding function. Consider the following numerical double frame:

$$\langle f[\mathcal{D}], f[\Theta], \mathcal{R}^* \rangle \tag{1}$$

[5] Already used in the previous essay, see page 51.

where $\mathcal{R}^*(f(Q), f(\phi))$ holds iff $\mathcal{R}(Q, \phi)$ holds. Clearly, (1) is a deeply infinite effective double frame. By the Recursive Jump Theorem (and given the assumed coding) there exists a denumerable family of infinite recursive sets of declaratives of the system each of which is not the set of nominal ppa's to an e-formula of the system.

Now suppose that Θ is finite. Since $\mathcal{D}$ is denumerable and the system is effective, there are denumerably many infinite recursive subsets of $\mathcal{D}$ (for there exist denumerably many infinite recursive subsets of an infinite recursive set). On the other hand, the assignment of sets of ppa's to e-formulas in a question-answer system is unique, and Θ is only finite. □

Theorem 1 shares the following assumption with Harrah's Theorem: the set of nominal ppa's to each question/e-formula comprises declaratives only and is denumerable. But this assumption is dispensable, as the next theorem illustrates.

THEOREM 2. *Let $\langle \Upsilon, \Theta, \mathcal{R} \rangle$ be an effective question-answer system such that:*

1. *the set of declaratives of the system is denumerable, and*
2. *the set of ω-questions of the system is r.e.*

There exists a denumerable family of infinite recursive sets of declaratives of the system such that no element of the family is the set of nominal ppa's to an e-formula of the system.

Proof. Let

$$\langle \Upsilon, \Theta, \mathcal{R} \rangle \tag{2}$$

be a question-answer system that fulfils the assumptions of the theorem and let Ω be the set of all ω-questions of the system. By assumption, Ω is r.e.

Suppose that Ω is denumerable. We define:

$$\hat{\mathcal{R}}(Q, \psi) \leftrightarrow \mathcal{R}(Q, \psi) \wedge Q \in \Omega.$$

Clearly, $\hat{\mathcal{R}}$ is r.e. Now consider the following question-answer system:

$$\langle \Upsilon, \Omega, \hat{\mathcal{R}} \rangle \tag{3}$$

Since $\hat{\mathcal{R}}$ is r.e, Ω is r.e., and, by assumption, the set of declaratives of the system (2) (and thus also of (3)) is recursive, the system (3) is effective. Moreover, Ω is denumerable and the set of declaratives of the system (3) is both denumerable and recursive. Thus, by Theorem 1, there exists a denumerable family of infinite recursive sets of declaratives of the system (3) such that no element of the family is the set of nominal ppa's to an e-formula of the system. But systems (2) and (3) do not differ with respect to their sets of declaratives or with regard to ω-questions, and, since Ω is the set of all ω-questions of the system (2), no e-formula of the system belonging to the set $\Theta \setminus \Omega$ is an ω-question. Hence, there exists a denumerable family of infinite recursive sets of declaratives of the system (2) such that no element of the family is the set of nominal ppa's to an e-formula of (2).

Now suppose that Ω is finite. Since, by assumption, the set of declaratives of the system (2) is denumerable, there are infinitely many denumerable recursive subsets of the set. On the other hand, only finitely many denumerable r.e. sets of sentences (or none, if Ω is empty) perform the role of sets of nominal ppa's of questions of Ω. □

From a formal point of view, the phenomenon pointed out by Theorem 2 is analogous to that which arises when any set of possible worlds is regarded as a proposition (see the previous essay): on the one hand there exist "inexpressible", yet recursive propositions, and, on the other, there exist recursive sets of declaratives which do not constitute sets of (nominal) ppa's to any e-formula/question of an *effective* question-answer system. This is not surprising, since both results are due to the Recursive Jump Theorem.

Theorem 2 provides an argument against the *identification* of questions with sets of declaratives, an attitude which is quite common among logicians who only occasionally enter the field of the logic of questions. But there is more in it. The following postulate set by Hamblin (1958):

H: *Knowing what counts as an answer is equivalent to knowing the question.*

for decades dominated ways of thinking about questions. Theorem 2 shows that there are cases in which sets of declaratives are "known" (that is, sets of their numerical codes are recursive), but there is no question corresponding to (i.e. having as nominal ppa's the elements of) the "known" sets.

8. Some further consequences. Observe that as an immediate consequence of Theorem 2 we get:

THEOREM 3. *If $\langle \Upsilon, \Theta, \mathcal{R} \rangle$ is a question-answer system such that:*

1. *the set of declaratives of the system is denumerable and recursive,*
2. *the set of e-formulas of the system is r.e., and*
3. *each infinite recursive set of declaratives of the system is the set of nominal ppa's to an e-formula of the system*

then the set of ω-questions of the system is not r.e. or the answerhood relation $\mathcal{R}$ of the system is not r.e. and hence the system is not effective.

Again, the situation is similar to that encountered in the case of propositions.

Yet, are there any reasons which justify us in saying that the assumption (3) of the above theorem pertains to natural languages? The answer is a conditional "Yes" – see Łupkowski & Wiśniewski (2011), p. 446. But, leaving this controversy aside, we can safely state that theorems 1–3 reveal limits of effectiveness of question-answer systems.

9. Final remark. As for effective question-answer systems, the "global" answerhood relation $\mathcal{R}$ is supposed to be r.e. Interestingly enough, one can get a result stronger than Theorem 2 but based on weaker assumptions. Call an ω-question Q *effective* if the set $\mathsf{d}Q$ (of nominal ppa's to Q) is an r.e. set. The following can be proven (we rephrase the theorem in the conceptual setting of this essay):

THEOREM 4. *Let $\mathcal{L}$ be a language such that:* (a) *among expressions of the language there are declaratives and e-formulas,* (b) *both declaratives and e-formulas of $\mathcal{L}$ can be coded by natural numbers,* (c) *the set of declaratives of $\mathcal{L}$ is denumerable and recursive, and* (d) *the set of all effective ω-questions of $\mathcal{L}$ is r.e. There exists an infinite family of infinite recursive sets of declaratives of $\mathcal{L}$ which are not sets of nominal ppa's to any e-formula of $\mathcal{L}$.*

For the proof and a discussion see Wiśniewski & Pogonowski (2010a). For applications see, e.g., Łupkowski & Wiśniewski (2011).

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