Semantics of Questions

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1 Introduction

The first attempts to apply the tools of modern formal logic to analyse questions date back to the 1920s. However, substantial progress took place from the 1960s on. The research of logicians resulted in formal systems for the representation of questions, as well as in many important conceptual distinctions. Although theorizing on questions had been initiated by logicians, linguists entered the game soon after in parallel with consecutive developments in formal (and then computational) linguistics. The borders between logic and formal linguistics are not sharp. However, the aims and methods are diverse. As a result, the flow of ideas is often limited: each community seems unaware (with notable exceptions, of course) that a lot of relevant work has been done in the neighbouring discipline.

This chapter is devoted to the semantics of questions. The perspective adopted here is that of a logician. A linguist might find the exposition incomplete. Yet, there is a hope that might be provided useful information or even inspiring insights.

2 Setting the Field

When speaking about questions, one can have in mind: (i) interrogative sentences (*interrogatives* for short), (ii) the meanings/semantic contents of interrogatives, or (iii) speech acts typically performed in uttering interrogatives. There are theories of questions which concentrate on (i) and/or (ii), and ignore (iii), while other theories overestimate (iii). In the majority of cases, however, (i), (ii) and (iii) are considered, although with emphasis put on one or other of them.

Theories of questions aim at modeling natural-language questions (hereafter: NLQ's) by providing their syntactic and/or semantic representations. Some theories provide syntax and semantics for interrogatives of artificial (including formal) languages as well.

No commonly accepted theory of questions has been elaborated so far. In section 3 we present the most influential proposals.

2.1 Questions vs. propositions

Interrogatives differ from declaratives syntactically and pragmatically. But is there any substantial difference at the semantic level? In particular, is it the case that interrogatives, contrary to appearance, in fact denote propositions?

Consider:

(1) Who likes Mary?
(2) Does John like Mary?

and

(1*) Peter knows/discovered/told us who likes Mary.
(2*) Peter knows/discovered/told us whether John likes Mary.

Since "knows", "discovers", "tells" and other so-called factive predicates select for propositions, one can argue that embedded interrogatives have a propositional denotation. Assuming that the denotation of an embedded interrogative equals the denotation of the corresponding interrogative, direct interrogatives would have denoted propositions as well. However, the matter is more complicated. The above conclusion would be generally binding if, first, there were no predicates which embed interrogatives but not declarative complements, and, second, each predicate that embeds declarative complements would also embed interrogative complements. Neither of these holds, however. Karttunen (1977) points out that there are predicates which embed interrogative complements, but not declarative complements, for example:

(1**) Peter asked/wondered/investigated/discussed who likes Mary.
($1^{\#}$) # Peter asked/wondered/investigated/discussed that John likes Mary.

Thus although, on some uses, interrogatives seem to denote propositions, it cannot be said that interrogatives always denote propositions. Ginzburg & Sag (2000) convincingly argue that the so-called true/false predicates ("believe,' "assert,', "deny," "prove," etc.) select for propositions, but are incompatible with interrogative complements. In their opinion, the best explanation of the fact that the true/false predicates do not select for interrogatives lies in assuming that interrogatives never denote propositions within an enriched ontology for concrete and abstract entities, inspired by Vendler (1972).

2.2 Answers and answerhood

Questions (semantically or pragmatically construed) come as a pair with *possible answers*. One should not identify answers with *replies*. In principle, every expression can serve as a reply to any question, viz.:

(1) Who likes Mary?
(3) It's rather cold outside.

After Grice, there is nothing surprising in that. But the Gricean-style reasoning of a questioner who has just received a nonrelevant reply is triggered by noticing that the reply is *transparently* non-relevant. This, in turn, requires a certain account of answerhood, maybe only a fuzzy one, but still definite enough to enable noticing that an interlocutor deliberatively does not obey the cooperation principle. So *possible answers* are not just possible replies. Furthermore, "being a possible answer" is semantically laden. The semantics of questions attempts to give account of answerhood in general as well as of different types of possible answers.

Principal possible answers (PPA's). Some possible answers seem "better" than others. For example, assume that the information request carried by (1) is something like:

(4) Please indicate all the persons that like Mary.

where "indicate" is a cover term for different ways of referring to a person or persons. Given the assumption, each of the following:

(5) John likes Mary
(6) John and Helen like Mary

is a possible answer to the question, though (leaving implicatures apart) an insufficient one. On the other hand, any of:

(7) Only John likes Mary.
(8) John and Helen like Mary, and nobody else does.

is a possible answer that is "optimal" in the sense that it provides information of the required kind and, at the same time, provides neither more nor less information than it is requested by the question. Answers of this kind are usually labelled *direct*. As Harrah (2002, p. 1) puts it, a direct answer:

($\mathbf{OA}_1$) *"gives exactly what the question calls for.* [...] *The label 'direct'* [...] *connotes both logical sufficiency and immediacy."*

Note that being true is not a prerequisite for being a direct answer.

However, (1) can also be understood as expressing a less demanding information request:

(9) Please indicate at least one person that likes Mary.

Assume that (1) is construed in this way. Now (5) and (6) are direct answers to the question under consideration, while (7) and (8) are not direct answers to it, since they provide more information than it is requested.

Some theorists label the "optimal" possible answers as *conclusive*, or *congruent*, or *proper*, or *exhaustive*, or *complete*, and so forth. Each of these names is a telling one, and the choices made reflect the underlying ideas of "optimality." Since the terminology is still diverse, we need a cover term. The expression *principal possible answers* (PPA's for short) seems appropriate here, so we will be using it as the cover term.

Other kinds of possible answers. Clearly, PPA's are not the only possible answers. For example, each of the following:

(10) John likes Mary or Helen likes Mary
(11) Neither John nor Helen likes Mary
(12) Nobody likes Mary

also constitutes a possible answer to the question whose information request is (9), though neither of them is a PPA. (10) is a *partial* answer: it does not resolve the question, but narrows down the space of possibilities. (11) is an *eliminative* answer: it excludes some possibilities. As for (12), opinions are divided. Some theorists would consider it as a PPA. As a consequence, the set of PPA's to the question would cover the space of all possibilities. Other theorists claim that the analyzed question is *loaded*: it presupposes that somebody likes Mary. On this account (12) is a *corrective* answer: it contradicts the presupposition and yields that no PPA to the question is true.

The categories of answers pointed out above do not exhaust the field. In particular, the following can also be regarded as possible answers to the analysed question:

(13) John and possibly Helen like Mary
(14) John and Helen
(15) Her fiancé

(14) and (15) are *short* answers. A short answer is expressed by a nonsentential utterance, that is, a fragmentary utterance which does not have the form of a complete sentence. Nevertheless, a short answer conveys information analogous to that carried by a sentence. The meaning of a short answer is co-determined by its direct semantic contribution and the question currently under discussion. Logicians tend to regard short answers as coded

sentential/propositional answers. The attitude of many linguists is different. In any case, a BNC corpus study (cf. Fernández et al. 2007) reveals that short answers constitute the second largest fraction of all nonsentential utterances in dialogues.

The terminology pertaining to answers other than PPA's, as well as their taxonomy depend on a theory. No commonly accepted, unified account has been elaborated yet. (The concepts of partial, eliminative and corrective answers used above will be explicated in subsection 5.4.)

2.3 Further issues

Let us now briefly mention some other question-related semantic phenomena.

Correctness and types of questions. It is highly dubious whether NLQ's have logical values. However, some NLQ's seem semantically, say, sound, while some others are "semantically faulty." For example, the following:

(16) Which natural number is smaller than 2?

is sound, in contradistinction to, for example:

(17) Which natural number is smaller than 0?
(18) Which natural number is smaller than itself?
(19) Have you done what you haven't done?

Besides soundness/unsoundness, NLQ's exhibit diverse semantic features worth being conceptualized in exact terms. We address this issue in Section 5, using the conceptual framework of Minimal Erotetic Semantics (MiES).

Semantic dependencies. There are many semantic dependency relations between questions. Consider:

(20) Does anybody like Mary?
(1) Who likes Mary?
(2) Does John like Mary?

(1) is dependent upon (20) in the following intuitive sense of the word: (1) *arises from* the affirmative answer to (20), while the negative answer to (20) *suppresses* (1). (2) is dependent upon (1), though in a different sense: the affirmative answer to (2) provides an answer to (1) and the negative answer to (2) eliminates an answer to (1). There are more semantic dependency relations between questions (e.g. "being equivalent", "being weaker than", etc.), as well as between (sets of) declaratives and questions. We address the dependency issue in sections 5 and 6.

Inferences and validity. Questions (semantically construed) are often *arrived at* on the basis of declaratives and/or questions, viz.:

(21) Either John or Helen likes Mary

———————————————

Does John like Mary?

(22) Does anybody like Mary?
Everybody dislikes Mary, possibly with the exception of John.

———————————————

Does John like Mary?

One can argue that arriving at a question resembles coming to a conclusion: there are premises involved and an inferential thought process takes place. Some inferences of this kind seem *intuitively valid*, while others are transparently nonvalid. One of the challenges to the semantics of questions is to give an account of the relevant concept of validity. In section 6 we show how the above issue is resolved by inferential erotetic logic.[1]

[1] The logic of questions is sometimes called *erotetic logic*, from Greek *erotema* which means "question".

3 Theories of Questions

In this section we show how questions are conceptualized in logic and formal linguistics. For the reasons of space we concentrate on the most influential theories and their basic claims.

3.1 Questions as sets of declaratives

As we pointed out above, the reduction of questions to propositions is problematic. An alternative solution is to treat the meaning of an interrogative as a *set* of propositions. When we are wary concerning ontological commitments, this reduces to: the meaning of an interrogative is a set of declaratives. The idea of conceiving questions as sets of expressions was introduced by one of the pioneers of erotetic logic, Gerold Stahl (cf. Stahl 1956, 1962).

But is *any* set of declaratives a question semantically construed? Saying "yes" immediately puts us into a trouble: there are many more questions than interrogatives. Really many more: when the set of declaratives is countably infinite, the set of questions becomes uncountably infinite. We obtain this result by Cantor's diagonal argument. (For this and more results of that kind see Harrah 1969). So maybe only recursive sets of declaratives constitute questions? But this solution gives rise to another difficulty. It can be shown that, given some reasonable assumptions, there exist recursive sets of declaratives (actually, denumerably many of them) of a language which are not assigned to any interrogative of the language (see Wiśniewski & Pogonowski 2010).

One can prefer generality over factivity and still sustain the claim that any set of declaratives is a question. The price to be paid is nonexpressibility of some questions of a language in the language. But usually a different strategy is adopted. Only some, but not all sets of declaratives are regarded as questions semantically construed. Sometimes questions are defined as sets of declaratives which are "available" from the corresponding interrogatives by means of certain syntactic transformations. Purely semantic solutions have also been proposed. For example, one can claim that a set of declaratives constitutes a question if the relevant declaratives are *mutually exclusive* (hereafter: ME); that is, the truth of an element of the set yields the falsity of all the other elements. This does not exclude, however, that all the elements are simultaneously false. So the *safety* requirement (SF) can additionally be imposed: it is impossible that all the relevant declaratives are simultaneously false.

The informal idea that lies behind conceiving questions as sets of declarative sentences is the reduction of questions to sets of PPA's. Let us consider the following set of declaratives:

(23) {John is both a philosopher and a logician, John is a philosopher, but not a logician, John is a logician, but not a philosopher, John is neither a philosopher nor a logician}

Both ME and SF hold for (23). So, assuming that ME and SF constitute, jointly, a sufficient condition for being a question semantically construed, (23) *is* a question and thus the meaning of some interrogative(s). But which interrogative(s)? Presumably, among others, the following one:

(24) Is John a philosopher and a logician?

However, what about the interrogative sentence:

(25) Is John a philosopher, or a logician?

Certainly, there is a reading of (25) under which it is synonymous with (24). But there is also a reading — and thus a meaning — of (25) such that the following constitutes the set of PPA's:

(26) {John is a philosopher, John is a logician}

As neither ME nor SF holds for (26), it is not a question semantically construed, that is, not a meaning of any interrogative.

So something went wrong. A refinement is needed. There are different rescue options possible. But, as no agreement has been reached yet, we stop here.

The reduction of questions to sets of declaratives is currently not mainstream in erotetic logic. We have paid disproportionate attention to it for two reasons. First, due to its generality and simplicity, the idea provides a temptation to logicians who only occasionally enter the area of questions and questioning. Second, similar difficulties emerge when questions are reduced to sets of propositions, where propositions are understood as meanings of declaratives.

3.2 Questions as epistemic imperatives

The underlying idea of the imperative-epistemic approach to questions is that the meaning of an interrogative is just the meaning of its imperative-epistemic paraphrase. Here are schemata of such paraphrases, proposed by Åqvist (1965) and Hintikka (1976), respectively:

(27) Let it be the case that I know ...
(28) Bring it about that I know ...

The ellipsis should be filled by an embedded interrogative sentence. So we have, for example:

(29) Let it be the case that I know who likes Mary.
(30) Bring it about that I know who likes Mary.

But what about meanings of the relevant embedded interrogatives? They are characterized in terms of epistemic logic. The meaning of the imperative operator used, in turn, is determined by a logic of imperatives.

Let us take Hintikka's account as an illustration. The expression that succeeds the imperative operator is a description of the epistemic state of affairs the questioner wants the respondent to bring about. It is called the *desideratum*. Desiderata of various questions involve such epistemic modalities as "know whether," "know where," "know who," "know when,", and so forth. The corresponding concepts of knowledge are explicated by Hintikka in terms of the concept of "knowing that." In doing this he makes use of his earlier results in epistemic logic, but also introduces some modifications and novelties to them.

Question-forming words (i.e. "which," "what," "where," "who, " etc.) are analyzed as a kind of ambidextrous quantifiers; that is, quantifiers which can be construed either existentially or universally. For example, the desideratum of "*Who likes Mary?*" can be either:

(31) $\exists x(x \text{ likes Mary} \wedge \exists y\ (y = x \wedge \mathrm{K}(y \text{ likes Mary})))$

or

(32) $\forall x(x \text{ likes Mary} \rightarrow \exists y\ (y = x \wedge \mathrm{K}(y \text{ likes Mary})))$

where K stands for the knowledge operator.

In Hintikka's theory the role of PPA's is performed by *conclusive answers*. Intuitively, a conclusive answer is

(OA$_2$) "*a reply which does not require further backing to satisfy the questioner.*" (Hintikka 1978, p. 287).

Possible replies and so-called conclusiveness-conditions are characterized first (in syntactic terms), and then a possible reply is regarded as a conclusive answer if this reply together with the description of the questioner's state of knowledge entails (by means of the underlying epistemic logic) the desideratum of the question. For example, "*John likes Mary*" is a conclusive answer to "*Who likes Mary?*" understood according to (31) on condition that a questioner knows who John is.

3.3 Questions as interrogative speech acts semantically construed

It is sometimes claimed that the meaning of an interrogative can be adequately characterized by a paraphrase that specifies the illocutionary act typically performed in uttering the interrogative. Roughly, questions are thus viewed as speech acts of a special kind, that is, as *interrogative acts*. The latter are modeled within a semantic framework, with the concept of a *success condition* playing a key role.

For example, in Vanderveken (1990), account an elementary illocutionary act has two semantic constituents: the *illocutionary force* and the *propositional content*. As for an interrogative act, the illocutionary force amounts to a request of the speaker to the hearer. What a hearer is supposed to do is specified by the propositional content of an interrogative act. Roughly, he or she is requested to perform a future speech act which conveys information of the required kind, that is, provides a resolving answer to the question under consideration.

Vanderveken's analysis works fine in the case of polar interrogatives. For example, when we have:

(2) Does John like Mary?

its analysis amounts, informally, to the following:

(33) I request that you assert that John likes Mary or deny that John likes Mary.

But constituent interrogatives raise difficulties. For an interesting discussion of Vanderveken's account see (Groenendijk & Stokhof 1997).

3.4 Questions as sentential functions

Let us compare the interrogative:

(1) Who likes Mary?

with the following condition:

(34) ... likes Mary

where the ellipsis is supposed to be filled with an expression referring to a person.

Neither (1) nor (34) has a content definite enough to enable an assignment of a truth value. In this sense they are both semantically incomplete. Moreover, it can be argued that they share a presupposition, namely "*Someone likes Mary*". This presupposition, however, works differently in either case. If nobody likes Mary, (34) cannot be *completed* to a true proposition saying that a given person likes Mary, while (1) cannot be *answered* with a true answer of this kind.

Conditions like (34) are modeled in classical logic as *sentential functions.* Syntactically, a sentential function is a well-formed formula with one or more free variable(s). Semantically, a sentential function expresses a condition that may be satisfied by some objects and not satisfied by others. One can claim that, on the semantic level, there is no difference between an interrogative and a sentential function. In brief: questions are sentential functions. Cohen (1929) is regarded as the first to put forward this idea.

The reduction of questions to (semantic counterparts of) sentential functions goes smoothly for constituent interrogatives, but the cases of polar interrogatives and coordinated interrogatives are more complex. Let us consider:

(2) Does John like Mary?
(35) Has John left for a while, or has he never lived here?

As for (2), the relevant condition is:

(36) ... John likes Mary.

and thus the corresponding sentential function should be:

(37) ξ John likes Mary.

But what is the semantic range of ξ? And in the case of (35): what is the condition?

3.5 From sentential functions to their interrogative closures

Let us consider the sentential function:

(38) x likes Mary

and let us close it by means of an interrogative operator "`Who` x". We get:

(39) `Who` x (x likes Mary)

The result is a semi-formal representation of "*Who likes Mary?*". It is syntactic. What about the semantic side? "`Who` x" determines the semantic range of x: the range equals the set of persons. The variable x is free in (38). But "`Who` x" binds x in (39) to the effect that its value must belong to the range determined by "`Who` x."

What we have sketched above is a part of the semantic analysis of interrogative sentences proposed by Ajdukiewicz (1926). "Where", "when" and "how" interrogatives are analyzed along similar lines. Interestingly enough, the general schema is applied to polar interrogatives as well. The semi-formal representation of:

(2) Does John like Mary?

amounts to:

(40) `O`ξ (ξ John likes Mary)

where `O`ξ is an interrogative operator that limits the possible values of variable ξ to the (extensional) operators of assertion and negation.

Ajdukiewicz is better known as the founding father of categorial grammar than as a pioneer in the logic of questions. His 1926 paper is a short note written in Polish, so its international impact was limited. But Ajdukiewicz's idea has found its developments in the work of Polish logicians, in particular Tadeusz Kubiński (discussed below) and Leon Koj (cf. Koj 1972, 1989).

3.6 Interrogative operators: Kubiński's account

The leading idea of Kubiński's analysis is that a question consists of an interrogative operator and a sentential function. Interrogative operators, in turn, comprise constants and variables. The only free variables in the sentential functions that occur in questions are the variables of the corresponding interrogative operators. These variables are “bound” by the relevant interrogative operators. The general schema of an interrogative[2] is:

$$\mathcal{O}\upsilon_1, ..., \upsilon_n A(\upsilon_1, ..., \upsilon_n)$$

where $\upsilon_1, ..., \upsilon_n$ are the only variables that occur free in $A(\upsilon_1, ..., \upsilon_n)$. Interrogative operators may contain (and thus bind) variables belonging to various syntactical categories. Generally speaking, the categories of variables indicate the (ontological) categories of objects which are asked about. For example, if an operator involves only individual variable(s), the corresponding questions ask about individuals. If the relevant variables run over sentential connectives, then the corresponding questions are about either the existence of some state(s) of affairs or some connection(s) between states of affairs. Moreover, there exist “mixed” interrogatives, that is, interrogatives whose operators contain variables belonging to two or more different categories.

For reasons of space, let us concentrate upon interrogative operators with individual variables. Here are examples of the so-called simple numerical interrogative operators:

Operator	Standard reading
$k \leq x$	*for which* [*at least* k] x
$k < x$	*for which* [*more than* k] x
kx	*for which* [*exactly* k] x
$\mathbf{C}x$	*for which* [*all*] x
$(k \leq)x$	*which are all* [*at least* k] x *such that*
$(k <)x$	*which are all* [*more than* k] x *such that*
$(k)x$	*which are all* [*exactly* k] x *such that*

where k is a numeral and x is an (individual) variable.

Simple numerical operators are not enough, however. Let us consider the following interrogative sentence:

(41) Which three boys love which two girls?

Clearly, different readings of (41) are possible. For example, (41) can be interpreted as expressing a question that asks about three boys and two girls such that each of these girls is loved by exactly one of the boys. But (41) can also be understood as a question that asks about three boys and two girls such that each of the boys loves the two girls, or as a question that asks about three boys and some (but at least two and no more than six) girls such that each of the boys loves two of the girls. Sometimes (41) can be construed as expressing a question that asks about a complete list of boys and girls fulfilling one of the above conditions. Other readings are also possible. Observe that

[2] In Kubiński’s terminology, this is a schema for *questions* rather than interrogatives. Kubiński speaks about *questions of formalized languages.*

the ambiguity effect is not only due to the presence of numerical expressions. It shows up in their absence as well, as in:

(42) Which boys love which girls?

By and large, *compound numerical interrogative operators* disambiguate multiple wh-questions. A compound numerical interrogative operator is not simply a string of interrogative quantifiers: it also includes some constants which, in effect, characterize the relevant scope dependencies.

Interrogatives are accompanied with sets of *direct answers*, defined purely syntactically. But direct answers are also:

($\mathbf{OA}_3$) "*these sentences which everybody who understands the question ought to be able to recognize as the simplest, most natural, admissible answers to the question.*" (Kubiński 1980, p. 12)

This is stated in pragmatic terms. However, recall the famous Hamblin (1958, p. 162) *dictum*:

> "*Knowing what counts as an answer is equivalent to knowing the question.*"

We can reverse the perspective: once we have an interrogative/question Q of a formal language, we also know what counts as a direct answer to Q. Furthermore, we can say that Q *represents* a NLQ Q^* *construed in such a way* that the "simplest, most natural, admissible answers" to Q^* have the syntactic form of direct answers to Q. Thus the relation between NLQ's and Kubiński's interrogatives/questions of formal languages is not 1–1. Interrogatives display possible readings of NLQ's. Needless to say, many (if not all) of the latter allow for multiple readings.

Kubiński did not provide any explicit semantic account of interrogative operators. But the remaining constituents of interrogatives, as well as direct answers, are expressions of semantically interpreted formal languages. Thus one can speak about entailment relation(s) between direct answers, and this, given the assignments of direct answers to interrogatives, enables a characterization of certain semantic relations between questions/interrogatives (such as being equipollent, being weaker than, being stronger than, etc.).

Remark. Kubiński was one of the founding fathers of erotetic logic (the story is told by Harrah 1997). His first paper in English devoted to the topic was presented at an international congress in 1958 (cf. Kubiński 1960). Unfortunately, his monograph in English, (Kubiński 1980), based on (Kubiński 1971), written in Polish, was published after the influential book by Belnap & Steel (1976). The issues addressed by Kubiński in the 1960s (possible readings, scope dependencies, quantifying into questions, etc.) were later, independently, analyzed by formal linguists, but presumably without knowledge of his proposals.

3.7 Subjects and requests: Belnap

The basic idea of Belnap's theory (cf. e.g. Belnap & Steel 1976) of questions is that a question offers a set of "alternatives" together with some suggestions or indications as to what kind of choice or selection among them should be made. The proposed account of the *logical* structure of interrogatives reflects this idea.

For brevity, let us concentrate upon the so-called elementary interrogatives. On Belnap's account, an elementary interrogative consists of the question mark "?", a *request*, and a *subject*.[3] The function of the subject is to offer the relevant (nominal) alternatives, whereas the function of the request is to characterize the required kind of selection. The role of the question mark is to assign the corresponding question (conceptualized as an abstract object defined set-theoretically) to the pair "request–subject" of an interrogative.

An elementary interrogative falls under the schema:

$$?\ \mathsf{scd}\ (\eta)$$

where:

1. η is the subject;
2. s is the *selection-size specification*: its role is to characterize the quantity of alternatives expected to occur in a direct answer (i.e. "exactly n", "at least n", "at most m", "at least n but at most m", etc.);
3. c and d are the *completeness claim* and the *distinctness claim*, respectively. Roughly, c specifies "how many" (in the qualitative sense) of the assumed as true alternatives should occur in a direct answer, while a (nonempty) d acts to the effect that a direct answer should involve semantically distinct alternatives.

As for elementary interrogatives, c can be either empty or maximal (notation: $-$ and $\forall$, respectively). Similarly, there are elementary interrogatives with nonempty d's (notation: $\neq$) and with empty d's (notation: $-$). The selection-size specification "select at least n, but at most m" ($n \geq 1$) is expressed by $\frac{m}{n}$; if the upper bound is undetermined, we write $\frac{-}{n}$.

When the constituents of an interrogative are specified, so are the corresponding *answerhood conditions*. As Belnap puts it:

> "*what counts as an answer to an interrogative arises out of the meanings of its constituents and its structure*" (Belnap 1990, p.13).

This, in particular, pertains to *direct answers* which perform the role of PPA's of Belnap's theory. The underlying intuition is that direct answers:

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[3] To be more precise, of a *lexical* subject and a *lexical* request. Belnap provides syntax for interrogatives, but also assigns abstract, nonlinguistic counterparts to the relevant syntactic items (the counterparts are labelled *real*). For the reasons of space we do not characterize them here.

($\mathbf{OA}_4$) "*are directly and precisely responsive to the question, giving neither more nor less information than what is called for*" (Belnap 1969*a*, p.124).

Direct answers to elementary interrogatives are defined purely syntactically. As for more complex interrogatives, both syntactic and semantic means are used. As in the case of Kubiński, there is no 1–1 correspondence between interrogatives and NLQ's.

Some examples may be helpful. Let us consider:

(25) Is John a philosopher, or a logician?

The subject of an interrogative that corresponds to (25) is:[4]

(43) (John is a philosopher, John is a logician)

However, (25) is equivocal. One can construe it as requesting a single choice between the alternatives given. This is reflected by the following request:

(44) $\frac{1}{1} - -$

Under this reading direct answers are simply the alternatives. However, since being a logician does not exclude being a philosopher, (25) can also be understood as allowing for a multiple choice; the corresponding request would be:

(45) $\frac{2}{1} - -$

and "*John is both a philosopher and a logician*" becomes a (new) direct answer. One can also require an exclusive choice among the alternatives. Now the request is:

(46) $\frac{1}{1}\forall -$

and the direct answers are: "*John is a philosopher, but not a logician*" and "*John is a logician, but not a philosopher*".

Let us come back to:

(1) Who likes Mary?

The subject (written semi-formally) is:

(47) (x is a person $\|x$ likes Mary)

The (nominal) alternatives are of the form "*b likes Mary*", where b is a name of a person. Depending on readings, the corresponding requests are diverse. The following:

(44) $\frac{1}{1} - -$

reflects the "give-me-one-example" reading; now each alternative is a direct answer. When we have:

[4] For transparency, we use natural language expressions in characterizing subjects.

(48) ${}^{-}_{1} - -$

the "mention-some" reading is the assumed one; direct answers are the alternatives and conjunctions of (nonequiform) alternatives. If, additionally, it is required that the persons mentioned are to be really distinct, the distinctness-claim is non-empty, viz.:

(49) ${}^{-}_{1} - \neq$

and each direct answer involving a conjunction of alternatives includes also a constituent saying that the names used refer to distinct persons.

It can also happen that it is the "complete list" of persons liking Mary which is requested. We would have:

(50) ${}^{-}_{1}\forall -$

or:

(51) ${}^{-}_{1}\forall \neq$

depending on whether the distinctness claim is empty or not. In both cases direct answers are supposed to include, additionally, completeness-clauses of the form:

(52) $\forall x$: if x is a person that likes Mary, then $x = b_1$ or $\ldots$ or $x = b_k$.

When the distinctness claim is nonempty and $k > 1$, a direct answer should also include a constituent saying that $b_1, \ldots, b_k$ are pairwise distinct.

Although the way we have presented examples obscures this, interrogatives are well-formed formulas of a formal language in which declarative well-formed formulas (d-wffs for short) also occur. The "declarative part" of such language is supplemented with an extensional semantics. Since direct answers are d-wffs, they have logical values. Belnap assigns logical values to interrogatives as well, though in an indirect manner. An interrogative is true in a model iff at least one direct answer to the interrogative is true in the model.[5] Once we have the concept of truth of interrogatives, the other semantic concepts (entailment, validity, etc.) apply to them as well. Moreover, one can define some semantic concepts that are specific only to interrogatives. We will make use of some of Belnap's proposals in section 5.

3.8 Questions as intensions of interrogatives: basic approaches

The development of Montague Semantics and intensional logics in general has resulted in elaborating intensional theories of questions.[6]

Probably the most influential papers on questions written within the intensional paradigm are (Hamblin 1973) and (Karttunen 1977). Generally speaking, in both cases the intension/meaning of an interrogative is a function from

[5] Cf. Belnap & Steel (1976, p. 119).

[6] For intensional logics see e.g. (Lappin 2015).

possible worlds to denotations, and the extension/denotation of an interrogative (in a world) is a set of propositions. The crucial difference between Hamblin and Karttunen lies in the fact that Hamblin regards the relevant propositions as expressing possible answers, while for Karttunen they express true answer(s). As an illustration, let us, again, come back to:

(1) Who likes Mary?

According to Karttunen, the denotation of (1) in the actual world is the set of true (w.r.t. the word) propositions that state of a person that he/she likes Mary. More generally, the denotation of (1) in a world w is the value of:[7]

(53) $\lambda p(\exists x(p = \lambda w(\texttt{likes-Mary}(w)(x))) \wedge p(w))$

for w, that is, roughly, is the set of all propositions which are true in w and "state" of some x that the value of x in w belongs to the extension of `likes-Mary` in w.

According to some intensional theories, the extension/denotation of an interrogative (in a world) is not a set of propositions, but, depending on the type of interrogative analyzed, a truth value, or an individual, or a set of individuals (cf. e.g. Tichy 1978). The so-called categorial (or functional) approach conceives meanings of interrogatives as functions from categorial answers to propositions. To be more precise, the meaning of an interrogative is a function that assigns propositions to the meanings of categorial answers, where categorial answers are either declarative sentences or nonsentential utterances/expressions: noun phrases, adverbs, etc. (cf. e.g. Hausser 1983). In view of the structured meaning approach (cf. e.g. Krifka 2001) the meanings of interrogatives are functions which, when applied to the meanings of "short" answers (cf. page 5), yield propositions. The relevant short answers refer to object belonging to a common ontological category and a formalization makes this explicit. For example, the meaning of (1) is characterized by:

(54) $\lambda w(\lambda x \in \texttt{Persons}(w)(\texttt{likes-Mary}(w)(x)))$

When w is the actual world, $\texttt{Persons}(w)$ is the set of persons. By dropping the references to a possible world we get a simpler schema:

(55) $\lambda x \in \texttt{Persons}(\texttt{likes-Mary}(x))$

A *full answer* is a proposition which, generally speaking, is the value of the (function characterizing the) meaning of an interrogative for an argument being a short/constituent answer.

3.9 Questions as partitions of the logical space: Groenendijk and Stokhof

Groenendijk & Stokhof (1984, 1997) also regard questions as intensions of interrogatives. However, the concept of intension used is different. Generally

[7] For brevity, we do not use the original PTG notation; moreover, "`likes-Mary`" is conceived as a one-place predicate. $p(w)$ says that proposition p is true in w.

speaking, on this account a question is viewed as a set of propositions (i.e. sets of possible worlds) which constitutes a partition of the logical space of possible worlds. The propositions belonging to a question are thus mutually exclusive and their union is the set of all possible worlds.[8] The idea of conceiving questions as partitions of "possible states of nature" is also present in other theories of questions, starting from (Higginbotham & May 1981). The "partition" approach to questions is currently among the most widely adopted (cf. Dekker et al. 2007).

For reasons of space let us only consider Groenendijk and Stokhof's account of yes-no questions. Interrogatives that correspond to yes-no questions have the form $?A$, where A is a declarative formula. Call a *model* a set of possible worlds. Let:

$$[A]_{\mathcal{M},w}$$

stand for the *extension* of A in world w of model $\mathcal{M}$, that is, the truth value assigned to A in world w of $\mathcal{M}$. The set of worlds:

$$[A]_{\mathcal{M}} = \{w \in \mathcal{M} : w(A) = \mathbf{1}\}$$

is the *intension* of A in $\mathcal{M}$. The *extension* of interrogative $?A$ in world w of model $\mathcal{M}$, $[?A]_{\mathcal{M},w}$, is the set of worlds:

$$\{w^* \in \mathcal{M} : [A]_{\mathcal{M},w^*} = [A]_{\mathcal{M},w}\}$$

that is, roughly, the set of all the worlds of $\mathcal{M}$ in which A has the same truth value as in w. The *intension* of interrogative $?A$ in $\mathcal{M}$, $[?A]_{\mathcal{M}}$, is the set of possible extensions of the interrogative in $\mathcal{M}$, i.e. the set:

$$\{[?A]_{\mathcal{M},w} : w \in \mathcal{M}\}$$

Now the *question* expressed by $?A$ in $\mathcal{M}$ is identified with the intension of $?A$ in $\mathcal{M}$.

A semi-formal example may be helpful. Given the definitions introduced above, we get:

$$[\textit{? Does John like Mary}]_{\mathcal{M}} = \{[\textit{John likes Mary}]_{\mathcal{M}}, \mathcal{M} \setminus [\textit{John likes Mary}]_{\mathcal{M}}\}$$

The intension/question is thus a two-element set of sets of possible worlds. Since a set of possible worlds is (or may be counted as) a proposition, a question is a set of propositions. Moreover, the propositions in the set are mutually exclusive, and they exhaust the logical space consisting of all the possible worlds in $\mathcal{M}$. To put it in the form of a slogan: *a yes-no question is a bipartition of the logical space.*

[8] This corresponds, in a sense, to the famous Hamblin's statement: "*The possible answers to a question are an exhaustive set of mutually exclusive possibilities.*" (Hamblin 1958, p. 162)

Other questions are viewed in a similar manner. They are still partitions, but not necessarily bipartitions. Of course, the notions of model, extension and intension are more complicated when wh-interrogatives and quantified formulas are taken into consideration.

Answerhood is defined as follows:

(i) ϕ is an answer to Q in model $\mathcal{M}$ if and only if for some $w \in \mathcal{M} : [\phi]_{\mathcal{M}} \subseteq [Q]_{\mathcal{M},w}$
(ii) ϕ is an answer to Q if and only if ϕ is an answer to Q in each model.

A *complete answer* is a proposition being a cell of the corresponding partition.

3.10 Questions as propositional abstracts: Ginzburg's account

Another influential approach to questions is based on situation semantics.[9] The old idea, according to which questions are akin to "open propositions", is conceptualized by Ginzburg (1995) (see also Ginzburg & Sag 2000) by conceiving questions as *propositional abstracts.*

At the start, one considers a universe which contains among its members a class of entities called *situations* (these are partial, temporally located, actual entities) and a class of entities called *states-of-affairs* (hereafter: SOA's). In addition, the universe comprises relations as well as possibilities, facts, and outcomes.

A SOA is a structured object constructed from a relation and an assignment of entities to the argument roles of the relation. (Relations are not conceived as sets of ordered tuples, but as unstructured atomic individuals.) Here is an example of SOA (the notation is self-explanatory):

(56) $\langle\langle \texttt{Like,\{likes:John,is liked:Mary\}} \rangle\rangle$

The role of SOA's is to designate properties that situations might possess. SOA's may be *supported* by situations; for instance, each situation in which John likes Mary supports the above SOA. There are positive and negative SOA's; the latter are constructed from the former via an adverbial-like operator. *Atomic propositions*, in turn, are defined in terms of situations and SOA's: PROP(s,α) is the proposition that s is a situation of the type designated by SOA α. The set of *propositions* includes atomic propositions and is closed under certain operations of negation, meet and join. The situation theory used by Ginzburg supports a form of abstraction. This abstraction differs from that of Lambda Calculus (and hence, Montague Semantics) in that it permits both simultaneous abstraction of more than one variable, and vacuous abstraction.

Questions are then conceived as *abstracts from propositions.* Thus, for example, we have:

[9] For situation semantics see, e.g., (Devlin 2006), (Seligman & Moss 2011), (Mares et al. 2011).

(57) $\lambda\{\ \}\mathrm{PROP}(s,\langle\langle\texttt{Like};\{\texttt{likes:John,is liked:Mary}\}\rangle\rangle)$
(58) $\lambda\{x\}\mathrm{PROP}(s,\langle\langle\texttt{Like};\{\texttt{likes:}x\texttt{,is liked:Mary}\}\rangle\rangle)$
(59) $\lambda\{x,y\}\mathrm{PROP}(s,\langle\langle\texttt{Like};\{\texttt{likes:}x\texttt{,is liked:}y\}\rangle\rangle)$

for "*Does John like Mary?*", "*Who likes Mary?*" and "*Who likes what?*" respectively.

The above idea can be also expressed in a much richer theoretical setting of Type Theory with Records, TTR for short. (For TTR see Cooper 2005, 2012, Cooper & Ginzburg 2015). Both situations and propositions can be modeled within the TTR ontology, by identifying them with certain types of records. TTR uses the standard notion of abstraction based on functional types. Given all this, one can reformulate the theory of questions as propositional abstracts in a way that avoids the nonstandard situation-theoretic notion of abstraction; see (Ginzburg 2005, 2012). Basic, noncompound questions are functions from records into propositions.

As for answerhood, Ginzburg proceeds as follows. There are information items which, intuitively speaking, are *about* a given NLQ. Some (but not all) of them can be regarded as those which *potentially resolve* the question: the relevant concept of a potentially resolving answer is characterized in detail for the types of NLQ's analyzed. However, one should not confuse potential resolvedness with *resolvedness*. The latter is relative to contextual factors, among which a purpose/goal of a questioner and her knowledge/belief state (including her deductive abilities) play crucial roles. A detailed analysis reveals what is a resolving answer in a given context. Since resolving answers are (and must be, as Ginzburg convincingly argues) contextually parametrized, answerhood does not serve as a basis in defining the meanings of interrogatives.

3.11 Questions in Inquisitive Semantics

Inquisitive semantics (INQ for short) originated from an analysis of questions, but currently evolves toward a general theory of meaning. The beginnings of INQ date to the late 1990s. INQ, however, is currently a research programme rather than a completed theory: alternative accounts are still being proposed.[10]

The basic idea of INQ can be briefly expressed as follows: the meaning of a sentence comprises two components, *informative content* and *inquisitive content*. The former is the information provided by a sentence, the latter is the issue raised by the sentence. By and large, if the information provided is sufficient to settle the issue that is raised, the sentence is an *assertion*. If, however, the information provided is insufficient to settle the raised issue, the sentence is *inquisitive*. For example, the sentence:

[10] For various versions of INQ see, e.g., (Groenendijk & Roelofsen 2009), (Ciardelli & Roelofsen 2011), (Ciardelli et al. 2013), (Ciardelli et al. 2015).

(8) John likes Mary

raises the issue whether John likes Mary and provides information that John likes Mary. (8) is thus an assertion. The following:

(60) Either John or Helen likes Mary

raises the issue who likes Mary, John or Helen, but the information provided by (60) amounts to the claim that one of the above possibilities holds and thus is insufficient to settle the raised issue. Hence (60) is inquisitive. Observe that inquisitive sentences are akin to questions in being carriers of information to be completed.

In what follows we concentrate upon the "basic," most often used system of INQ, labelled InqB, and we remain at the propositional level only. An interesting feature of InqB is that questions are indistinguishable from declaratives *syntactically*.[11] Being a question is a semantic property that some declarative sentences/formulas possess.

Let $\mathcal{L}_{\mathsf{InqB}}$ be a propositional language over a set of propositional variables $\mathcal{P}$. The primitive logical constants of $\mathcal{L}_{\mathsf{InqB}}$ are: $\bot, \vee, \wedge, \rightarrow$. Well-formed formulas (wffs) are defined as usual. $\mathcal{L}_{\mathsf{InqB}}$ is supplemented with a possible-world semantics; a possible world is conceived either as a subset of $\mathcal{P}$ or as a binary valuation of $\mathcal{P}$. Regardless of which of these accounts is adopted, the set $\mathcal{W}$ of suitable worlds for the language is determined.

A *state* is a subset of $\mathcal{W}$. States are thus sets of possible worlds. One can think of such sets as modelling information states. Singleton sets/states correspond to information states of maximal consistent information, while $\mathcal{W}$ corresponds to the ignorant state, i.e. an information state in which no possible world is excluded. $\emptyset$ represents the absurd state.

Both "ingredients" of meaning, informative content and inquisitive content, are defined in terms of *support*, being a relation between states and wffs. In the case of InqB support, $\vDash$, is defined as follows (the letters σ, τ are metalanguage variables ranging over states, and $\mathtt{p}$ runs over $\mathcal{P}$):

(a) $\sigma \vDash \mathtt{p}$ if and only if for each $w \in \sigma$: $\mathtt{p}$ is true in w,
(b) $\sigma \vDash \bot$ if and only if $\sigma = \emptyset$,
(c) $\sigma \vDash (A \wedge B)$ if and only if $\sigma \vDash A$ and $\sigma \vDash B$,
(d) $\sigma \vDash (A \vee B)$ if and only if $\sigma \vDash A$ or $\sigma \vDash B$,
(e) $\sigma \vDash (A \rightarrow B)$ if and only if for each $\tau \subseteq \sigma$: if $\tau \vDash A$ then $\tau \vDash B$.

Inquisitive negation is introduced by:

$$\neg A =_{df} (A \rightarrow \bot)$$

Thus we get:

(neg) $\sigma \vDash \neg A$ if and only if for each $\tau \subseteq \sigma$ such that $\tau \neq \emptyset : \tau \nvDash A$.

[11] This does not hold for INQ in general; see e.g. (Ciardelli et al. 2015).

The definition of support by a state generalizes the standard definition of truth in a world. To see this, it suffices to put $\sigma = \{w\}$ and then replace "$\{w\} \vDash A$" with "A is true in w". We get the usual clauses defining truth of a wff in a world. However, the generalization is nontrivial. Support by a state does not amount to truth in each world of the state: the clauses for disjunction and implication (and also negation) are more demanding.

The *informative content* of a wff A, $\mathsf{info}(A)$, is defined as follows:

$$\mathsf{info}(A) = \bigcup\{\sigma \subseteq \mathcal{W} : \sigma \vDash A\}$$

while the *inquisitive content* of a wff A, $\mathsf{inqct}(A)$, is defined by:

$$\mathsf{inqct}(A) = \{\sigma \subseteq \mathsf{info}(A) : \sigma \vDash A\}.$$

A wff A is *inquisitive* if and only if

$$\mathsf{info}(A) \notin \mathsf{inqct}(A)$$

that is, the informative content of A does not support A. Let:

$$\mathsf{tset}(A) = \{w \in \mathcal{W} : A \text{ is true in } w\}.$$

INQ labels $\mathsf{tset}(A)$ as the *truth set* of A. Speaking in more traditional terms, $\mathsf{tset}(A)$ is simply the proposition expressed by A.[12]

In the case of InqB (but not INQ in general) we have:

$$\mathsf{info}(A) = \mathsf{tset}(A)$$

and hence A is inquisitive if and only if:

$$\mathsf{tset}(A) \notin \mathsf{inqct}(A)$$

Here are examples of inquisitive wffs:

(61) $p \vee r$

(62) $p \vee \neg p$

(61) is inquisitive since $\mathsf{tset}(p \vee r) = \mathsf{tset}(p) \cup \mathsf{tset}(r)$, and neither p nor r is supported by $\mathsf{tset}(p) \cup \mathsf{tset}(r)$. (62) is inquisitive because $\mathsf{tset}(p \vee \neg p) = \mathcal{W}$, but $\mathcal{W} \nvDash p$ as well as $\mathcal{W} \nvDash \neg p$. Speaking in general terms: the information carried by the disjunction (61) is insufficient to settle the issue which of the disjuncts hold(s). As for (62), $p \vee \neg p$ carries no factual information and hence the issue of whether p holds remains unresolved.

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[12] However, the concept of proposition used in InqB is "lifted": a proposition is a set of sets possible worlds. More precisely, a proposition expressed by wff A is $\mathsf{inqct}(A)$.

An interesting property of InqB-inquisitiveness is that each inquisitive wff expresses at least two *possibilities*, where a possibility is a maximal state closed under support. More precisely, a state σ is a possibility for a wff A just in case $\sigma \vDash A$ and for each $w \notin \sigma : \sigma \cup \{w\} \nvDash A$.

The property of *being a question* is not identified with inquisitiveness, but with noninformativeness. A wff A is *noninformative* if and only if

$$\mathsf{tset}(A) = \mathcal{W}.$$

Noninformative wffs are just classical tautologies. Therefore each wff having the property of being a question is a classical tautology, and each tautology is (more precisely: has the property of being) a question. Hence, similarly as in the case of the partition approach (see subsection 3.9), a question "covers" the whole logical space. A substantial difference lies in the fact that the possibilities expressed by an inquisitive question (i.e. a wff which is both noninformative and inquisitive) need not be disjoint. (62) is an inquisitive question. Here are further examples of inquisitive questions:

(63) $p \rightarrow r \vee \neg r$

(64) $(p \vee \neg p) \vee (r \vee \neg r)$

Observe that both (63) and (64) have constituents which are questions themselves.

The interrogative operator "?" is introduced by the following definition:

$$?A =_{df} A \vee \neg A$$

Thus (62), (63) and (64) can be respectively written as:

(65) $?p$

(66) $p \rightarrow ?r$

(67) $?p \vee ?r$

One should not confuse the above definition with a syntactic characterization of questions/interrogatives. Also, it is not claimed that each question is a yes-no question. For example, neither (63) nor (64) falls under the schema $A \vee \neg A$. (63) represents a conditional NLQ calling for answers of the forms $p \rightarrow r$ and $p \rightarrow \neg r$. (64), in turn, represents an alternative question that calls for the following answers: p, $\neg p$, r, $\neg r$.

3.12 General remarks. E-formulas

Let us end this section with some metatheoretical remarks.

A formal theory of questions, logical or linguistic, has to "incorporate" interrogatives into a formal language which, initially, had not been designed for representing NLQ's. Generally speaking, this can be done in two ways.

I. (THE "DEFINE WITHIN" APPROACH.) One can *define* questions in the metalanguage of the language considered, and then regard as interrogatives these (already present) meaningful expressions of the language which express questions just defined. For convenience, one can then operate with some specific interrogative formulas but they are only abbreviations of the corresponding expressions of the language, and, what is more important, their semantics is just the semantics of the corresponding expressions.

II. (THE "ENRICH WITH" APPROACH.) One can *enrich* a language with interrogatives. In order to achieve this, one extends the vocabulary and then introduces interrogatives syntactically, as a new category of well-formed formulas. The new category is distinct from the remaining ones both syntactically and semantically.

Regardless of which approach is adopted, one ends with a class of *erotetic formulas* or *e-formulas* for short.

Let us stress: e-formulas need not be defined or determined purely syntactically. They can be characterized in semantic terms, for example as the well-formed formulas that "correspond" to questions semantically construed, or have the semantic property of being a question, etc. We use the term "e-formula" instead of "erotetic well-formed formula" (or "e-wff") in order to avoid the suggestion that the relevant formulas have to be defined inductively.

The general picture is the following:

$$Form_{\mathcal{L}} = \mathcal{D}_{\mathcal{L}} \cup \mathcal{E}_{\mathcal{L}} \cup \ldots$$

where $\mathcal{D}_{\mathcal{L}}$ is the set of declarative well-formed formulas (d-wffs for short) of a language $\mathcal{L}$, $\mathcal{E}_{\mathcal{L}}$ is the set of e-formulas of the language, and the ellipsis indicates that the set of formulas of a language under consideration can also include well-formed/meaningful expressions of other kind or kinds (optatives, imperatives, exclamatives, ...). Recall that, on some accounts, $\mathcal{E}_{\mathcal{L}}$ is not disjoint with the remaining item(s).

Characterizing e-formulas is only the starting point. The crucial issue is to assign *principal possible answers* (PPA's) to them. As we have illustrated above, this can be (and is!) done in many ways, and with different basic intuitions in mind.

Most (if not all) e-formulas considered within a theory are objects supposed to represent the corresponding NLQs. Similarly, PPAs to e-formulas that represent NLQs should represent PPAs to the corresponding NLQs. However, the concept of principal possible answer to a NLQ is vague. The underlying intuitions are expressed by using, among others, pragmatic terms.[13] Furthermore, NLQs permit multiple readings. Or, to put it differently, in many cases contextual and/or pragmatic factors co-determine what is "directly and precisely responsive to the question, giving neither more nor less information than what is called for," or what is a just-sufficient (i.e. immediate and sufficient) pos-

[13] Cf., e.g., OA_1 (p. 5), OA_2 (p. 10), OA_3 (p. 14), OA_4 (p. 16).

sible answer, etc. Thus, as a matter of fact, e-formulas and NLQs are linked as follows:

(♠) *An e-formula Q represents a NLQ Q^** CONSTRUED IN SUCH A WAY *that possible answers to Q^* having the desired semantic and/or pragmatic properties are represented by PPAs to Q.*

Of course, a theory can be rich enough to give its own accounts of the desired properties, as, for example, Hintikka's theory does (here the conclusiveness issue can be resolved by means of an epistemic logic). But NLQs still permit multiple readings and thus one has to decide in advance what exactly is to be modeled. On Hintikka's account, question-forming words are akin to ambidextrous quantifiers (see subsection 3.2) and one can "calculate" conclusive answers only after deciding which reading of a NLQ is taken into consideration.

4 Minimal Erotetic Semantics: Basics and Tools

Minimal erotetic semantics (MiES for short) enables an introduction of some important semantic notions pertaining to e-formulas. In defining these notions, one makes use of the existence of an already established assignment of PPAs to e-formulas, and of semantic concepts pertaining to d-wffs of a language. Minimal erotetic semantics combines some ideas present in Belnap's erotetic semantics (cf. Belnap & Steel 1976, chap. 3) with certain insights to be found in (Shoesmith & Smiley 1978). Needless to say, MiES goes beyond them.

Minimal erotetic semantics remains neutral in two respects. It does not presuppose what is *the* logic of d-wffs. The general conceptual framework of MiES is compatible with classical logic as well as with nonclasical logics. Moreover, MiES is applicable regardless of whether — and if yes, how — questions themselves have been previously conceptualized semantically. If they were, MiES provides a "second collection" of semantic concepts pertaining to e-formulas. Otherwise MiES allows us to speak about e-formulas in semantic terms even if their semantic content has been left undefined.

In what follows we present only some elements of MiES. We focus on concepts relevant to the issues mentioned in subsection 2.3. A more detailed exposition of MiES can be found in (Wiśniewski 2013, chap. 3 and 4).[14]

4.1 Partitions, admissible partitions, and entailment

The precondition of applicability of MiES is the following: we deal with a formal language in which both d-wffs and e-formulas are present. Languages fulfilling the precondition are diverse. Moreover, their "declarative parts" are governed by different logics. So we need a general, uniform framework which relies neither on the choice of logic for d-wffs nor on syntactic forms of e-formulas. The framework, however, should enable us to operate with adequate (at least with respect to scopes) counterparts of entailment and entailment-related semantic notions defined in the "original" semantics of the declarative part of a language.

Notation. In what follows we use the letters A, B, C, D, with or without subscripts, as metalanguage variables for d-wffs, and the letters X, Y, Z, with subscripts if needed, as metalinguistic variables for sets of d-wffs.

For conciseness, we abbreviate "if and only if" as "iff."

We start with a technical concept of partition of a language. Let $\mathcal{L}$ be a formal language of the kind considered here. First, we define the concept of partition of the set $\mathcal{D}_{\mathcal{L}}$ of d-wffs of $\mathcal{L}$.[15]

[14] For a model-theoretic variant of MiES see (Wiśniewski 1997a) or (Wiśniewski 1995, chap. 4).

[15] Definition 1 mirrors the definition of the concept of partition of the set of wffs of a "question-free" formal language, provided by (Shoesmith & Smiley 1978, p. 15).

Definition 1. (PARTITION OF THE SET OF D-WFFS) *By a partition of* $\mathcal{D}_{\mathcal{L}}$ *we mean an ordered pair:*

$$\mathsf{P} = \langle \mathsf{T}_{\mathsf{P}}, \mathsf{U}_{\mathsf{P}} \rangle$$

where $\mathsf{T}_{\mathsf{P}} \cap \mathsf{U}_{\mathsf{P}} = \emptyset$ *and* $\mathsf{T}_{\mathsf{P}} \cup \mathsf{U}_{\mathsf{P}} = \mathcal{D}_{\mathcal{L}}$.

Intuitively, T_{P} consists of all the d-wffs which are "true" in P, and U_{P} is made up of all the d-wffs which are "untrue" in P. But "true" is used here as a general cover term which, as we will see, is construed differently in different cases and, in particular, is not synonymous with "true in the actual world". For stylistic reasons, however, it is convenient to speak about truths and untruths of a partition.

Besides d-wffs, $\mathcal{L}$ has also e-formulas among its meaningful expressions. But MiES remains neutral in the controversy whether e-formulas/questions have truth values. So we put:

Definition 2. (PARTITION OF A LANGUAGE) *By a partition of* $\mathcal{L}$ *we mean a partition of* $\mathcal{D}_{\mathcal{L}}$.

Thus what is "partitioned" is the set of d-wffs only. Note that we use the term "partition" in a sense different from that adopted in the Groenendijk and Stokhof's theory (see subsection 3.9).

The above concept of partition is very wide and admits partitions which are rather odd from the intuitive point of view. For instance, $\langle \mathcal{D}_{\mathcal{L}}, \emptyset \rangle$ is a partition, and, for any d-wff A, $\langle \mathcal{D}_{\mathcal{L}} \setminus \{A\}, \{A\} \rangle$ is a partition. In order to avoid oddity on the one hand, and to reflect some basic semantic and/or logical facts concerning (the declarative parts of) a language under consideration, we distinguish the class of *admissible partitions*, being a non-empty subclass of the class of all partitions of the language. By and large, an admissible partition splits the set of d-wffs into disjoint sets of "truths" and "untruths" in a way that "agrees" with a semantics of the declarative part of a language considered and/or by the logic built in it.

Let us stress: we do not assume that a language has only one admissible partition. There usually exist many such partitions (see below).

Let $X \subseteq \mathcal{D}_{\mathcal{L}}$ and $A \in \mathcal{D}_{\mathcal{L}}$. *Entailment in* $\mathcal{L}$, symbolized by $\models_{\mathcal{L}}$, can be defined in terms of admissible partitions, according to the following pattern:

Definition 3. (ENTAILMENT) $X \models_{\mathcal{L}} A$ *iff there is no admissible partition* $\mathsf{P} = \langle \mathsf{T}_{\mathsf{P}}, \mathsf{U}_{\mathsf{P}} \rangle$ *of* $\mathcal{L}$ *such that* $X \subseteq \mathsf{T}_{\mathsf{P}}$ *and* $A \in \mathsf{U}_{\mathsf{P}}$.

Depending on a language and its admissible partitions, we get either entailment determined by classical logic or by a nonclassical logic.

4.2 Admissible partitions and entailment: examples

Let us illustrate the above general considerations with some examples. We start with some propositional languages/logics, and then we turn to the first-order case.

Terminological remark: e-formulas and questions. From now on, for purely stylistic reasons, we will be speaking of questions instead of e-formulas. The reader is kindly requested to remember that "question" below means, unless otherwise stated, only "e-formula."

Enriching propositional languages with questions

D-wffs of the propositional languages with questions considered below are simply the wffs of the initial propositional languages. Questions are introduced according to a common pattern. We enrich the vocabulary with the signs: ?, {, }, and the comma. A question (i.e. e-formula) is an expression of the form:

(68) $?\{A_1, \ldots, A_n\}$

where $n > 1$ and $A_1, \ldots, A_n$ are nonequiform, that is, pairwise syntactically distinct, d-wffs of the language under consideration.

If (68) is a question, then each of $A_1, \ldots, A_n$ is a *principal possible answer* (PPA) to the question, and these are the only PPA's to it.

Note that questions are not sets of d-wffs, but object-level language expressions of a strictly defined form. In particular, $?\{p, q\} \neq ?\{q, p\}$.

The schema (68) is general enough to capture most (if not all) of propositional questions studied in the literature. Any question of the form (68) can be read: "*Is it the case that* A_1*, or ... , or is it the case that* A_n*?*". However, sometimes more specific readings can be recommended. For instance, the following:

(69) $?\{A, \neg A\}$

(70) $?\{A \wedge B, A \wedge \neg B, \neg A \wedge B, \neg A \wedge \neg B\}$

can be read: "*Is it the case that* A*?*" and "*Is it the case that* A *and is it the case that* B*?*" respectively. For conciseness, we will be abbreviating (69) by

(71) $?A$

and (70) by:

(72) $? \pm |A, B|$.

Language $\mathcal{L}^{?}_{\mathsf{CPL}}$

Let $\mathcal{L}_{\mathsf{CPL}}$ be the language of classical propositional logic (CPL). We enrich $\mathcal{L}_{\mathsf{CPL}}$ with questions in the manner described above. The enriched language is labelled by $\mathcal{L}^{?}_{\mathsf{CPL}}$.

A *CPL-valuation* is a function from the set of d-wffs of $\mathcal{L}^{?}_{\mathsf{CPL}}$ (i.e. wffs of $\mathcal{L}_{\mathsf{CPL}}$) into the set of truth values $\{\mathbf{1}, \mathbf{0}\}$, defined in the usual way.

Definition 4. (Admissible partitions of $\mathcal{L}^{?}_{\mathsf{CPL}}$) *A partition* $\mathsf{P} = \langle \mathsf{T}_{\mathsf{P}}, \mathsf{U}_{\mathsf{P}} \rangle$ *of* $\mathcal{L}^{?}_{\mathsf{CPL}}$ *is admissible iff for some CPL-valuation* v*:*

(1) $\mathsf{T_P} = \{A \in \mathcal{D}_{\mathcal{L}^?_{\mathsf{CPL}}} : v(A) = \mathbf{1}\}$, *and*
(2) $\mathsf{U_P} = \{B \in \mathcal{D}_{\mathcal{L}^?_{\mathsf{CPL}}} : v(B) = \mathbf{0}\}$.

Thus the set of "truths" of an admissible partition equals the set of d-wffs which are true under the corresponding valuation.

Observe that all the usual semantic properties are retained, but now they can be rephrased in terms of admissible partitions. Note that entailment in $\mathcal{L}^?_{\mathsf{CPL}}$ reduces to CPL-entailment.

Language $\mathcal{L}^?_{\mathsf{S4}}$

Modal propositional languages with questions are constructed similarly as the language $\mathcal{L}^?_{\mathsf{CPL}}$. The difference lies in taking the language of a modal propositional logic ℓ as the point of departure.

As an illustration, let us consider the case of $\mathsf{S4}$. The d-wffs of the relevant language $\mathcal{L}^?_{\mathsf{S4}}$ are the wffs of the language $\mathcal{L}_{\mathsf{S4}}$ of $\mathsf{S4}$. Questions are introduced according to the pattern presented above. We make use of the standard relational semantics of $\mathsf{S4}$. A $\mathsf{S4}$-model is an ordered triple:

$$\langle W, R, V\rangle$$

where $W \neq \emptyset$, $R \subseteq W \times W$ is both reflexive and transitive in W, and $V : \mathcal{D}_{\mathcal{L}_{\mathsf{S4}}} \times W \mapsto \{\mathbf{1}, \mathbf{0}\}$ satisfies the usual conditions.

Definition 5. (Admissible partitions of $\mathcal{L}^?_{\mathsf{S4}}$) *A partition* $\mathsf{P} = \langle \mathsf{T_P}, \mathsf{U_P}\rangle$ *of* $\mathcal{L}^?_{\mathsf{S4}}$ *is admissible iff for some* $\mathsf{S4}$*-model* $\langle W, R, V\rangle$ *and for some* $w \in W$*:*

(1) $\mathsf{T_P} = \{A \in \mathcal{D}_{\mathcal{L}^?_{\mathsf{S4}}} : V(A, w) = \mathbf{1}\}$ *and*
(2) $\mathsf{U_P} = \{B \in \mathcal{D}_{\mathcal{L}^?_{\mathsf{S4}}} : V(B, w) = \mathbf{0}\}$.

Thus the set of "truths" of an admissible partition consists of all the d-wffs (i.e. wffs of $\mathcal{L}_{\mathsf{S4}}$) which are true in the corresponding world of a given model. Entailment in $\mathcal{L}^?_{\mathsf{S4}}$ reduces to the so-called local entailment in $\mathsf{S4}$.

Language $\mathcal{L}^?_{\mathsf{InqB}}$

Let us enrich the language $\mathcal{L}_{\mathsf{InqB}}$ (of the basic system of INQ; see subsection 3.11) with questions. Recall that a state is a subset of an already given set of possible worlds, and that the set of possible worlds is determined by $\mathcal{L}_{\mathsf{InqB}}$. Admissible partitions are defined in terms of support.

Definition 6. (Admissible partitions of $\mathcal{L}^?_{\mathsf{InqB}}$) *A partition* $\mathsf{P} = \langle \mathsf{T_P}, \mathsf{U_P}\rangle$ *of* $\mathcal{L}^?_{\mathsf{InqB}}$ *is admissible iff for some state* σ*:*

(1) $\mathsf{T_P} = \{A \in \mathcal{D}_{\mathcal{L}^?_{\mathsf{InqB}}} : \sigma \models A\}$ *and*
(2) $\mathsf{U_P} = \{B \in \mathcal{D}_{\mathcal{L}^?_{\mathsf{InqB}}} : \sigma \not\models B\}$.

When we combine Definition 6 with Definition 3, we get entailment understood in a somewhat non-standard way, as the "transmission of support". Yet, this is how entailment is in fact construed in InqB. One can define $\models_{\mathsf{InqB}}$ in a more traditional manner, reflecting the idea of "transmission of truth", but this requires an introduction of a (non-standard) concept of truth. For details see Wiśniewski (2014).

Language $\mathcal{L}^{?}_{\mathsf{MCL}}$

Let $\mathcal{L}_{\mathsf{MCL}}$ be the language of Monadic Classical Logic with Identity (MCL for short). For simplicity, let us assume that the vocabulary of $\mathcal{L}_{\mathsf{MCL}}$ contains an infinite list of individual constants, but does not contain function symbols. Wffs of $\mathcal{L}_{\mathsf{MCL}}$ are defined in the standard way. A sentential function is a wff in which a free variable occurs; otherwise a wff is a *sentence*.

We construct a language $\mathcal{L}^{?}_{\mathsf{MCL}}$ which has a declarative part and an erotetic part. The declarative part of $\mathcal{L}^{?}_{\mathsf{MCL}}$ is $\mathcal{L}_{\mathsf{MCL}}$ itself. As for the erotetic part, we add the following signs to the vocabulary of $\mathcal{L}_{\mathsf{MCL}}$: $?, \{, \}, \mathbf{S}, \mathbf{U}$, and the comma. Questions of $\mathcal{L}^{?}_{\mathsf{MCL}}$ fall under the following schemata:

(68) $?\{A_1, \ldots, A_n\}$

where $n > 1$ and $A_1, \ldots, A_n$ are nonequiform (i.e. pairwise syntactically distinct) sentences of $\mathcal{L}_{\mathsf{MCL}}$, being the PPA's to the question;

(73) $?\mathbf{S}(Ax)$,
(74) $?\mathbf{U}(Ax)$

where x stands for an individual variable and Ax is a sentential function of $\mathcal{L}_{\mathsf{MCL}}$ which has x as the only free variable. (73) can be read "*Which x is such that Ax?*", while (74) reads: "*What are all of the x's such that Ax?*".

PPA's to (73) are instantiations (by individual constants) of Ax. Thus the PPA's are sentences of the form $A(x/c)$, where c is an individual constant. PPA's to (74) have the form:

(75) $A(x/c_1) \wedge \ldots \wedge A(x/c_n) \wedge \forall x(Ax \rightarrow x = c_1 \vee \ldots \vee x = c_n)$

where $n \geq 1$ and $c_1, \ldots, c_n$ stand for distinct individual constants.[16]

[16] The symbols $\mathbf{S}$ and $\mathbf{U}$ belong to the vocabulary of the object-level language $\mathcal{L}^{?}_{\mathsf{MCL}}$. However, we can introduce them to the metalanguage as well, yet with different meanings. We can assume that on the metalanguage level $\mathbf{S}(Ax)$ designates the set of all the sentences of the form $A(x/c)$, whereas $\mathbf{U}(Ax)$ designates the set of all the sentences of the form (75). Now we are justified in saying that each question of $\mathcal{L}^{?}_{\mathsf{MCL}}$ consists of the sign ? followed by an (object-level language) expression which is equiform to a metalanguage expression that designates the set of PPA's to the question. (Observe that this is not tantamount to a *reduction* of questions to sets of d-wffs. Questions are still linguistic expressions of a strictly defined form.) What we gain is transparency: it is easy to say what counts as a PPA to a question.

As for the semantics of $\mathcal{L}^?_{\mathsf{MCL}}$, we make use of the model-theoretic semantics of $\mathcal{L}_{\mathsf{MCL}}$ that operates with the concepts characterized below. A *model* of $\mathcal{L}^?_{\mathsf{MCL}}$ is an ordered pair $\mathcal{M} = \langle M, f \rangle$, where M is a nonempty set, and f is a function which assigns an element of M to each individual constant of $\mathcal{L}^?_{\mathsf{MCL}}$, and a subset of M to each unary predicate of $\mathcal{L}^?_{\mathsf{MCL}}$. An $\mathcal{M}$-*valuation* is a denumerable sequence of elements of M. The concepts of value of a term under an $\mathcal{M}$-valuation, and of satisfaction of a d-wff by an $\mathcal{M}$-valuation are defined in the standard manner. A d-wff A is *true* in a model $\mathcal{M} = \langle M, f \rangle$ iff A is satisfied by each $\mathcal{M}$-valuation.

We define the class of normal models. Roughly, a model $\mathcal{M} = \langle M, f \rangle$ of $\mathcal{L}^?_{\mathsf{MCL}}$ is *normal* just in case all the elements of M are named by individual constants of $\mathcal{L}^?_{\mathsf{MCL}}$. To be more precise, by a normal model of $\mathcal{L}^?_{\mathsf{MCL}}$ we mean a model $\mathcal{M} = \langle M, f \rangle$ of the language such that for each $y \in M$ we have: $y = f(c_i)$ for some individual constant c_i of $\mathcal{L}^?_{\mathsf{MCL}}$.

As long as normal models are concerned, the truth of an existential generalization, $\exists x Ax$, warrants the existence of a true PPA to the corresponding question, $?\mathbf{S}(Ax)$. This is why we have distinguished these models here.

Admissible partitions are defined in terms of normal models:

Definition 7. (Admissible partitions of $\mathcal{L}^?_{\mathsf{MCL}}$) *A partition* $\mathsf{P} = \langle \mathsf{T_P}, \mathsf{U_P} \rangle$ *of* $\mathcal{L}^?_{\mathsf{MCL}}$ *is admissible iff for some normal model* $\mathcal{M}$ *of* $\mathcal{L}^?_{\mathsf{MCL}}$*:*

(1) $\mathsf{T_P} = \{A \in \mathcal{D}_{\mathcal{L}^?_{\mathsf{MCL}}} : A \text{ is true in } \mathcal{M}\}$, *and*
(2) $\mathsf{U_P} = \{B \in \mathcal{D}_{\mathcal{L}^?_{\mathsf{MCL}}} : B \text{ is not true in } \mathcal{M}\}$.

Hence the set of "truths" of an admissible partition equals the set of d-wffs which are true in the corresponding normal model.

One can prove that in the case of finite sets of d-wffs, entailment in $\mathcal{L}^?_{\mathsf{MCL}}$ reduces to entailment determined by classical logic. The situation changes, however, when infinite sets of d-wffs are taken into consideration.

Remarks. The reference to normal models is the key feature of the above construction. We have distinguished them for "erotetic" reasons. However, when we deal with a first-order (or a higher-order) language enriched with questions, normal models can be distinguished for many reasons and in different manners. For example, one can define them as models of a theory expressed in the declarative part of the language (that is, models of the language in which all the theorems are true), or as models which make true some definition(s). It is also permitted to consider all models as normal. Each decision determines the corresponding entailment relation.

Nothing prevents us from taking a richer first-order language (or a higher-order language) as the point of departure, and from introducing other categories of wh-questions according to the above pattern. For possible developments see (Wiśniewski 1995, chap. 3).

4.3 A digression: the minimalistic method of determining admissible partitions

So far we have determined admissible partitions by making use of already given semantics of the "declarative parts" of the analyzed languages. However, there exists a more general method of determining admissible partitions.

Let $\mathcal{L}$ be a language of the analyzed kind such that the declarative part of $\mathcal{L}$ is the language of a logic ℓ; the d-wffs of $\mathcal{L}$ are just the wffs of the language of ℓ.

A logic determines the corresponding consequence relation; it is a binary relation between sets of wffs on the one hand and individual wffs on the other. Let $\vdash_\ell$ stand for the consequence relation determined by ℓ.

Assume that $\vdash_\ell$ is not universal (i.e. $\vdash_\ell \neq \wp(\mathcal{D}_\mathcal{L}) \times \mathcal{D}_\mathcal{L}$, where $\wp(\mathcal{D}_\mathcal{L})$ is the power set of $\mathcal{D}_\mathcal{L}$). First, we introduce:

Definition 8. (PROPER PARTITIONS) *A partition* $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ *of* $\mathcal{L}$ *is improper iff for some set* X *of d-wffs of* $\mathcal{L}$ *and some d-wff* A *of* $\mathcal{L}$ *such that* $X \vdash_\ell A$ *we have:* $X \subseteq \mathsf{T}_\mathsf{P}$ *and* $A \in \mathsf{U}_\mathsf{P}$*; otherwise* P *is called proper.*

The second step is to define the class of *admissible partitions* of $\mathcal{L}$ as the subclass of the class of all proper partitions of the language that fulfils a certain condition.

Definition 9. (THE CLASS OF ADMISSIBLE PARTITIONS) *The class* $\Pi_\mathcal{L}$ *of admissible partitions of* $\mathcal{L}$ *is the greatest class of proper partitions of the language that is closed under the following condition:*

($\heartsuit$) *if* $X \not\vdash_\ell A$*, then for some partition* $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ *in* $\Pi_\mathcal{L}$: $X \subseteq \mathsf{T}_\mathsf{P}$ *and* $A \in \mathsf{U}_\mathsf{P}$

for any set of d-wffs X *of* $\mathcal{L}$ *and any d-wff* A *of the language.*

We get:

Corollary 1. $\vdash_\ell \; = \; \models_\mathcal{L}$.

Thus entailment in $\mathcal{L}$ amounts, set-theoretically, to the consequence relation determined by the logic ℓ. This facilitates possible applications.

Let us stress that the above construction permits that ℓ is a nonclassical logic (but, still, a monotonic logic.)

Terminology. Let $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ be a partition. By saying that wff A is *true* in P we mean that A belongs to T_P.

4.4 Multiple-conclusion entailment

It is natural to think of questions which have well-defined sets of PPA's as offering sets of "possibilities" or "alternatives", among which some selection or choice is requested to be made. When we are interested in relations between questions and contexts of their appearance, some notion of, to speak generally, "entailing a set of possibilities" is needed. There is a logic, however, within which such notion is elaborated on: it is multiple-conclusion logic (see Shoesmith & Smiley 1978). This logic generalizes the concept of entailment, regarding it as a relation between *sets* of d-wffs. The entailed set is conceived as, intuitively speaking, setting out the field within which the truth must lie if the premises are all true. The concept of multiple-conclusion entailment is one of the main conceptual tools of MiES.

Let $\mathcal{L}$ be a language of the kind considered here, and let X and Y be sets of d-wffs of $\mathcal{L}$. The relation $||\!\!=_{\mathcal{L}}$ of *multiple-conclusion entailment in* $\mathcal{L}$ is defined as follows:

Definition 10. (Multiple-conclusion entailment) *$X \,||\!\!=_{\mathcal{L}} Y$ iff there is no admissible partition $\mathsf{P} = \langle \mathsf{T_P}, \mathsf{U_P} \rangle$ of $\mathcal{L}$ such that $X \subseteq \mathsf{T_P}$ and $Y \subseteq \mathsf{U_P}$.*

Thus X multiple-conclusion entails (*mc-entails* for short) Y iff there is no admissible partition in which all the d-wffs in X are true and no d-wff in Y is true. In other words, mc-entailment between X and Y holds just in case the truth of all the d-wffs in X warrants the presence of some true d-wff(s) in Y: whenever all the d-wffs in X are true in an admissible partition P, then at least one d-wff in Y is true in the partition P.

Definition 10 offers a nontrivial generalization of the concept of entailment. It happens that a set of d-wffs X mc-entails a set of d-wffs Y, but no element of Y is entailed by X. For example, we have:

$$\{p,\ p \to q \vee r\} \,||\!\!=_{\mathcal{L}^?_{\mathsf{CPL}}} \{q, r\}$$

on the one hand, and neither

$$\{p,\ p \to q \vee r\} \models_{\mathcal{L}^?_{\mathsf{CPL}}} q$$

nor

$$\{p,\ p \to q \vee r\} \models_{\mathcal{L}^?_{\mathsf{CPL}}} r$$

on the other hand.[17] So one cannot define mc-entailment of a set of d-wffs as ("single-conclusion") entailment of an element of the set. But, intuitively speaking, mc-entailment between X and Y amounts to (single-conclusion) entailment of a "disjunction" of all the Y's from a "conjunction" of all the X's. However, we cannot use this idea as a basis for a definition of mc-entailment. There are languages of the considered kind in which disjunction does not occur or is understood differently than in Classical Logic. Moreover, Y may be an infinite set and an infinite "disjunction" need not be expressible in a language (similarly for X and conjunction).

Note that entailment understood according to Definition 3 can be conceived as a special case of mc-entailment, namely as mc-entailment of a singleton set.

[17] The letters p, q, r stand, here and below, for propositional variables.

5 Minimal Erotetic Semantics: Questions

The "erotetic" concepts of MiES introduced in this section, as well as in Section 6, are applicable only to e-formulas/questions for which sets of sentential PPA's are defined. A sentential PPA is a PPA which has the form of a d-wff.

From now on, we will be considering (unless otherwise stated) an arbitrary but fixed formal language $\mathcal{L}$ of the analyzed kind; by d-wffs and questions we will mean d-wffs and e-formulas of the language, respectively. Language $\mathcal{L}$ is supposed to satisfy the following conditions: (a) it has questions and d-wffs among well-formed expressions, (b) for any question of the language, the set of PPA's to the question is defined; the set has at least two elements, (c) PPA's are d-wffs, and (d) the class of admissible partitions of $\mathcal{L}$ is defined.

Terminology and notation. For the sake of brevity, in what follows we omit the specifications "of $\mathcal{L}$" and "in $\mathcal{L}$". Similarly, we write $\models$ instead of $\models_{\mathcal{L}}$, and $|\!\models$ instead of $|\!\models_{\mathcal{L}}$. We omit curly braces when referring to singleton sets of premises.

We use $\mathbf{d}Q$ for the set of all the PPAs to question Q.

5.1 Soundness of a question

MiES does not presuppose that questions are true or false. Instead, the concept of soundness of a question is used.

The underlying intuition is: a question Q is sound iff at least one principal possible answer (PPA) to Q is true. So, for example, the question: "*Who is the only author of Principia Mathematica?*" is not sound, whereas the question: "*Who were the authors of Principia Mathematica?*" is sound.[18] Similarly, the question: "*What is the smallest natural number?*" is sound, but the question: "*What is the greatest natural number?*" is not sound.

Of course, when a formal language is concerned, the concept of soundness needs a relativization.

Definition 11. (Soundness of a question) *A question Q is sound in a partition* P *iff* $\mathbf{d}Q \cap \mathsf{T}_{\mathsf{P}} \neq \emptyset$.

Thus a question is sound in a partition if at least one PPA to the question is true in the partition. The basic idea underlying the above definition was suggested by Bromberger (1992, p. 146).

Remark: NLQs and truth values. It is highly disputable whether a NLQ can be assigned truth or falsity understood in the literal sense. But one can try to assign to NLQ's some other "truth values". For example, Nelken & Francez (2002) opt for **resolved** and **unresolved**, and develop an interesting extensional semantics for languages with questions. The crucial difference between their

[18] Recall that *Principia Mathematica* was written by two authors.

proposal and the MiES account lies is using a three-valued logic to create tripartitions of the set of d-wffs (the third value is labeled "unknown"). In view of (Nelken & Francez 2002) a yes-no question, $?A$, gets the value **resolved** if A is either true or false, and gets the value **unresolved** if the value of A is "unknown."

Safety and riskiness

It can happen that a question is sound in one admissible partition and is not sound in some other(s). If, however, a question is sound in each admissible partition of a language, we call it a safe question. More formally:

Definition 12. (SAFETY) *A question Q is safe iff $\mathbf{d}Q \cap \mathsf{T}_{\mathsf{P}} \neq \emptyset$ for each admissible partition P.*

It is obvious that the set of PPAs to a safe question is mc-entailed by the empty set. Observe, however, that a question can be safe although no PPA to it is valid, that is, true in each admissible partition of a language. For example, the following questions of $\mathcal{L}^{?}_{\mathsf{CPL}}$ are safe, but no PPA to them is valid:[19]

(76) $?p$
(77) $? \pm |p, q|$

A question which is not safe is called risky.

Definition 13. (RISKINESS) *A question Q is risky iff $\mathbf{d}Q \cap \mathsf{T}_{\mathsf{P}} = \emptyset$ for some admissible partition P.*

Thus a risky question is a question which has no true PPA in at least one admissible partition of the language. Here are simple examples of risky questions of $\mathcal{L}^{?}_{\mathsf{CPL}}$:

(78) $?\{p, q\}$
(79) $?\{p \wedge q, p \wedge \neg q\}$

A language of the considered kind usually involves both safe and risky questions. There are notable exceptions, however.

The above concepts of safety and riskiness originate from Belnap's erotetic semantics (cf. Belnap & Steel 1976, p. 130).[20] We have rephrased Belnap's definitions in MiES terms. Safety and riskiness correspond to non-informativeness resp. informativeness in the sense of Groenendijk & Stokhof (1997). In view of their analysis, each question (semantically construed) is noninformative. In general, when questions are conceptualized semantically as partitions of the logical space, there is no room for risky questions.

[19] Recall that $?p$ abbreviates $?\{p, \neg p\}$, and $? \pm |p, q|$ abbreviates $?\{p \wedge q, p \wedge \neg q, \neg p \wedge q, \neg p \wedge \neg q\}$.

[20] Belnap writes that the notion of safety is due to Harrah.

5.2 Presuppositions and prospective presuppositions

Presuppositions are conceptualized differently in different theories (for an overview see Potts 2015). MiES adopts Belnap's account:[21]

Definition 14. (Presupposition) *A d-wff B is a presupposition of a question Q iff $A \models B$ for each $A \in \mathbf{d}Q$.*

Thus a presupposition of a question is a d-wff which is entailed by each PPA to the question. For instance, the following:

(80) $p \vee q$

is an example of a presupposition of question (78). Here are examples of presuppositions of question (79):

(81) $(p \wedge q) \vee (p \wedge \neg q)$
(82) $q \vee \neg q$
(83) p

Observe that each presupposition of a question which is sound (in an admissible partition) is true (in the partition), and that a question which has a false presupposition cannot be sound (again, with respect to a given admissible partition). On the other hand, the truth of a presupposition of a question need not warrant the soundness of the question. For instance, r is a presupposition of the following question of $\mathcal{L}^?_{\mathsf{CPL}}$:

(84) $?\{p \wedge r, q \wedge r\}$

but the question is not sound in an admissible partition in which r is true and both p and q are not true.

A presupposition whose truth warrants soundness of the question is called a prospective presupposition. More precisely:

Definition 15. (Prospective presupposition) *A presupposition B of question Q is prospective iff $B \,|\!\!\models \mathbf{d}Q$.*

For example, the following d-wff:

(85) $r \wedge (p \vee q)$

is a prospective presupposition of question (84), whereas the d-wff p is a prospective presupposition of the question (79). Note that these are not the only prospective presuppositions of the analyzed questions. In general, a prospective presupposition of a question of the form $?\{A_1, \ldots, A_n\}$ (of $\mathcal{L}^?_{\mathsf{CPL}}$ or $\mathcal{L}^?_{\mathsf{MCL}}$) is either a disjunction of all the PPAs to the question or a d-wff which is equivalent to such disjunction (by "equivalence" we mean here mutual entailment in a language). One can show that the existential generalization $\exists x A x$ is

[21] "*A question,* q*, presupposes a sentence,* A*, if and only the truth of* A *is a logically necessary condition for there being some true answer to* q"(Belnap 1969*b*, p. 29).

a prospective presupposition of the corresponding existential which-question $?\mathbf{S}(Ax)$ of $\mathcal{L}^?_{\mathsf{MCL}}$. The remaining prospective presuppositions are equivalent to the existential generalization.

We do not claim, however, that each question of any language has prospective presuppositions. For instance, the following general which-question of $\mathcal{L}^?_{\mathsf{MCL}}$:

(86) $?\mathbf{U}(\mathtt{P}x)$

where $\mathtt{P}$ is a predicate, has no prospective presupposition, because the condition "there exist finitely many x's such that $\mathtt{P}x$" is inexpressible in $\mathcal{L}^?_{\mathsf{MCL}}$.

Notation. The set of presuppositions of a question Q will be referred to as $\mathbf{Pres}Q$, whereas the set of prospective presuppositions of Q will be designated by $\mathbf{PPres}Q$.

5.3 Types of questions

Normal questions

The soundness of a question yields that each presupposition of the question, if there is any, is true. Yet, the converse need not hold. If it (nonvacuously) holds, MiES labels the question as normal.

Definition 16. (Normal question) *A question Q is normal iff* $\mathbf{Pres}Q \neq \emptyset$ *and* $\mathbf{Pres}Q \mid\models \mathbf{d}Q$.

The clause "$\mathbf{Pres}Q \neq \emptyset$" is dispensable when valid (i.e. true in each admissible partition) d-wffs occur in a language.[22]

Regular questions

Regular questions constitute a subclass of normal questions. By and large, a regular question is a question which is normal due to its prospective presupposition(s).

Definition 17. (Regular question) *A question Q is regular iff there exists $B \in \mathbf{Pres}Q$ such that $B \mid\models \mathbf{d}Q$.*

There is no warranty that each question of any language has prospective presuppositions. Thus there may exist questions which are normal but not regular.

[22] Yet, the clause is not superfluous. There exist "tautology-free" logics. Kleene's "strong" three-valued logic is a classical example here.

Remark. Regularity and normality are semantic concepts. One cannot say that a question (syntactically construed) is normal/regular in an "absolute" sense. For instance, existential which-questions are normal (and regular) in $\mathcal{L}^?_{\mathsf{MCL}}$, but would cease to be normal when admissible partitions were determined by all models of the language. If we enriched $\mathcal{L}^?_{\mathsf{MCL}}$ with a quantifier "there exist finitely many", general which-questions would become regular (provided that admissible partitions were defined as before).

Self-rhetorical questions and proper questions

Generally speaking, a question is proper iff the truth of all its presuppositions warrants the existence of a true PPA to it, but does not warrant the truth of any single PPA to the question. A question which fulfils the latter condition may be called self-rhetorical. More precisely:

Definition 18. (SELF-RHETORICAL QUESTION) *A question Q is self-rhetorical iff* $\mathbf{Pres}Q \models A$ *for some* $A \in \mathbf{d}Q$.

Thus a question is self-rhetorical just in case some PPA to the question is entailed by a presupposition or presuppositions of the question. Or, to put it differently, the question is already resolved by some of its presuppositions.

Remark. Observe that questions can be self-rhetorical for diverse logical reasons. If a question has a valid d-wff among its PPAs, it is self-rhetorical. If all the PPAs to a question are equivalent and hence the "choice" offered by the question is only apparent, the question is self-rhetorical as well. A much weaker condition is also sufficient for self-rhetoricity: there is a PPA which is entailed by all the other PPAs. If this is the case, the sets of PPAs and of presuppositions overlap. Here is an example of a self-rhetorical question having this property:

(87) $? \{p, q, p \vee q\}$

Proper questions are defined by:

Definition 19. (PROPER QUESTION) *A question Q is proper iff Q is normal, but not self-rhetorical.*

5.4 Types of answers

So far we have operated with only one category of answers, that is, PPAs. However, the conceptual apparatus of MiES allows us to define further types of answers.

Just-complete answers

The following definition introduces a concept that is not superfluous when PPA's are defined syntactically.

Definition 20. (Just-complete answer) *A d-wff B is a just-complete answer to a question Q iff $B \notin \mathbf{d}Q$, and for some $A \in \mathbf{d}Q$, both $B \models A$ and $A \models B$ hold.*

Just-complete answers are equivalent to PPAs, but are not PPAs. It is convenient to introduce the following notational convention:

$$[[\mathbf{d}Q]] = \{B\text{: } \textit{for some } A \in \mathbf{d}Q, B \models A \textit{ and } A \models B\}$$

The set $[[\mathbf{d}Q]]$ comprises the PPAs to Q and the just-complete answers to Q.

Partial answers

As for MiES, a partial answer is a d-wff that is neither a PPA nor a just-complete answer, but which is true iff a true PPA belongs to some specified *proper subset* of the set of all the PPAs to the question. This condition is supposed to hold for each admissible partition. More formally:

Definition 21. (Partial answer) *A d-wff B is a partial answer to a question Q iff $B \notin [[\mathbf{d}Q]]$, but for some non-empty proper subset Y of $\mathbf{d}Q$:*

(1) $B \mid\models Y$, and
(2) for each $C \in Y$: $C \models B$.

Examples of partial answers will be given below.

Eliminative answers

Generally speaking, an eliminative answer, if true, eliminates at least one PPA to the question. In languages in which negation occurs "eliminates" amounts to "entails the negation." Yet, since negation need not occur in a language of the kind considered here, we need a more general concept of elimination.

Definition 22. (Eliminating)

(1) A d-wff B eliminates a d-wff C iff for each admissible partition $\langle \mathsf{T_P}, \mathsf{U_P} \rangle$: if $B \in \mathsf{T_P}$, then $C \in \mathsf{U_P}$.
(2) A d-wff B eliminates a set of d-wffs Y iff B eliminates each element of Y.

MiES defines eliminative answers as follows:

Definition 23. (Eliminative answer) *A d-wff B is an eliminative answer to a question Q iff*

(1) $B \notin [[\mathbf{d}Q]]$, and

(2) $B \in \mathsf{T}_{\mathsf{P}}$ for some admissible partition P, and
(3) there exists $A \in \mathbf{d}Q$ such that B eliminates A.

When the classical negation occurs in a language, an eliminative answer can also be defined as a contingent d-wff which entails the negation of at least one PPA, but is not equivalent to any PPA.

Eliminative answers versus partial answers. There are eliminative answers which are not partial answers, and there are partial answers which are not eliminative. For consider the following question of $\mathcal{L}^{?}_{\mathsf{CPL}}$:

(88) $?\{p, q, r\}$

The d-wff:

(89) $p \vee q$

is a partial answer to question (88), but is not an eliminative answer to the question. The d-wff:

(90) $\neg r$

is an eliminative answer to question (88), but is not a partial answer to it. So one cannot *identify* partial answers with eliminative answers. These categories are not disjoint, however. For example, in the case of the following question of $\mathcal{L}^{?}_{\mathsf{CPL}}$:

(77) $? \pm |p, q|$

each of the d-wffs p, $\neg p$, q, $\neg q$ is both a partial answer and an eliminative answer to the question.

Corrective answers

Roughly, a corrective answer is a contingent d-wff which eliminates all the PPAs to the question. In MiES we express this intuition by:

Definition 24. (Corrective answer) *A d-wff B is a corrective answer to a question Q iff*

(1) $B \notin [[\mathbf{d}Q]]$, and
(2) $B \in \mathsf{T}_{\mathsf{P}}$ for some admissible partition P, and
(3) B eliminates $\mathbf{d}Q$.

If the classical negation occurs in a language considered, clause (*3*) can be replaced with "B entails the negation of a presupposition of Q."

Clearly, each corrective answer is an eliminative answer (as a matter of fact, a "maximal" one: it eliminates all the PPA's). On the other hand, there are eliminative answers that are not corrective in the sense of Definition 24. As an illustration, let us consider the following question of $\mathcal{L}^{?}_{\mathsf{CPL}}$:

(91) $?\{p \wedge q, p \wedge r\}$

The d-wffs $\neg q$ and $\neg r$ are eliminative answers to question (91), but are not corrective answers to the question. Here are examples of corrective answers to question (91):

(92) $\neg p$

(93) $\neg(q \vee r)$

One can easily show that the set of partial answers to a question and the set of corrective answers to the question are disjoint.

5.5 Dependencies

One can define some dependency relations between questions in terms of relations between their sets of PPAs. For the reasons of space let us only consider two relations of this kind.

Equipollence and being weaker

Definition 25. (Equipollence of questions) *A question Q is equipollent with a question Q_1 iff there exists a bijection $f\colon \mathbf{d}Q \mapsto \mathbf{d}Q_1$ such that for each $A \in \mathbf{d}Q$: A is equivalent to $f(A)$.*

Thus Q and Q_1 are equipollent if there exists a 1–1 mapping between their sets of PPAs such that the corresponding PPA's entail each other.

Definition 26. (Being weaker than) *A question Q is weaker than a question Q_1 iff Q and Q_1 are not equipollent, but there exists a surjection $f : \mathbf{d}Q_1 \mapsto \mathbf{d}Q$ such that for each $A \in \mathbf{d}Q_1$: A entails $f(A)$.*

The above definitions are, in principle[23], due to Kubiński (1971, p. 51). Note that in both cases it is required that the relevant mapping is a function. By abandoning this requirement one gets more general notions. For example, as long as being weaker is concerned, one can only require each PPA to Q_1 to entail some PPA to Q. The resultant concept of being weaker becomes akin to that of *containment* in the sense of Hamblin (1958): a question Q_1 contains a question Q if from each answer to Q_1 one can deduce some answer to Q. A similar idea underlies the definition of *interrogative entailment* proposed by Groenendijk & Stokhof (1997, p. 1090).

Relative soundness

Let us now introduce an auxiliary concept that is specific to MiES.

Definition 27. (Relative soundness)

(1) A question Q is sound relative to a set of d-wffs X iff $X \mid\!\models \mathbf{d}Q$,

[23] In principle, since Kubiński applies a concept of entailment that warrants that contradictory d-wffs entail only contradictory d-wffs.

(2) A question Q_1 is sound relative to a question Q along with a set of d-wffs X iff for each $B \in \mathbf{d}Q : X \cup \{B\} \mid\models \mathbf{d}Q_1$.

Thus Q is sound relative to X iff Q has a true PPA in every admissible partition in which all the d-wffs in X are true. In other words, if only X consists of truths, Q must be sound. Clause (*2*) of Definition 27 amounts to: Q_1 is sound in each admissible partition in which Q is sound and X consists of truths.

A warning is in order. One should not confuse relative soundness with soundness in a partition. Relative soundness is always a semantic *relation*. Soundness in a partition is a *property* which a question has or does not have.

6 Erotetic Inferences and How Questions Arise

There are inferential thought processes in which questions are arrived at and thus perform the role of "conclusions." We often pass from declarative premise(s) to a question, as in:

(94) Mary is Peter's mother.
If Mary is Peter's mother, then John is Peter's father or George is Peter's father.

Who is Peter's father: John or George?

We also pass from questions to questions on the basis of declaratives, e.g.:

(95) What airline did Andrew travel by: BA, Ryanair, or Air France?
Andrew travelled by BA or Air France iff he arrived in the morning, and by Ryanair iff he arrived in the evening.

When did Andrew arrive: in the morning, or in the evening?

(96) Is Andrew lying?
Andrew lies iff he speaks very slowly.

Is Andrew speaking very slowly?

Declarative premises are not always needed. For instance:

(97) Is Andrew silly and ugly?

Is Andrew silly?

The above examples illustrate that there exist *erotetic inferences* (e-inferences for short) of at least two kinds. The key difference between them lies in the type of premises involved. In the case of e-inferences of the first kind the set of premises comprises declaratives only. The premises of an e-inference of the second kind consist of a question and possibly some declarative(s). Observe that questions involved in e-inferences, both as premises and as conclusions, are direct questions.

Some e-inferences are *intuitively valid*, while others are not. The following can serve as a preliminary test of intuitive validity: put the expression "So the question arises:" just before the conclusion. If the resultant description of an e-inference is undoubtedly true, the inference can be regarded as intuitively valid. Observe that (94), (95), (96), (97), as well as (21) and (22) (see page 7) pass the test.

Inferential erotetic logic (hereafter: IEL) is a logic which analyses e-inferences and proposes criteria of validity for these inferences. For IEL see e.g. the monographs (Wiśniewski 1995, 2013) or the introductory papers (Wiśniewski 1996, 2001).

The key concepts of IEL are *evocation of questions* and *erotetic implication.* By defining the concept "a set of d-wffs evokes a question" one explicates the concept: "a question arises from a set of declaratives"(cf. Wiśniewski 1995, chap. 1). By defining erotetic implication one explicates the intuitive notion: "a question arises from a question on the basis of a set of declaratives"(again, cf. Wiśniewski 1995, chap. 1). *Validity* of e-inferences of the first kind is then defined in terms of evocation, while validity of e-inferences of the second kind is defined by means of erotetic implication. Thus, although the concepts of evocation and erotetic implication are distinct, there is a common idea that underlies the analysis of validity of e-inferences provided by IEL: a question that is the conclusion of a valid e-inference arises from the premises.

Remark. In what follows we will define evocation and erotetic implication by using the conceptual apparatus of MiES. As for IEL, however, the desired property of possible answers to NLQ's (see subsection 3.12) is *just-sufficiency*, where "just-sufficient" means "satisfying the request of a question by providing neither less nor more information than it is requested".[24] Thus an e-formula/question Q whose set of PPA's is $\mathbf{d}Q$ is supposed to represent a NLQ read as its just-sufficient possible answers were simply the sentences formalized by the d-wffs in $\mathbf{d}Q$. One should bear this in mind when thinking about intuitive contents of the relevant concepts.

6.1 Evocation of questions

The basic intuition which underlies the concept of evocation is very simple. Let X be a set of declarative sentences/d-wffs. If the truth of all the sentences/d-wffs in X guarantees the existence of a true PPA to a question Q, but does not warrant the truth of any single PPA to Q, we say that X evokes Q. For instance, the (singleton set made up of the) sentence: "*Somebody likes Mary*" evokes the question: "*Who likes Mary?*". Similarly, the set of premises of (94) above evokes the question which is the conclusion.

We define question evocation as follows:

Definition 28. (EVOCATION OF QUESTIONS) *A set of d-wffs X evokes a question Q (in symbols: $\mathbf{E}(X, Q)$) iff*

(1) $X \mid\models \mathbf{d}Q$*, and*
(2) for each $A \in \mathbf{d}Q : X \mid\not\models \{A\}$.

Clause (2) is formulated in terms of mc-entailment for uniformity only, since X does not mc-entail $\{A\}$ iff X does not entail A. Clause (1) requires the evoked question to be sound relative to the evoking set. Clause (2), in turn, requires any PPA to an evoked question to be carrier of information that cannot be (deductively) extracted from the evoking set only.

[24] Just-sufficient possible answers are often called *direct answers* and papers devoted to IEL follow this convention.

One can easily show that a normal question Q is evoked by a set of d-wffs X iff each presupposition of Q is entailed by X, but X entails no PPA to Q. When Q is a regular question, the situation is even simpler: X evokes Q just in case X entails a prospective presupposition of Q without entailing any PPA to Q.[25] We also have:

Corollary 2. *A question Q is proper iff* $\mathbf{Pres}Q \neq \emptyset$ *and* $\mathbf{E}(\mathbf{Pres}Q, Q)$.

Examples of evocation

We write $\mathbf{E}_\ell$ to indicate that the underlying logic of d-wffs is ℓ. For brevity, we use object-level language expressions instead of their metalinguistic names, and we simply list the elements of evoking sets.

(98) $\mathbf{E}_{\mathsf{CPL}}(p \vee \neg p, ?p)$
(99) $\mathbf{E}_{\mathsf{CPL}}(p \vee q, ?p)$
(100) $\mathbf{E}_{\mathsf{CPL}}(p \vee q, ?\{p, q\})$
(101) $\mathbf{E}_{\mathsf{CPL}}(p \rightarrow q \vee r, p, ?\{q, r\})$
(102) $\mathbf{E}_{\mathsf{CPL}}(p \wedge q \rightarrow r, \neg r, ?\{\neg p, \neg q\})$

The symbols $\Diamond$ and $\Box$ stand for the modal operators of possibility and necessity, respectively.

(103) $\mathbf{E}_{\mathsf{S4}}(\Diamond p, \Diamond \neg p, ?p)$
(104) $\mathbf{E}_{\mathsf{S4}}(\neg \Box p, ?\{\Box \neg p, \Diamond p\})$
(105) $\mathbf{E}_{\mathsf{S4}}(p, ?\Box \Diamond p)$
(106) $\mathbf{E}_{\mathsf{S4}}(\neg \Box p, ?\Box \neg \Box p)$

The letters $\mathtt{P}$, $\mathtt{R}$ stand for one-place predicates, and the letters $\mathtt{a}$, $\mathtt{b}$ for individual constants of $\mathcal{L}^?_{\mathsf{MCL}}$.

(107) $\mathbf{E}_{\mathsf{MCL}}(\exists x(\mathtt{P}x \wedge (x = \mathtt{a} \vee x = \mathtt{b})), ?\{\mathtt{Pa}, \mathtt{Pb}\})$
(108) $\mathbf{E}_{\mathsf{MCL}}(\forall x(\mathtt{P}x \vee \mathtt{R}x), ?\{\mathtt{Pa}, \mathtt{Ra}\})$
(109) $\mathbf{E}_{\mathsf{MCL}}(\exists x \mathtt{P}x, ?\mathbf{S}(\mathtt{P}x))$
(110) $\mathbf{E}_{\mathsf{MCL}}(\forall x(\mathtt{P}x \leftrightarrow x = \mathtt{a}) \vee \forall x(\mathtt{P}x \leftrightarrow (x = \mathtt{a} \vee x = \mathtt{b})), ?\mathbf{U}(\mathtt{P}x))$

Note that evocation is strongly dependent on the underlying logic of d-wffs. For example, (the counterparts of) (105) and (106) do not hold for $\mathsf{S5}$. When we take InqB as a basis, neither (99) nor (102) is true. (109) does not hold in view of classical logic, but holds in $\mathcal{L}^?_{\mathsf{MCL}}$ because we consider only normal models of the language of MCL.

For further examples of evocation, in particular evocation in first-order languages enriched with questions see e.g. (Wiśniewski 1995, chap. 5) or (Wiśniewski 2013, chap. 6).

[25] The relevant types of questions are defined in subsection 5.3.

6.2 Erotetic implication

The e-inference described in (95) (see page 44) is a paradigmatic example of intuitively valid e-inference of the second kind. The conclusion: "*When did Andrew arrive: in the morning, or in the evening?*" is erotetically implied by the premises: "*What airline did Andrew travel by: BA, Ryanair, or Air France? Andrew travelled by BA or Air France iff he arrived in the morning, and by Ryanair iff he arrived in the evening.*"

The conceptual apparatus of MiES allows us to define erotetic implication in the following way:

Definition 29. (Erotetic implication) *A question Q implies a question Q_1 on the basis of a set of d-wffs X (in symbols: $\mathbf{Im}(Q, X, Q_1)$) iff:*

(*1*) *for each $A \in \mathbf{d}Q : X \cup \{A\} \mid\models \mathbf{d}Q_1$, and*

(*2*) *for each $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \mid\models Y$.*

Clause (*1*) of the above definition amounts to the following: if the implying question, Q, is sound (in an admissible partition) and all the d-wffs in X are true (in the partition), then the implied question, Q_1, is sound (in the partition) as well. In other words, clause (*1*) requires question Q_1 to be sound relative to question Q along with X. Clearly, the question: "*When did Andrew arrive: in the morning, or in the evening?*" is sound relative to the premises of (95). Clause (*2*), in turn, amounts to the fact that *each* set made up of a PPA to the implied question and the declarative premises narrows down the "space of possibilities" initially offered by the set of PPA's to the implying question. Coming back to the example. If the answer "*Andrew arrived in the morning*" is true and the declarative premise is true, then the *proper subset*:

(111) {Andrew travelled by BA, Andrew travelled by Air France}

of the set of PPA's to the implying question must contain a true PPA to the question. If, however, the second PPA to the implied question, namely "*Andrew arrived in the evening*" is true and the declarative premise is true, the following PPA to the implying question:

(112) Andrew travelled by Ryanair

must be true. In other words, the true PPA to the implying question belongs to the following proper subset of the set of PPA's to the question:

(113) {Andrew travelled by Ryanair}

which happens to be a singleton set.

The peculiarity of erotetic implication is its goal-directedness: an implied question is semantically grounded in the implying question and, at the same time, facilitates answering the implying question.

Narrowing down vs. answering

Observe that clause (*2*) of Definition 29 is satisfied, int. al., when the following condition holds:

(dp) *for each* $B \in \mathbf{d}Q_1$: $X \cup \{B\}$ *entails a PPA to* Q *or a partial answer to* Q.

On the other hand, the clause (*2*) yields (**dp**) given that mc-entailment in the language is compact (i.e. X mc-entails Y iff there exist finite subsets X_1 of X and Y_1 of Y such that X_1 mc-entails Y_1) and the language includes disjunction classically construed.

One can easily prove:

Corollary 3. *If a language includes classical disjunction and conjunction, mc-entailment in the language is compact, and* Q *as well as* Q_1 *are normal questions, then* Q *implies* Q_1 *on the basis of* X *iff:*

(1) a prospective presupposition of Q *entails, together with* X, *a prospective presupposition of* Q_1, *and*
(2) each PPA to Q_1 *entails, together with* X, *a PPA to* Q *or a partial answer to* Q.

Note that the initial assumptions of Corollary 3 are satisfied when the underlying logic of declaratives is classical logic (both propositional and first-order). But, for example, mc-entailment in $\mathcal{L}^?_{\mathsf{MCL}}$ is not compact and thus in the general setting one cannot define erotetic implication by using the clauses (*1*) and (*2*) of Corollary 3.

Some properties of erotetic implication

Mutual soundness. An implied question need not be sound relative to the declarative premises themselves, and similarly for the implying question. However, the following holds:

Corollary 4. *Let* $\mathbf{Im}(Q, X, Q_1)$. *Then* $X \parallel\!\models \mathbf{d}Q$ *iff* $X \parallel\!\models \mathbf{d}Q_1$.

As an immediate consequence of Corollary 4 we get:

Corollary 5. *Let* $\mathbf{Im}(Q, X, Q_1)$ *and let* $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ *be an admissible partition of the language such that* $X \subseteq \mathsf{T}_\mathsf{P}$. *Then* Q_1 *is sound in* P *iff* Q *is sound in* P.

Table 1 displays possible connections. It shows that erotetic implication behaves in a somewhat non-standard way. Given that X consists of truths, sound questions are implied only by sound questions, and unsound questions imply only unsound questions.

Table 1. *From implying question to implied question*

Q	X	Q_1
sound in P	$X \subseteq \mathsf{T_P}$	*sound in* P
unsound in P	$X \subseteq \mathsf{T_P}$	*unsound in* P
sound in P	$X \nsubseteq \mathsf{T_P}$	*sound in* P *or unsound in* P
unsound in P	$X \nsubseteq \mathsf{T_P}$	*sound in* P *or unsound in* P

The transitivity issue. Erotetic implication is not "transitive": it happens that $\mathbf{Im}(Q, X, Q_1)$ and $\mathbf{Im}(Q_1, X, Q_2)$ hold, but $\mathbf{Im}(Q, X, Q_2)$ does not hold. Here is a simple example taken from language $\mathcal{L}^?_{\mathsf{CPL}}$. We have:

(114) $\mathbf{Im}_{\mathsf{CPL}}(?p, ?\{p \wedge q, p \wedge \neg q, \neg p\})$

(115) $\mathbf{Im}_{\mathsf{CPL}}(?\{p \wedge q, p \wedge \neg q, \neg p\}, ?q)$

but we do not have $\mathbf{Im}_{\mathsf{CPL}}(?p, ?q)$. As the example illustrates, the lack of transitivity is a virtue rather than a vice.

Another interesting feature of erotetic implication is its ampliativity. For instance,

(116) $?q$

is not implied by:

(117) $?p$

on the basis of:

(118) $q \rightarrow p$

but one can reach (116) from (117) and (118) in two steps due to:

(119) $\mathbf{Im}_{\mathsf{CPL}}(?p, q \rightarrow p, ?\{p, \neg p, q\})$

(120) $\mathbf{Im}_{\mathsf{CPL}}(?\{p, \neg p, q\}, ?q)$

A *regular erotetic implication* is "transitive", however. One gets the definition of regular erotetic implication by replacing clause (*2*) of Definition 29 with:

(**rg**) *for each* $B \in \mathbf{d}Q_1$ *there exists* $C \in \mathbf{d}Q$ *such that* $X \cup \{B\} \models C$*.*

Examples of erotetic implication

In presenting examples we adopt analogous conventions as in the case of evocation (see page 46).

(121) $\mathbf{Im}_{\mathsf{CPL}}(?\{p, q, r\}, ?\{p, q \vee r\})$

(122) $\mathbf{Im}_{\mathsf{CPL}}(?\{p, q \vee r\}, ?\{p, q, r\})$

Note that (122), unlike (121), is regular, i.e. the clause (**rg**) holds for it.

(123) $\mathbf{Im}_{\mathsf{CPL}}(?\{p, q\}, p \vee q, ?p)$

The disjunction $p \vee q$ is indispensable in (123); if it were dropped, erotetic implication would not hold.

(124) $\mathbf{Im}_{\mathsf{CPL}}(?\{p, q, \neg(p \vee q)\}, ?p)$

(125) $\mathbf{Im}_{\mathsf{CPL}}(?p, p \leftrightarrow q, ?q)$

(126) $\mathbf{Im}_{\mathsf{CPL}}(? \pm |p, q|, ?p)$

(127) $\mathbf{Im}_{\mathsf{CPL}}(?p, ? \pm |p, q|)$

Observe that (126) is not regular, but is analytic[26], while (127) is not analytic, but is regular.

(128) $\mathbf{Im}_{\mathsf{CPL}}(?(p \otimes q), ? \pm |p, q|)$

where $\otimes$ is any of the connectives: $\wedge, \vee, \rightarrow, \leftrightarrow$.

(129) $\mathbf{Im}_{\mathsf{CPL}}(? \pm |p, q|, p, ?q)$

(130) $\mathbf{Im}_{\mathsf{MCL}}(?\mathbf{S}(\mathtt{P}x), \forall x(\mathtt{P}x \leftrightarrow \mathtt{R}x), ?\mathbf{S}(\mathtt{R}x))$

(131) $\mathbf{Im}_{\mathsf{MCL}}(?\mathbf{U}(\mathtt{P}x), \forall x(\mathtt{P}x \leftrightarrow \mathtt{R}x), ?\mathbf{U}(\mathtt{R}x))$

(132) $\mathbf{Im}_{\mathsf{MCL}}(?\mathbf{S}(\mathtt{P}x), \forall x(\mathtt{R}x \rightarrow \mathtt{P}x), \exists x \mathtt{R}x, ?\mathbf{S}(\mathtt{R}x))$

(133) $\mathbf{Im}_{\mathsf{MCL}}(?\mathbf{S}(\mathtt{P}x), \exists x \mathtt{P}x, ?\mathtt{Pa})$

(134) $\mathbf{Im}_{\mathsf{MCL}}(?\mathbf{S}(\mathtt{P}x), \forall x(\mathtt{P}x \leftrightarrow \mathtt{R}x \wedge \mathtt{T}x), \exists x(\mathtt{R}x \wedge \mathtt{T}x)), ? \pm |\mathtt{Ra}, \mathtt{Ta}|)$

(135) $\mathbf{Im}_{\mathsf{MCL}}(?\{\mathtt{Pa}, \mathtt{Pb}\}, \forall x(\mathtt{P}x \leftrightarrow \mathtt{R}x \wedge \mathtt{T}x), \mathtt{Ra}, \mathtt{Tb}, ?\{\mathtt{Rb}, \mathtt{Ta}\})$

As in the case of evocation, erotetic implication is strongly dependent upon the underlying logic of declaratives. For instance, the counterparts of (123), (124), (126), (127) and (128) do not hold in InqB, while (132), (133) and (134) would not hold if each model of $\mathcal{L}_{\mathsf{MCL}}$ had been regarded as normal.

For further examples, properties, and types of erotetic implication see Wiśniewski (1994, 1996, 2001); see also Wiśniewski (1995, chap. 7) or Wiśniewski (2013, chap. 7).

[26] $\mathbf{Im}(Q, X, Q_1)$ is *analytic* iff $X = \emptyset$ and each immediate subformula of a PPA to Q_1 is a subformula or negation of a subformula of a PPA to Q.

7 Other Developments

For decades, research on questions focused on their structure, semantic representations and the answerhood problem. These topics are still relevant, but, since the late 1980s, are becoming overshadowed by a focus on the role of questions in inquiry, reasoning, issue management and dialogue, and so forth.

This change in perspective was initiated by Jaakko Hintikka and his *Interrogative Model of Inquiry* (hereafter: IMI).[27] IMI considers an inquiry as an interrogative game, played by an Inquirer and an external source of information. The Inquirer is permitted to perform deductive moves as well as interrogative moves. The latter amount to putting auxiliary questions; the answers received serve as premises in further moves. The choice between admissible moves, however, is a matter of strategy, and interrogative moves are not viewed as inferences.

A theory of inferences which have questions as conclusions is proposed by IEL (see Section 6, also for references). IEL is not just an addition to IMI, but differs from it conceptually. Problem-solving and question answering are modelled in IEL in terms of *erotetic search scenarios* (see Wiśniewski 2003, 2013). As for proof theory, IEL gave rise to the method of Socratic proofs (cf. e.g. Wiśniewski 2004, Leszczyńska-Jasion et al. 2013) and the Synthetic Tableaux Method (cf. e.g. Urbański 2001).

Question raising, erotetic inferences and interrogative problem-solving attracted attention of some researchers working within the paradigm of adaptive logics (cf. e.g. Meheus 1999, 2001, De Clercq 2005).

Questions have been extensively analyzed by means of tools taken from dynamic semantics (cf. the collection of papers Aloni et al. 2007).

Recently questions became fully fledged categories in dynamic epistemic logics (cf. e.g. van Benthem & Ş. Minică 2012, Peliš & Majer 2011) and belief revision theory (see e.g. Enqvist 2010).

Current research on dialogues has shed new light on different aspects of questions and questioning (see e.g. Ginzburg 2012).

[27] Hintikka's papers devoted to IMI written in the 1980s and 1990s are collected in (Hintikka 1999). See also (Hintikka 2007).

8 Further Readings

The survey paper (Harrah 2002) provides a comprehensive exposition of logical theories of questions elaborated till late 1990s. Supplementary information about more linguistically oriented approaches can be found in (Groenendijk & Stokhof 1997) (reprinted as Groenendijk & Stokhof 2011), (Lahiri 2002), (Fiengo 2007), and (Krifka 2011).

Theories of questions and answers proposed by Harrah, Åqvist, Belnap, Kubiński, and Hintikka are concisely presented in (Wiśniewski 1995, chap. 4); for a more detailed exposition of Kubiński's theory see (Wiśniewski 1997). For Hintikka's IMI and the underlying approach to questions see (Hintikka et al. 2002).

Ginzburg (2011a) provides a survey of recent developments in the research on questions, both in logic and in linguistics.

All the items recommended above contain extensive bibliographies.

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